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AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

EFFECT OF DOWEL-BAR MISALIGNMENT ACROSS CONCRETE PAVEMENT JOINTS

By Arthur R. Smith,[1] M. Am. Soc. C. E., and Sanford W. Benham,[2] Assoc. M. Am. Soc. C. E.

Synopsis

The subject of joints in concrete pavements, to provide for expansion and contraction, and the means of transferring load across joints, is of particular interest to those engaged in highway construction. Common dowel bars, passing through the joint and embedded in the two adjacent slabs, have long been used for this purpose. The importance of installing the bars correctly, that is, perpendicular to the cross-section of the pavement, will be recognized since dowels, incorrectly installed, will not slip freely in the concrete. This paper describes an investigation to determine the accuracy with which road-building contractors were installing dowel bars, and another investigation which was conducted to determine, by experiment, the degree of precision necessary to assure satisfactory functioning of joints.

Introduction

In accordance with the specifications of the State Highway Commission of Indiana, 1-in. expansion joints are installed in concrete pavement at 80-ft intervals, with a transverse contraction joint in the center of each 80-ft section. As a means of transferring load across joints, dowel bars of intermediate grade steel (0.75 in. in diameter by 24 in. long) at 12-in. centers, are placed in the pavement across all contraction joints and across some types of expansion joints. To prevent bond, the bars are coated with an asphalt paint and are also oiled. Fig. 1 illustrates an assembly of twenty dowel bars wired to two spacer bars, as installed across expansion and contraction joints in 20-ft pavement.

Note.—Discussion of this paper will be closed in **October, 1937**, *Proceedings*.

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The importance of placing the bars correctly, that is, perpendicular to the cross-section of the pavement, will be readily appreciated when it is realized that if they are imperfect in this respect, binding between them and the concrete will occur when the joint opens or closes. As a consequence, either the bars will become distorted or the concrete will be damaged in the vicinity of the bars.

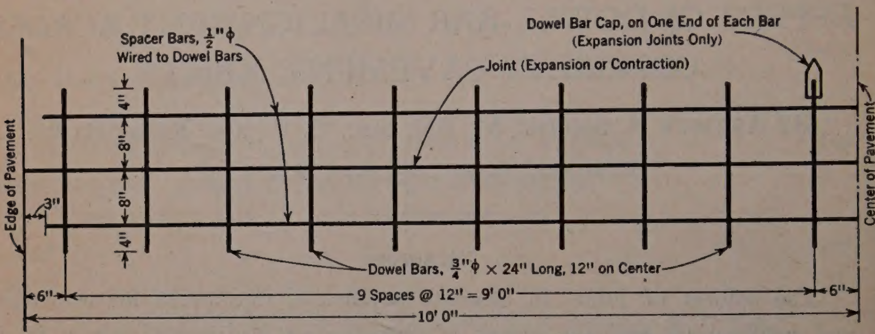


FIG. 1.—TYPICAL DOWEL-BAR ASSEMBLY IN 20-FOOT PAVEMENT.

A faulty installation of dowel bars across a contraction joint constitutes a serious hazard. When contraction of the pavement opens the joint cracking or spalling of the concrete may occur. If the concrete is strong enough to resist such damage, however, the joint, instead of being a plane of weakness, may offer more resistance to movement than the tensile strength of the unimpaired cross-section of the pavement. In this case the contraction joint will not fulfill its function and cracks will occur in the pavement at random as if no provision had been made to locate them. In like manner, a faulty installation of dowel bars across an expansion joint will cause damage to the concrete, the bars, or to both, when expansion of the pavement reverses the process and closes the joint. In this case, also, if the concrete and dowel bars offer sufficient resistance to such damage, the joint will not fulfill its function and there will be no provision for expansion. Therefore, to avoid damage to the pavement in the vicinity of expansion and contraction joints the dowel bars must be placed in their proper positions and held in those positions by some method that will prevent their movement during the placing and finishing of the concrete.

During the summer of 1935, the Bureau of Materials and Tests of the State Highway Commission of Indiana, conducted a number of field tests (Special Investigation *A*-7) to determine the accuracy with which dowel bars were being installed by various contractors. When a dowel bar is in perfect position, that is, perpendicular to the cross-section of the pavement, the projected length of its axis on the cross-section is zero. The projected length is equal to the hypotenuse of a right triangle, the altitude of which is the vertical error of the bar, that is, the difference in elevation between the two ends of the bar with respect to the surface of the pavement. The base of the triangle is the horizontal error of the bar, that is,

the difference between the distances from the edge of the pavement to the two ends of the bar. Thus, dowel-bar error which, in this paper, is defined as the projected length on the cross-section of the pavement per 22 in. of dowel-bar length, is equal to the square root of the sums of the squares of the horizontal and vertical errors. Although the bars are 24 in. long, the field tests were conducted more easily by choosing points 1 in. inward from each end of the bar to represent the ends of the bar. In conducting the tests, data were obtained, by means of the apparatus described herein under "Methods of Test", from which it was possible to compute the error of each dowel bar in each installation checked. Several different methods of installing the bars were being utilized, namely:

(1) Sheet metal chairs, of the pin type, driven into the sub-grade, with the upper end bent around the dowel bar or spacer bar (see Fig. 2).

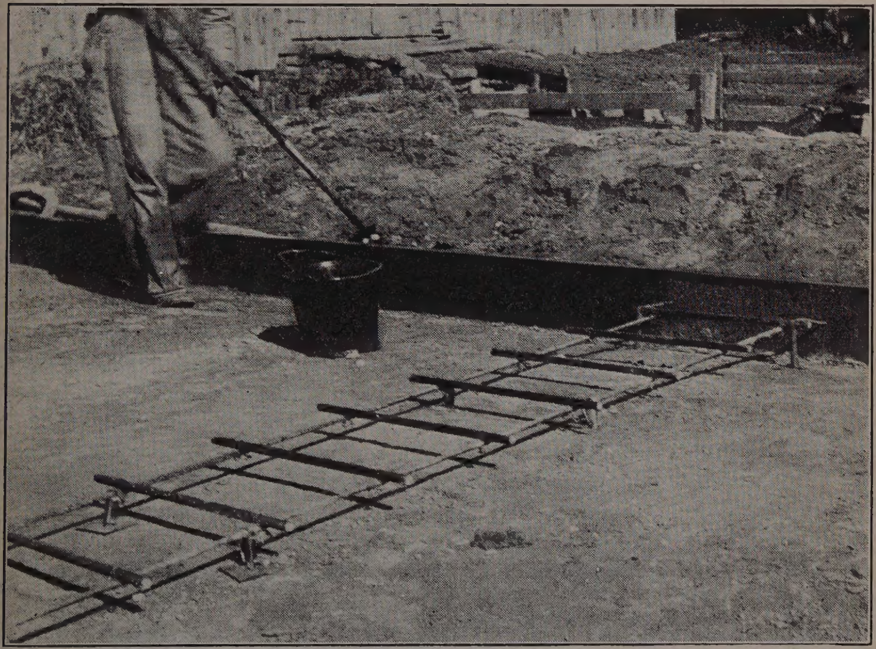


FIG. 2.—SHEET METAL CHAIRS OF THE PIN TYPE DRIVEN INTO THE SUB-GRADE (TEST NO. 6).

(2) Welded assemblies, consisting of heavy, wire, **U**-shaped chairs welded to the spacer bars. The upper ends of the chairs are bent around the dowel bars and the horizontal part of the **U** rests on the sub-grade (see Fig. 3).

(3) Specially designed holders, which span the sub-grade and are supported by the forms. The dowels are thereby held during the placing of the concrete and are left "floating" in the concrete after the removal of the holder. One type of holder is shown in Fig. 4. No satisfactory photograph is available of one that is supported entirely from the forms (independent of the sub-grade).

(4) Combinations of Methods (1) and (3).

Two claims of superiority for Method (3) have been advanced: First, the holder is supported by the forms, which determine the position of the cross-section of the pavement, thereby eliminating the objections to methods in which the positions of the dowels are governed by the contour of the sub-grade, which is never perfect; and, second, the installation of dowel bars is accomplished more quickly and easily with a holder than with chairs. The first claim, of course, is based on the assumption that the holder is made accurately, is sufficiently strong to resist distortion in handling, and that it is placed on the forms in the proper position.

Methods of Test

Installations of dowel bars across nine contraction joints and across five expansion joints were checked for accuracy. These fourteen installations included the methods of dowel-bar support described in the "Introduction".

The contour of a concrete pavement and the location of the plane representing its cross-section are determined by the tops of the forms, since these lines are the paths of travel of the finishing machine screed. For that reason horizontal and vertical readings were taken on both ends of each bar from a reference line the position of which was definitely fixed with respect to the tops of the forms. This reference line, described briefly, consisted of a steel tape graduated to 0.01 ft, riveted to the lower flange of a 4-in. aluminum I-beam, spanning the sub-grade and resting on the tops of the forms, the points of support being three sharp pointed steel pegs.

In order to determine the rigidity with which installing devices held the dowel bars during the placing and finishing of the concrete the initial errors of the bars (before concrete was placed), and also their final errors (after the concrete had been placed and finished), were measured. To obtain the necessary data for the computation of the initial and final error of each dowel bar in a given installation and the movements of both ends of each bar, that occurred during construction operations, readings were taken as soon as the installation was completed and pronounced satisfactory by the inspector. The operations of placing and finishing concrete and the extent to which workmen stepped on the bars, were observed closely in an effort to determine whether or not movement could be attributed to carelessness. As soon as the finishing machine had passed over the joint for the last time, sufficient concrete was removed from the newly finished surface of the pavement to expose both ends of each dowel bar and permit final readings.

Discussion of Results

The results of Special Investigation *A*-7 revealed that, in some of the installations checked, the bars were in extremely faulty positions. Twelve of the fourteen installations tested were in 20-ft pavement and two were in 10-ft pavement (widening projects). Consequently, the errors of 260 dowel bars were determined. Of these 260 bars, one had an error of 1.5

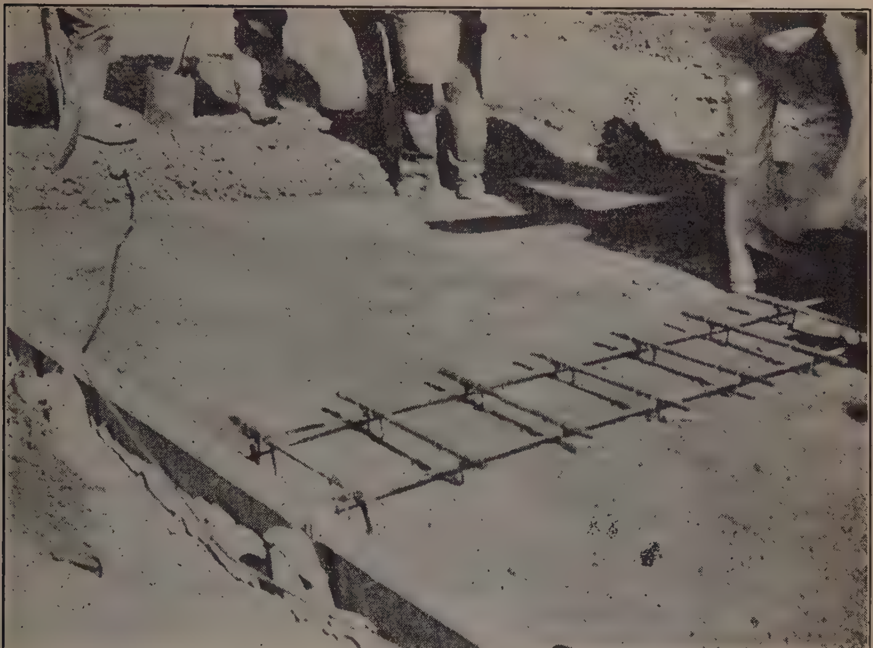


FIG. 3.—U-SHAPED CHAIRS RESTING ON THE SUB-GRADE AND WELDED TO THE SPACER BARS (TEST NO. 12).

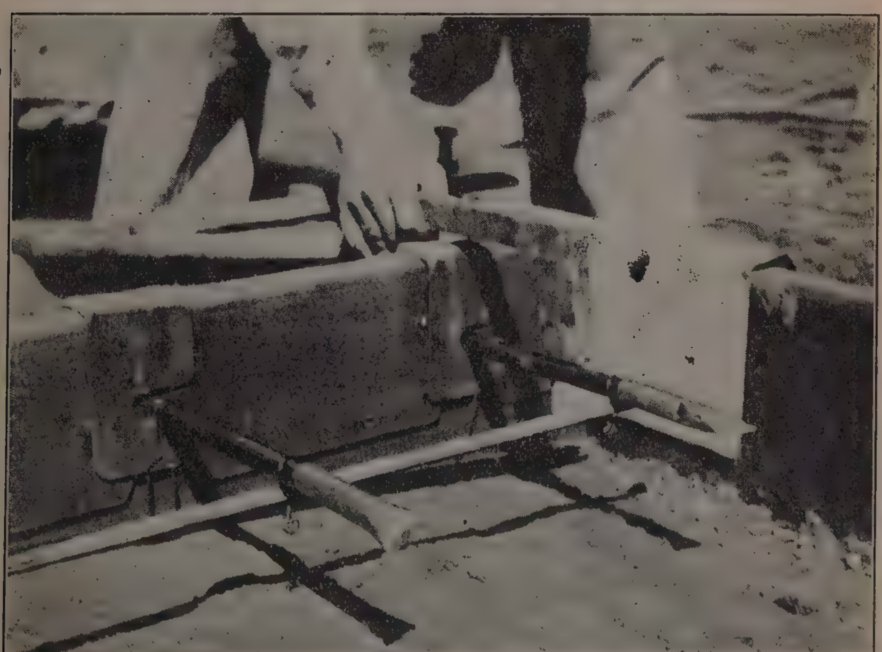


FIG. 4.—VIEW SHOWING THE USE OF A HOLDER (TEST NO. 14).

in., and one bar was found to be in perfect position. The results of this investigation, a summary of which is shown in Table 1, are expressed as the maximum, minimum, and average error (both initial and final) and maximum, minimum, and average movement of all bars in each installation tested. Movement is defined as the sum of the movements of the two ends of a bar. In some cases the two ends of a bar were moved in such directions that the final error was less than the initial, that is, the movement improved the position of the bar. In general, however, movement had the opposite effect. Movement should not be confused with the difference between the initial and final error since, because of the directions of movement of the two ends, a bar may move and yet have a final error equal to its initial error. The converse is not true, however. If a bar does not move, its initial and final errors are, of course, the same.

TABLE 1.—SUMMARY OF RESULTS, SPECIAL INVESTIGATION *A*-7

Test No.*	Dowel-Bar Error, in Inches						Dowel-Bar Movement, in Inches			Description of joint and method of installing bars
	Initial			Final						
	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	
1	0.47	0.04	0.18	1.05	0.08	0.41	0.68	0.14	0.43	Contraction joints. Bars installed with holders.
2	0.51	0.06	0.31	0.45	0.06	0.31	1.01	0.10	0.37	
3	0.68	0.10	0.25	0.97	0.21	0.55	1.14	0.60	0.87	
4	0.75	0.35	0.49	0.85	0.28	0.54	0.93	0.23	0.51	
9	0.75	0.11	0.37	0.85	0.03	0.45	1.16	0.60	0.89	
Average	0.63	0.13	0.32	0.83	0.13	0.45	0.98	0.33	0.61	......
6	0.60	0.08	0.27	0.65	0.09	0.29	0.37	0.09	0.23	Contraction joints. Bars mounted on chairs.
7	0.45	0.30	0.21	0.78	0.08	0.32	0.53	0.12	0.29	
8	0.67	0.16	0.34	0.63	0.16	0.33	0.95	0.22	0.48	
12†	0.50	0	0.21	0.50	0.03	0.21	0.40	0.23	0.30	
Average	0.56	0.07	0.26	0.64	0.09	0.29	0.56	0.16	0.32	......
5	0.50	0.12	0.29	0.54	0.07	0.30	0.40	0.15	0.31	Expansion joints. Bars installed with holders unless otherwise noted.
10	0.66	0.17	0.35	1.50	0.10	0.51	1.82	0.18	0.59	
11†	1.44	0.76	1.16	1.40	0.79	1.12	0.17	0.04	0.12	
13‡	0.80	0.06	0.41	1.15	0.27	0.75	1.89	0.24	0.63	
14	0.35	0.06	0.18	0.29	0.02	0.15	0.55	0.13	0.31	
Average	0.75	0.23	0.48	0.98	0.25	0.57	0.97	0.15	0.39	......
Average of all tests	0.65	0.14	0.36	0.82	0.16	0.45	0.84	0.21	0.45	......

* Tests were numbered in chronological order. † U-shaped chairs resting on sub-grade and welded to spacer bars. ‡ Chairs, of the pin type, were used (one to each dowel bar), in addition to the holder.

It will be noted that the best installation tested (Test No. 14, Table 1) was an assembly of dowel bars, across an expansion joint, held in position during the placing of concrete by means of a holder. The average initial error was 0.18, the average final error, 0.15, and the average movement, 0.31 in. This is an instance in which the movements of the bars improved their positions. The poorest installation tested (Test No. 11, Table 1) was across an expansion joint in which the bars were mounted on U-shaped chairs welded to the spacer bars. These chairs rested on the sub-grade. The average error was 1.16 in., initial, and 1.12 in., final. Quite paradoxically this was the best installation so far as movement was concerned

(0.12 in.). The poorest installation so far as movement is concerned was across a contraction joint in which the bars were installed with a poorly designed holder (Test No. 9, Table 1). The average movement was 0.89 in., and the directions of movement were such that the initial error was increased.

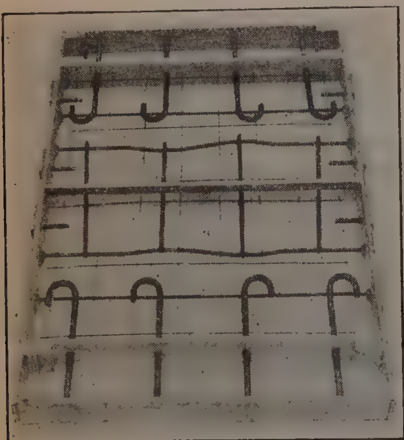
The results of the investigation indicated the need of information concerning the effect on the pavement of given degrees of error in dowel-bar placement. Since it is not feasible to require that every dowel bar in every installation be in perfect position, some figure specifying the permissible error was desired. To arrive at such a figure by calculation is an indeterminate problem because of the complication of stresses induced both in the concrete and in the bars, due to binding between the concrete and the bars, when the joint opens or closes. The problem is made more intricate by the fact that, in an assembly of twenty dowel bars, the errors vary greatly not only in magnitude but also in direction. An attempt at a solution of such a problem would immediately require a number of assumptions, the validity of which would be subject to question.

Permissible Error Determined by Experiment

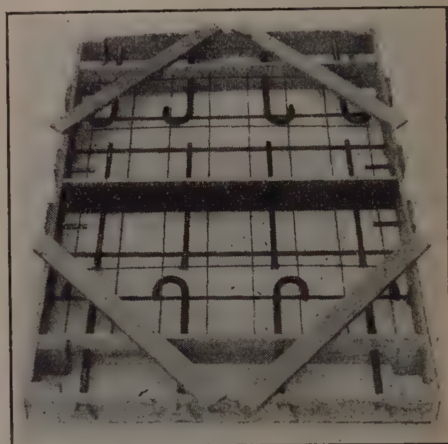
Because of the apparent impossibility of an analytical solution Special Investigation *A*-8 was conducted to determine the permissible error by experiment. The size and shape of test specimens decided upon, were slabs, 4 ft wide by 4 ft 8 in. long by 6 in. thick, cast on the ground, with a transverse contraction joint in the center (parallel to the 4-ft dimension) through which four dowel bars were placed at 12-in. centers. The slabs were made in groups of five, all five being poured from the same batch of ready-mixed concrete. In one slab the dowel bars were installed in perfect position (with zero error); in the other four slabs, they were purposely set at errors of 0.25 in., 0.5 in., 1 in., and 1.5 in. The second group of slabs, made on a different day, were duplicates of the first group except that 1-in. air cushion expansion joints were provided instead of contraction joints. Groups 3 and 4 were duplicates respectively of Groups 1 and 2 except for thickness, which was reduced to 5 in. In all, the test specimens consisted of twenty experimental slabs, each containing four dowel bars, including two slab thicknesses for each of five errors, for both contraction and expansion joints.

The design of the experimental slabs and the directions of dowel-bar errors will be more readily undertsood by referring to the photograph of an empty form (Fig. 5(*a*)). This form is for the fabrication of a 6-in. slab containing a contraction joint, the dowel bars having an error of 1.5 in. The two outside dowel bars have no error vertically; that is, they are level, but a plan view would show that each one has a projected length on the cross-section of the slab of 1.5 in., and that they converge toward the reader. A plan view would show the two interior bars in perfect position but, due to the difference in elevation between the two ends of each bar, each one has a projected length of 1.5 in. Note the relative positions of these two bars. The end toward the reader, of the bar on the left, is 1.5 in.

higher than the opposite end. The end toward the reader, of the bar on the right, is 1.5 in. lower than the opposite end. As this contraction joint is opened the two interior bars tend to rotate the slab (clockwise from the reader's position), which tendency is resisted by the two outside bars which produce compression (horizontally) in the half of the slab toward the reader and tension in the opposite half. Consequently, opening the joint produces stresses in the concrete and dowel bars which are not only severe but of a complicated distribution. This same arrangement of dowel-bar positions was used in all slabs containing dowel bars in error.



(*a*) CONTRACTION JOINT; DOWEL-BAR ERROR = 1.5 INCHES.



(*b*) EXPANSION JOINT; DOWEL-BAR ERROR = 0.

FIG. 5.—FORM FOR THE FABRICATION OF A 6-INCH SLAB.

The method of forming the contraction joint is also shown in Fig. 5(*a*)). The pre-molded material with which contraction joints are usually filled is simulated by a piece of wood, 0.75 in. thick, having a vertical dimension equal to one-third the slab thickness. A weakened plane was created by installing a piece of No. 24 gage galvanized iron, extending from the wood strip downward to the sub-grade. The dowel bars pass through semi-circular openings, 3 in. in diameter, in the sheet metal. Expansion joints were formed by installing a cork filler, 1 in. thick, as shown in Fig. 5(*b*), which also shows the corner bracing used on all forms to prevent distortion during the placing of the concrete. To give the cork sufficient rigidity against bending during the placing of the concrete, it was backed up with a board which was removed after the form was filled. Before the slabs were tested the cork was removed with a chisel. In Fig. 5(*b*) the dowel bars are in perfect position with respect to the cross-section of the slab—that is, zero error. Anchor bars were installed in both ends of the slabs at the time of fabrication, as shown in Fig. 5, for attaching the testing equipment.

In addition to the twenty experimental slabs containing common dowel bars, thirteen slabs were made for the study of another type of load-transfer device. For the purpose of this paper, however, the discussion of

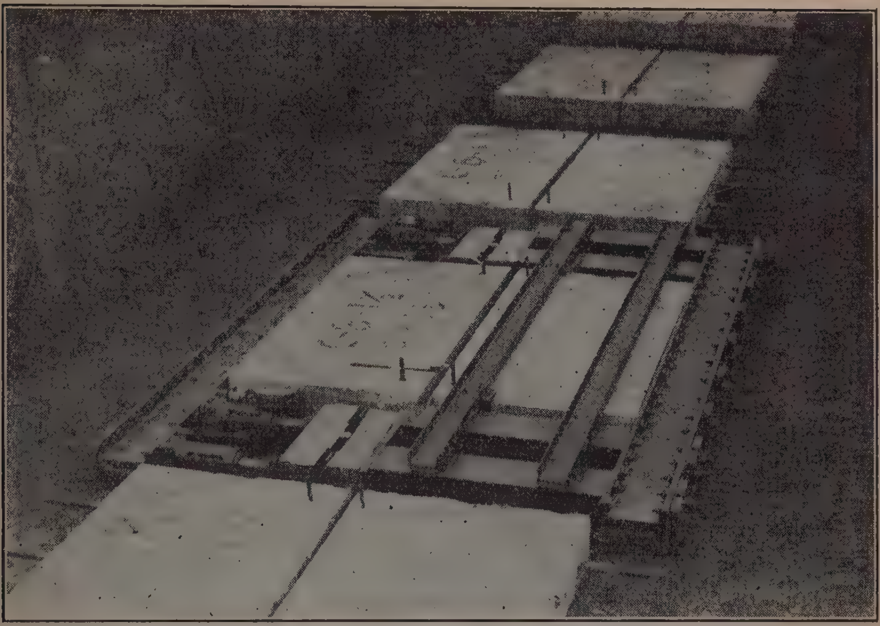


FIG. 6.—EQUIPMENT FOR OPENING A CONTRACTION JOINT.

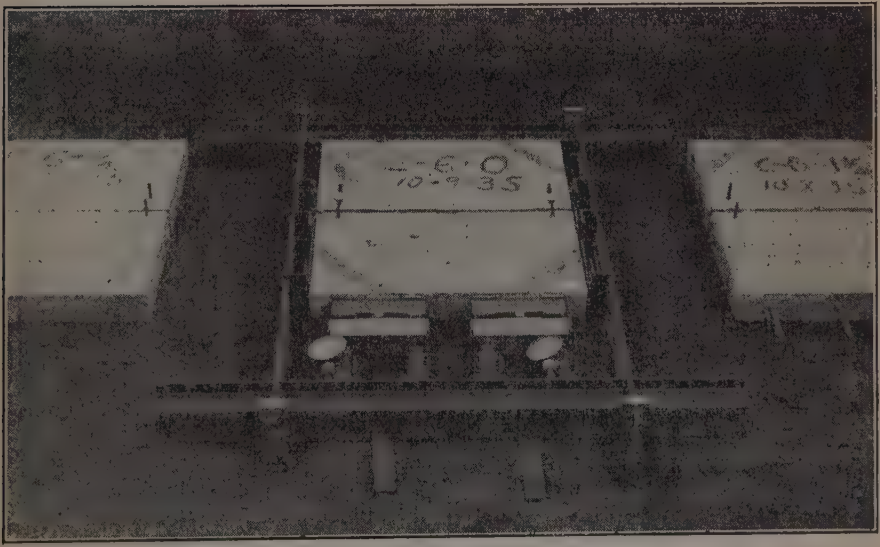


FIG. 7.—EQUIPMENT FOR CLOSING AN EXPANSION JOINT.

results is confined to those obtained with common dowel bars since they are in so much more general use than any proprietary device. The entire investigation required the fabrication and testing of thirty-three experimental slabs.

In the fabrication and curing of the slabs every effort was made to duplicate field conditions. The concrete was of the same cement content and consistency as that used in practice and the slabs were reinforced with standard wire mesh. The sub-grade was tamped and sprinkled before placing the concrete. All slabs were cured under wet burlap during the first ten days, after which they were exposed to the weather. From each batch of concrete, beams and cylinders were made which were cured in the same manner as the corresponding slabs and tested at the same time the slabs were tested, thereby determining the flexural and compressive strength of the concrete in the slabs at the time of test.

At the age of 28 days all contraction joints were opened a distance of 0.75 in. and all expansion joints were closed. The equipment used for opening and closing joints (shown in Figs. 6 and 7) was operated by hydraulic jacks fitted with gages for determining the necessary loads. The test data included records of slip, corresponding load, and notes, supplemented by photographs, describing the damage done to the slab for each increment of slip. Within a few days after all contraction joints were opened and all expansion joints closed, the process was reversed and the contraction joints were closed and the expansion joints opened to their original positions, thus completing one cycle of movement. At the age of one year all slabs were opened and closed nine more times, completing ten cycles of movement, and then pulled entirely apart in order that the cracking and spalling on the faces of the joints could be photographed.

Results and Conclusions

The results of the tests indicate that common dowel bars across contraction joints in 5-in. pavement will cause a very slight spalling of the concrete around the bars if the errors are as great as 0.25 in. In 6-in. pavement the errors may be as great as 1 in. without causing spalling. These damages occurred when the joints were opened to a width of 0.75 in. If it can be assumed safely that a contraction joint will never open more than 0.5 in., little concern need be felt about the accuracy with which the dowel bars are installed. When opened to this width (0.5 in.) the only slab that showed any spalling was the 5-in. slab in which the bars were in error 1.5 in.

The refinement with which dowel bars are placed is more important in expansion joints than in contraction joints. An error of 0.25 in. caused slight spalling and when the joint spaces were closed 0.75 in., to a width of 0.25 in. (which may occur in practice), errors of 1 in. caused the slabs to crack and spall. Typical cracking and spalling are shown in Figs. 8 and 9, photographs of a 5-in. slab, containing an expansion joint, in which the dowel bar error was 1.5 in., after one cycle of movement (Fig. 8) and after ten cycles of movement (Fig. 9).

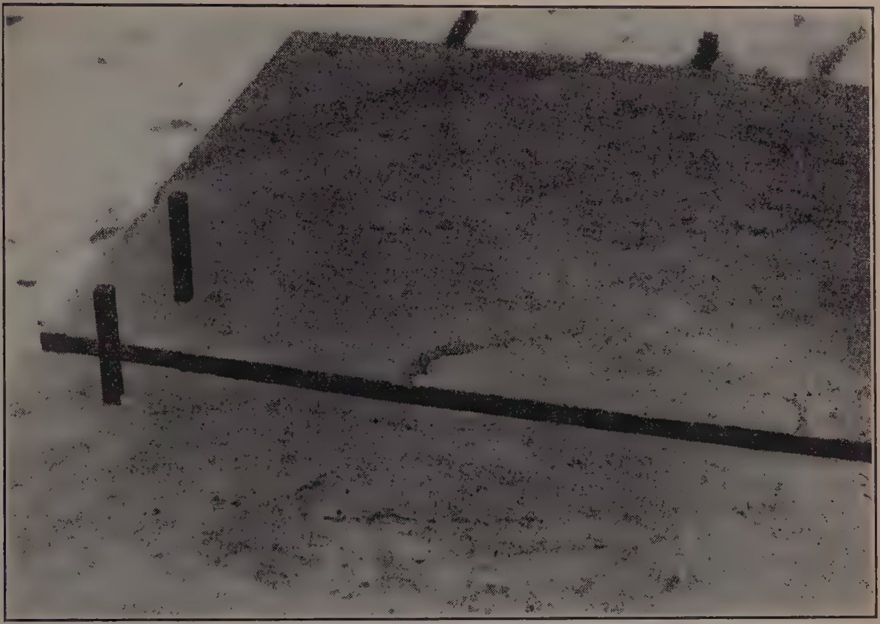


Fig. 8.—Damage Done to Test Slab by One Cycle of Movement (Thickness of Slab, 5 Inches; and Dowel-Bar Error, 1.5 Inches.)

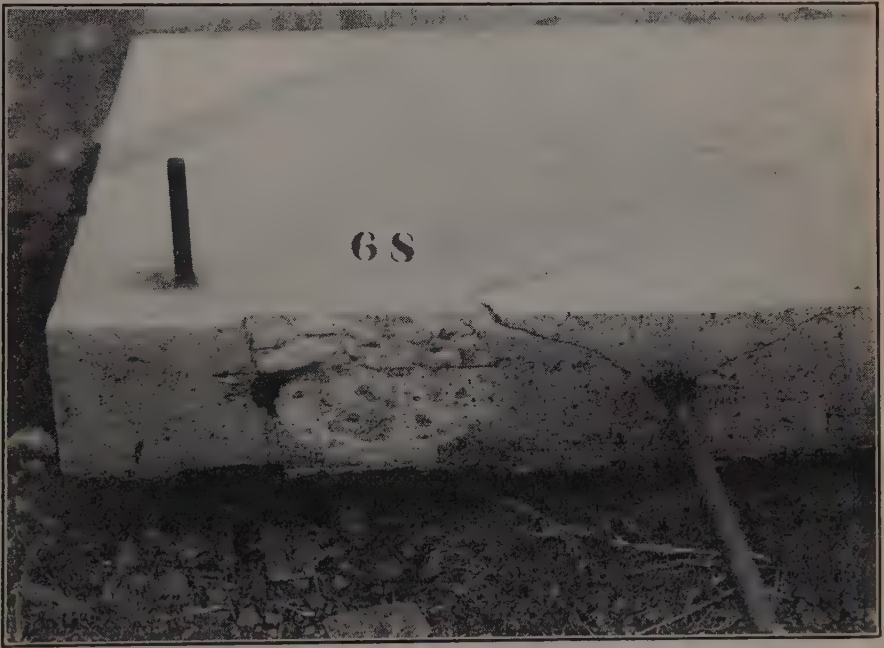


Fig. 9.—Same Slab as Shown in Fig. 8, After Ten Cycles of Movement.

Both the compressive and flexural strengths of the concrete, at the time the slabs were tested, were unusually high. The flexural strength of all test specimens was considerably greater than 550 lb per sq in., the State requirement for opening pavement to traffic in Indiana. The weakest concrete tested had a 28-day modulus of rupture of 670 lb per sq. in., and a compressive strength of 4 010 lb per sq in. Consequently, no cracking nor spalling of the slabs can be attributed to inferior concrete.

The loads required to open or close either the contraction or the expansion joints were less than anticipated and in no case were they great enough to stress the slab dangerously either in direct tension or compression. All damage done to the slabs was plainly due to stresses caused when the joints were being opened or closed, by the wedging and twisting effect of bar misalignment. The loads necessary to open and close the joints were not great enough to produce tensile or compressive stresses approaching the strength of the interior of the slabs—that is, the parts in which no dowel bars nor anchor bars were present. This statement is proved not only by computation, but by observation, since no transverse cracks occurred. Because the slabs were capable of transmitting forces so far in excess of those required to open and close the joints, even when the dowel bars were in the most faulty positions, the loads observed in this investigation seemed to be of no practical significance and are not given in this paper. As a matter of general interest, however, the load necessary to close an expansion joint to a width of 0.25 in. in no case exceeded 4 000 lb per bar. To open a contraction joint to a width of 0.5 in. required a load which, in no instance, exceeded 3 000 lb per bar.

Based on the results reported herein, it is recommended that common 24-in. dowel bars be placed with a degree of accuracy such that the error of no one bar in an entire installation shall exceed 1 in. This degree of precision will provide assurance that the relative positions of all the bars will form a less severe geometric arrangement than that formed by the bars that were placed at 1-in. errors in the experimental slabs, since only the maximum error will reach 1 in. and many of the bars will have errors considerably less. The results of the field tests (Special Investigation *A*-7) show that a requirement for accuracy to this degree will impose no hardship on the contractor. The vertical position of each bar can be checked quickly with a carpenter's level, as shown in Fig. 4, and adjusted if necessary, until the bubble position is approximately the same as when the level is set on the forms. The horizontal position can be made perfect by adjusting each bar so that the distances from each end to one of the forms are equal. Since the total error (projected length on the cross-section of the pavement) of a bar is equal to the square root of the sums of the squares of its vertical and horizontal errors, the vertical and horizontal error could each be as great as 0.707 in. and still comply with this specification. If some type of holder is used to install the bars, it should be adjusted at the beginning of the job and, provided it is always set on the forms in correct position and does not become damaged in handling, all installations should be correct.

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PAPERS

ESSENTIAL CONSIDERATIONS IN THE STABILIZATION OF SOIL[1]

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SYNOPSIS

All soil mixtures commonly used for road surfaces and speedways are highly stable at some moisture content. This is equally true of cohesionless beach sands, friable glacial silts, and highly plastic clays. Hence, every possible combination of sand, silt, and clay may serve satisfactorily as aggregate in stabilized soil, provided the right binder is used to cement the soil particles together. Furthermore, the moisture which binds soil mixtures into masses stable enough to withstand the abrasion produced by racing motor cars must have surface tension and viscosity greater than those of free water. On these two facts the entire theory of soil stabilization is based.

The underlying principles involved in soil stabilization and the possible means of its accomplishment are discussed without attempting to evaluate the practical aspects or the relative values of the various methods. Particularly stressed is the application of the collodial phenomena of adsorption and base exchange (21)[4] as they affect: (*a*) Particles of soil, sand, crushed rock, gravel, slag, etc., coated with films of air, water, soluble chemicals, and binders not soluble in water; (*b*) the relative adhesion between solids and films; and (*c*) the effect of the chemical composition of aggregate and binders and the ions on the surfaces of the solid particles.

Construction methods, details of design, and methods of test have been discussed generally in another paper (1*a*) and will not be included herein. Reports listed in the Bibliography (which forms an Appendix to the paper) furnish more detailed information on the manner of apply-

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[4] Numbers in parentheses refer to the Bibliography in the Appendix.

ing the various treatments and making required tests as follows: Graded soil mixtures (2); calcium chloride (3); sodium chloride (4); asphalt emulsion (5), (6); asphalt, sub-oiling method (7); asphalt, mixed method (6), (8); tar (9), (10); Portland cement (11), (12); sulfite liquor (13); molasses (13), (14); calcium silicate (15); electro-chemical treatment (16); application of heat (17); mechanical analysis (18), (19); routine soil tests (18), (19); and Proctor tests (20).

Adsorption

Some soil particles have high attraction for air and, under proper conditions, become coated with air films; others have high attraction for moisture and become coated with moisture films. Dried soil clods retain moisture enough to bind the particles into hard and, in some instances, enormously strong masses. On the other hand, air is often adsorbed on soil solids so strongly after a period of drought that drops of rain will roll along on dust without wetting it.

The power that plastic clays possess of retaining their plasticity when mixed with sand or other non-plastic material, is caused by adsorption of one solid by another. The clay is not distributed uniformly through the pores or interstices of the coarse particles, but most of it forms a coating on the non-plastic material and many of the pores remain unoccupied even if there is more than sufficient clay to fill them.

For water or other binder to wet soil particles coated with air, it must displace the air, which means that the particles must attract the liquid more strongly than they attract the air. In like manner clay adsorbs different liquids to different extents; that is, its power of adsorption is selective and under suitable conditions one liquid will displace another in contact with the clay.

Bancroft (21), (22) calls attention to thicknesses of adsorbed moisture films which have been found on different materials as follows (in millionths of an inch): On glass of one kind, 5.1; on glass of another kind, 3.1; on sand (10) mesh, 11.0; on sand (60 mesh), 4.5; and on plastic wet clay, 2.0. The stationary film on copper was observed to be 0.000150 to 0.000225 in. when water flowed rapidly and about three times as much when it flowed more slowly over the copper. The air film between two pieces of glass compressed at 7 grams per sq cm (0.099 lb per sq in.), was found to be 0.000120 in. thick.

Film Moisture in Soil

In completely saturated soil, every particle is covered with a coating of film moisture attracted to it more strongly than by the force of gravity. Until enough water is added to the soil to satisfy the condition of equilibrium between attraction of gravity and of the soil particle for water, all the moisture is arranged around the particles as films, with the interstices between the films filled with air.

It is impossible to remove all the film moisture from soils by enormous centrifugal forces exerted by powerful centrifuges. Therefore, it is concluded that much of the film moisture is held on the soil particles by adhesion equal to, or greater than, the enormous forces that fail to remove it. As a result of this great adhesion, film moisture is more viscous or glue-like than water in bulk.

The chemical composition of the soil particles, the nature of ions adsorbed on their surfaces, the moisture content, degree of compaction, and temperature of the soil control the thicknesses of moisture films, which vary widely.

Chemical composition of soil is indicated by the ratio of silica to the combined iron and aluminum oxides. This ratio is termed the silica sesquioxide ratio and is designated by the symbol, $\frac{Si\,O_2}{R_2\,O_3}$. Clays high in silica, termed "podsols," consist principally of the highly water-adsorbent, scale-like particles productive of high plasticity and shrinkage. Soils high in iron and alumina, termed "laterites," consist more of the bulky or spherical particles which do not attract water so strongly.

A colloid which holds hydrogen adsorbed on its surface is termed a hydrogen-ionized or H-colloid. One with calcium adsorbed on its surface is termed a calcium-ionized or Ca-colloid. If a substance like hydrated lime, $Ca(OH)_2$, is leached through soil containing H-colloids, the calcium replaces the hydrogen to form Ca-colloids, and the hydrogen thus released combines with the $(OH)_2$ to form water. Such exchange of ions is termed "base exchange."

The potassium ion is representative of those ions which undergo small volume change when alternately wetted and dried. It has a true diameter of 0.000000009 in. and in suspension becomes associated with 16 molecules of water, thus attaining an apparent diameter of 0.000000021 in. The lithium ion is representative of those ions which undergo great volume change when alternately wetted and dried. It has a true diameter of 0.000000006 in. and becomes associated with more than 120 molecules of water thus attaining an apparent diameter of 0.000000040 in. Between potassium and lithium other of the more common metallic ions can be arranged in the order of their attraction for water (36). Therefore, a potassium clay would be expected to undergo the least, and a lithium clay the greatest, volume change under climatic variations. Change of ions on the surfaces of the clay and colloidal particles effects a change in shrinkage, swell, and like properties on which the stability of the soil depends.

The concentration of the hydrogen ions determines the relative acidity of alkalinity. It is indicated by the pH-value which may be defined as the reciprocal of the logarithm of the grams of ionized hydrogen per liter of solution or suspension. The lower the pH-value is below 7.0, the greater is the acidity of the liquid and the greater is its base-exchange capacity.

The moisture content of a clay soil controls the thicknesses of film on the colloids. This, in turn, determines the density to which a particular pressure will compact the soil.

The relation between dry-weight density per cubic foot of samples compressed at equal pressures, to the moisture contents in percentage of the total soil and moisture volumes, at which the samples were compressed, was found by C. A. Hogentogler, Jr. (28) to consist of a series of straight lines with different slopes, as shown by the full lines in Fig. 1. The broken line in Fig. 1 shows the relation the density would have to moisture content if the samples contained no air. At any density, the difference in moisture contents indicated by the two lines represents the percentage of moisture, by volume, required to replace the contained air.

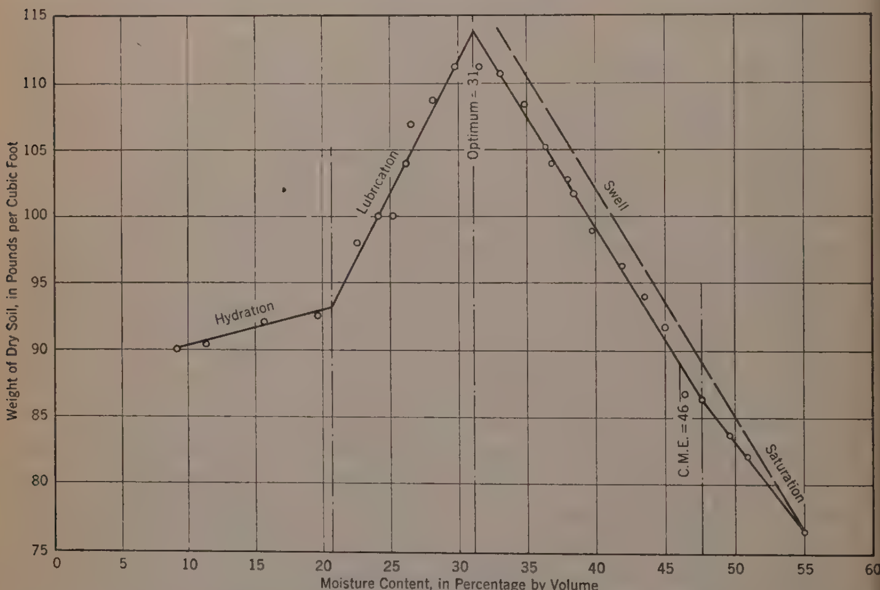

FIG. 1.—RELATION BETWEEN DENSITY AND MOISTURE CONTENT.

The moisture contents at which the straight lines intersect, indicate the limits of four distinct stages of wetting which the compressed samples undergo before their pores become completely filled with water. Wetting up to a moisture content of 20.7% (Fig. 1) may be termed the state of hydration, during which part of the contained water is absorbed by the soil particles, and the remainder is adsorbed on their surfaces in the form of cohesive films.

Moisture contents ranging from 20.7 to 31.1% indicate the stage of lubrication. Part of the contained moisture now acts as a lubricant to facilitate the re-arrangement of particles being compacted without, however, excluding all the air. Water in excess of 31.1% causes the soil mass to swell, although the air content is not further appreciably decreased until a moisture content of 47.7% is reached. Moisture contents between 47.7

and 54.3% represent the stage of saturation, during which practically all the air is displaced and the soil becomes truly saturated.

The moisture content (31.1% in this case) at which maximum density is attained is the optimum suggested by R. R. Proctor, M. Am. Soc. C. E. (20) for use in connection with the construction of earth embankments and dams.

Temperature is an influencing factor because in cold weather the adsorbed films are in general thicker than in warmer weather (1*b*).

Effect of Film Character on Properties of Soil

The free water, or pore water, is that which fills the interstices remaining between the outer surfaces of the adsorbed moisture films and not the pores between the surfaces of the soil particles as such. The flow of water through soil likewise occurs between the films adsorbed on the particles. As a result the speed at which water percolates through soil, as well as the moisture contents indicative of some specific state of stability of soil, varies wherever the thicknesses of the adsorbed films change, in some cases, when the viscosity of the free water changes.

Winterkorn (23), (24), (25) has shown that, due to the change of the kind of adsorbed ions on the particles of one type of soil, the liquid limit varied between 39 and 57, the plasticity index between 16 and 40, and the percentage of colloids between 11 and 28. The moisture content increase, indicating the swell of the samples in the Terzaghi compression device when the pressure was reduced from 3.2 kg per sq cm to 0, varied between 1 and 5 per cent.

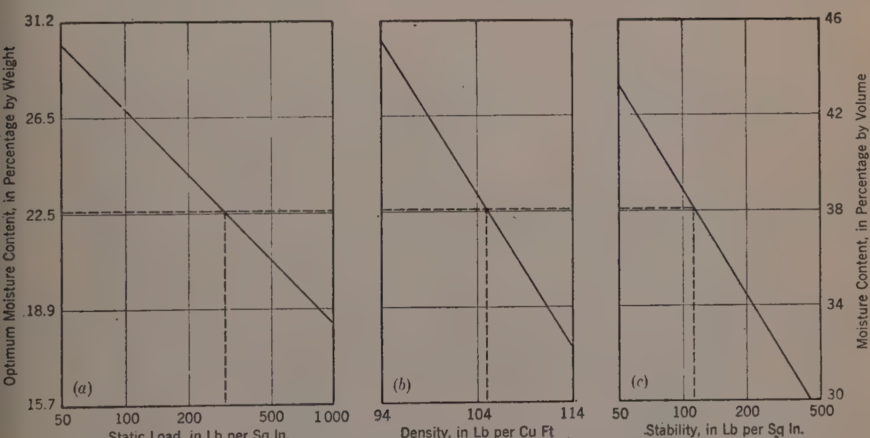

Fig. 2.—Control Curves.

The optimum moisture content (mentioned previously) at which maximum density is attained is not a constant for a particular soil, but varies as the compacting pressure is changed. The relations of optimum moisture content to compacting pressures, attained densities, and stabilities, as indicated by the Proctor plasticity needle for one soil investigated at George Washington University, are shown in Fig. 2.

Change of temperature alone, when other conditions are constant, causes considerable variation in the indicated properties furnished by tests. The test data furnished by A. M. Wintermyer, Assistant Highway Engineer, United States Bureau of Public Roads, and shown in Table 1 disclose no effect of temperature on the liquid limit of Sample S-9616, which is pumice having no plasticity.

TABLE 1.—Effect of Temperature on Plasticity Test

Sample No.	Tested at 40° F			Tested at 90° F		
	Liquid limit	Plastic limit	Plasticity index	Liquid limit	Plastic limit	Plasticity index
S-9616	40	..	0	40	..	0
S-9617	57	30	27	49	23	26
S-9618	36	24	12	32	23	9

The liquid limits and plastic limits of both plastic soils (Samples S-9617 and S-9618, Table 1) were higher when the tests were made at the lower temperature. However, the corresponding variations in the plasticity indexes, which are most used for field control in the construction of stabilized roads, are relatively small.

The results of tests performed by Messrs. R. C. Thoreen and F. A. Robeson, of the U. S. Bureau of Public Roads, and shown in Table 2, reveal the relative influence of both viscosity and adsorbed film thickness on coefficients of permeability.

TABLE 2.—Effect of Temperature on Coefficient of Permeability

Soil	Temperature at which test was made, in degrees Fahrenheit	Coefficient of Permeability	
		At test temperature, in centimeters per second	Corrected for viscosity to 68° F, in centimeters per second
Sand	85	0.0481	0.0390
Passing No. 60 sieve and retained on No. 80 sieve	85	0.0222	0.0375
Clay soil	72	6.47×10^{-8}	6.13×10^{-8}
	50	3.49×10^{-8}	4.54×10^{-8}

The coefficient of permeability of the sand was found to be 0.0481 cm per sec at 85° F, as compared with 0.0222 cm per sec at 35° F. Applying the usual correction for difference in viscosity of water (33), these coefficients become 0.0390 cm per sec and 0.0375 cm per sec, respectively, at a temperature of 68° F.

The coefficient of permeability of the clay soil was found to be 6.47 $\times 10^{-8}$ cm per sec when tested at 72° F and 3.49 $\times 10^{-8}$ cm per sec when tested at 50° F. These coefficients, when corrected for viscosity of water at 68° F, become 6.13 $\times 10^{-8}$ cm per sec and 4.54 $\times 10^{-8}$ cm per sec. Permeability tests made on samples of a similar clay at various voids ratios indicate that the decrease in permeability due to the greater film

thicknesses at the lower temperatures, shown in Table 2, is equivalent to that produced by reducing the voids ratio from 0.97 to 0.89, which would indicate an increase in the effective volume of the solid particles of about 4 per cent.

The effect of temperature variation on the stability of soil is illustrated by tests on a clay soil compacted in the Proctor cylinder and penetrated with the plasticity needle (20). The data showed (1*f*) that the stability was 1 400 lb per sq in. when compacted at 42° F; 960 lb per sq in. when the temperature of the sample was raised to 130° F; 1 490 lb per sq in. when the temperature was then reduced to 33° F; and, finally, 1 100 lb per sq in. when the temperature was again raised to 118° F.

Types of Soil Stabilization

In the application of information now available on soil stabilization the methods to be used depend largely on the availability of required materials. In some locations, such as the southeastern part of the United States, Nature provided deposits of sand clay and top-soil having the grading and character required in the best of road soils.

In some locations, binder soils and aggregates are available for producing mixtures having the properties of the best of the naturally good soils. In other locations, materials are available for the graduations required in roads, but the binder soil may be of inferior quality. Finally, there are locations where there is deficiency of the aggregate required for properly graded mixtures.

Studies of the best of natural road soils indicate that the design requirement of stable soil mixtures should include the following:

(1) The aggregate should be hard and durable enough to resist weathering, traffic abrasion, and crushing. Sound, tough particles or fragments of gravel, stone, slag, or combinations of them, crushed to the proper size, should prove suitable. Certain types of shales and similar materials that break and weather rapidly when alternately frozen and thawed, or wetted and dried, should not be used.

(2) The soil fines should be of a character such as to provide graded mixtures with the proper balance of capillarity and adhesion without risk of detrimental volume change. It is particularly essential that the fines do not swell enough in the presence of moisture to cause the clay to become a lubricant instead of a binder.

(3) When local materials are available for the proper proportioning of aggregate and binder soil, but the natural clay does not have the binder value required in highly stable road surfaces, a number of admixtures may be used singly or in some combination. There are, first, the chemical salts, calcium chloride, sodium chloride, and magnesium chloride; and second, the waste products of industry, such as sulfite liquor from the manufacture of wood pulp, "blackstrap" from the molasses industry, and huge acumulations of waste sizes of the mineral aggregate industry.

(4) When only fine or poorly graded materials are available, the asphaltic, tar, and Portland cement binders may be utilized to provide stable base courses to be covered with bituminous surfacings. In isolated cases heat treatment has been used.

In the stabilization of fills the Proctor method or some modification is receiving considerable attention and for foundation soils, injection of chemicals and electro-chemical treatment furnish possibilities.

Admixtures Affect Soils in a Number of Ways

Among the ways that admixtures may assist in stabilizing soils are the following:

(1) By absorbing moistures from the air and reducing evaporation from the road surface, some chemicals may serve to keep the roads in a damp condition. This reduces the dust nuisance and facilitates the compaction of the surfaces under traffic.

(2) By electro-chemical action, chemicals and the waste products may serve to increase the density and stability of road mixtures.

(3) By chemical action, the same materials may combine with each other, or with certain of the natural clay constituents, and thus form water insoluble cements of high binding value.

(4) The Portland cement and bituminous materials serve the double purpose of binders and for eliminating the destructive water-absorbent properties of the clay binders.

It is difficult to distinguish between the electro-chemical and the purely chemical actions of admixtures for increasing the stability of soils. However, singly or in some combination they seem to explain not only the performances of soil roads in service, but also suggest relatively simple procedures for future construction.

As primes and fillers, certain soaps, etc., and admixtures of hydrated lime, and certain types of stone dust seem admirably suited for increasing the adhesion of bituminous binders for certain soils.

In addition to their deliquescent and water retentive actions, the chloride salts, by electrolytic action, may cause the compacted mixture to attain densities greater than untreated soils, and they minimize the volume change of the binder which is productive of disintegration of the untreated surfaces with changing moisture conditions.

Mixtures of acidic and neutral materials seem to attain greater stability than that of either one separately. Limestone dust and certain slags seem admirably suited for use as pre-treatments, or with other admixtures to neutralize acid soils. This may be especially desired in the case of siliceous clays. In some base courses now (1937) being contructed, an acid clay is mixed with non-acidic limestone aggregate.

In connection with the formation of natural colloidal cements two types of either rock powders or clays must be considered: Those which on soaking in water or watery solutions soften to the extent that they

become, to ever so slight a degree, glue-like, and those materials which do not. To a small extent many of the sandstone, trap, and limestone powders are thus made glue-like by water alone and more so by water-soluble inorganic salts (13).

Experiments have shown that the cementing value of mixtures of the two types of materials (those which soften and those which do not) is considerably greater than the binding value of either material alone. Mr. A. S. Cushman and Prévost Hubbard, Affiliate, Am. Soc. C. E., found (13): (*a*) That by combining a granite with a cementing value of 3 and a limestone with a cementing value of 27, they obtained a cementing value of 110 for the mixture; (*b*) that a granite powder which had a cementing value of 10 when mixed with water alone, had a cementing value of 44 when mixed with lime water; and (*c*) that an amorphous chert which had a cementing value of 6 when mixed with water alone, had a cementing value of 22 when mixed with 4% of calcium oxide and water, and more than 2 000 when mixed with 13% of calcium oxide.

As a result it has been found possible to construct low type surfaces of only waste limestone or slag screenings treated with the chloride salt solutions. With granite screenings the addition of a small quantity of limestone also has been found to be beneficial.

Admixtures of Portland Cement and Bituminous Materials

Efforts to stabilize fine-grained soils by admixtures of lime and cement materials were made in Iowa and South Dakota as early as 1924 and in Ohio several years later. The results of this early work were not particularly promising, but they should not be considered as indicating the possibilities of such treatments, because the requirements of the thorough distribution of the admixture, the high degree of compaction, and the protective surface treatment now deemed necessary, were not recognized in the earlier work.

More recently (11), (12) research on the use of Portland cement for stabilizing soil bases has been performed in South Carolina by the State Highway Department. The purpose of this work has been to develop a base material that could be constructed at less cost than that required to provide one of properly graded sand clay or top-soil.

The results from these experiments were so promising that a number of similar roads are being constructed in other States. Experimental base courses constructed with available soil materials and the various tars (9), asphaltic emulsions (5), and cut-backs, and oils (6, 8) show similar promise.

Use of Other Adhesives

Waste Sulfite Liquor.—Products prepared from the crude liquor have been marked from time to time under various trade names, for use as binding mediums in road construction. As early as 1910 Mr. Hubbard (13) published results obtained by the U. S. Bureau of Public Roads, showing the increase in compressive strength obtained by treating various rock

powders with products prepared from waste sulfite liquors. However, the binding base of the adhesives tested was soluble in water, and, as a result, it was found that frequent rains tended to destroy the bond and remove the lignin from the road surface. In order to offset this difficulty attempts were made, at that time, to waterproof or make insoluble the residual base without destroying its binding value by admixtures of asphaltic road oils and deliquescent chemicals, such as calcium chloride.

In 1936 waste sulfite liquor, marketed under a trade name, was used for the treatment of gravel surfaces in New Jersey and experimented with by the State Highway Department of Washington for use primarily as a dust palliative, although the possibilities for its use as a stabilizing agent, especially in well-graded mixtures, have not been overlooked.

Molasses Residues.—A thick, sirupy liquid by-product obtained in the manufacture of sugar from sugar cane is known as "blackstrap" which, when treated with quick lime, forms compounds of high binding value known as calcium sucrates. Blackstrap, in combination with oil and lime, was applied as an experiment by the U. S. Bureau of Public Roads on a road at Newton, Mass., during the summer of 1908 (13), and a molasses-lime mixture was applied as a surface treatment on a section of the Bradley Lane Experimental Road, Chevy Chase, Md., in December, 1911 (29). Within a year, however, rains removed all traces of the treatment, causing the road to take on the appearance of an adjoining section which had been maintained as untreated macadam.

In January, 1936 (14), attention was called to a practice in India in the olden days, when a kind of coarse sugar, made by evaporation from the sap of palm trees, was added to lime to produce an improved quality of mortar. With this as a background, a short length of road surface formed of lime kankar was treated and the results were encouraging. (Kankar is a kind of limestone usually occurring as nodules in top-soil, used for making lime and in building roads in India.) In order to prevent the molasses from being washed away during rains, slaked lime was mixed with it, the claim being that the addition of burnt lime to molasses produces tri-calcium sucrate, which is insoluble in water.

Calcium Humate.—When lime is added to a soil containing humus (24), the calcium replaces the acidic hydrogen in humic acid, forming the more stable so-called calcium humate or neutral humus.

Hilgard (30) points out that there are several humates (of lime, magnesia, and iron) which assist in producing and maintaining crumb structure. When fresh, these substances are colloidal (jelly-like), like clay itself, but, unlike the latter, when once dried, they do not resume their plastic form by wetting. The crumbs thus formed are, therefore, quite permanent and contribute to the looseness of uncompacted soils rich in humus. One part of lime humate is said to be equal in cementing power to eleven parts of clay.

Calcium Silicate.—The insoluble calcium silicate binder is produced by treating soils first with sodium silicate followed by the application of calcium chloride according to the general principle used in the old English

method of manufacturing artificial stone (31). Its use in soil stabilization has been principally in connection with increasing the stability of granular soils in connection with the construction of buildings and other structures (15).

Treatment Without Admixtures

Electro-Chemical Treatment.—In a method utilizing electricity for stabilizing soils, described by Endell and Hoffmann (16), metal electrodes are introduced into the soil at appropriate distances, aluminum always being used for the anode and copper for the cathode. Direct current is then allowed to pass between electrodes until the soil is hardened. Experiments were described in which a clay specimen had been brought into its container with an almost liquid consistency, the water content being about 80 per cent. After treatment for some time with a current of 8 to 14 amperes, at 300 to 500 volts, the clay had become so hard that an iron rod, 1 sq cm in cross-section, failed to penetrate it when placed under a load of 10 kg (22 lb). Specimens cut from the side of this block nearest the aluminum electrode neither disintegrated nor swelled during several months of immersion in water.

Application of Heat.—The essential features of this method of treatment are illustrated by a method used in the so-called "black-soil" areas in Australia, in localities where the haul for gravel or crushed rock is excessive (17). A slow-moving, down-draft furnace with a speed of about 10 ft per hr was developed at Sydney, New South Wales, for baking the soil in place and converting it into a brick-like material. The machine (see Fig. 3) is wood-fired, of the air-gas producer type, on a chassis on road wheels, and is propelled under its own power.

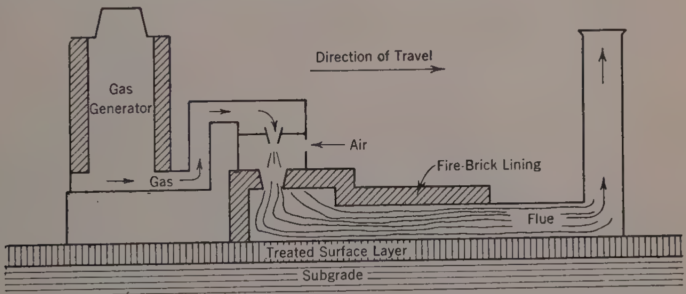

Fig. 3.—Diagram of Furnace for Heating Soil in Place.

Prior to heating, the road is graded to the desired cross-section and the strip to be treated is scarified to a depth of about 4 in. As the machine advances over a particular area, the soil is first subjected to the exhaust gases and then gradually increasing temperatures until the full intensity of the heat of the gases issuing from the generator is reached. Work proceeds continuously day and night, a machine 6 ft wide completing about 500 ft of roadway 18 ft wide per week. The effect of the treatment is noticed to a depth of about 6 in.

When traversed by roller the treated clods are broken down fairly readily. The clay binder is applied to the broken material, the road is shaped up and rolled again, and is then ready to be opened to traffic.

Effect of Grading and Admixtures on Density

The effects of grading and admixtures on stabilized soils compacted by the Proctor method (20) have been described in some detail elsewhere (1), the gradings and physical properties of red clay, Iredell, Arlington, and Manor soils being shown in tabular form (1*e*).

Grading.—The effect of the different gradings on the densities obtained at equal compactive efforts and fairly constant temperatures has been discussed in the earlier study (1*f*) (1*g*).

The red clay (Sample 1, Fig. 4, and Table 3) contains 68% of clay, 20% of silt, and 12% of sand. At the optimum moisture content of 16.7%, the dry weight was 113.0 lb per cu ft as compared with a dry weight of 129.8 lb per cut ft,

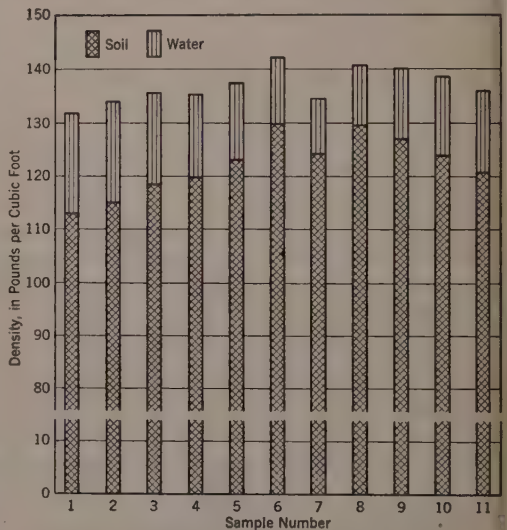

Fig. 4.—Effect of Grading on Densities of Compacted Samples.

TABLE 3.—Grading of Compacted Samples (See Fig. 4).

Classification	Grading (Percentages), for Sample Nos.:										
	1	2	3	4	5	6	7	8	9	10	11
Clay	68	34	21	17	13	8	0	18	18	18	18
Silt	20	60	37	30	23	15	13	5	5	5	5
Sand	12	6	42	53	64	77	87	77*	77†	77‡	77§
Total	100	100	100	100	100	100	100	100	100	100	100

* 24% retained on a No. 20 sieve. † Nothing retained on a No. 20 sieve. ‡ Nothing retained on a No. 40 sieve. § 1% retained on a No. 60 sieve.

at 9.6% optimum moisture content for the excellent soil mortar, Sample No. 6.

Samples Nos. 2, 3, 4, and 5, Table 3, were obtained by adding silt and sand to the red clay represented by Sample No. 1. An attempt was made to obtain proportions representative of the different fractions of Sample No. 6. Thus, Sample No. 5 represents approximately the grading of the fraction of material of Sample No. 6 that passes the No. 40 sieve; Sample No. 4 represents the fraction that passes the No. 60 sieve; Sample No. 3 represents the fraction that passes the No. 100 sieve; and Sample No. 2 represents the fraction that passes the No. 270 sieve.

Fig. 4, with Table 3, shows that as the total sand is decreased from that of Sample No. 6 to that of Sample No. 2, the weight of the compacted mixtures decreases and the optimum moisture contents increase. In Sample No. 7 an attempt was made to approximate the grading of the fraction of Sample No. 6 that was larger than 0.005 mm. As would be expected in the absence of the clay fraction, the optimum moisture content is smaller than that of Sample No. 6, being only 8.5 per cent. The weight of dry soil at maximum density, however, is only 124.1 lb per cu ft, which is 5.7 lb per cu ft less than that of the excellently graded material (Sample No. 6).

In the four samples, Nos. 8, 9, 10, and 11, Table 3, the sand, silt, and clay fractions are constant, but the grading of the sand fraction is varied. The maximum density of Sample No. 8 (with 5% of silt and 18% of clay, and approximately the same sand gradation as Sample No. 6) is slightly lower than that of Sample No. 6. As the coarser sand fractions are eliminated in Samples Nos 9, 10, and 11, and the mixtures approach a more nearly uniform grain size, their densities decrease.

Electrolytes.—In Fig. 5(a) are shown the effects of different electrolytes on the densities of compacted samples of the Arlington, Manor, and the Iredell soils. Using the Arlington samples for illustration, it will be noted that admixtures of sodium hypo-sulfite, sodium chloride, ferric chloride, and calcium chloride, effect an increase in weight of dry soil per cubic foot over that of the natural soil with water alone, whereas the weight of the sample treated with sodium silicate is less than that of the sample without admixtures.

Fillers and Adhesives.—In Fig 5 (b) are shown the effects produced by other admixtures on the densities of different samples. The fact that films of water-soluble adhesives might be thicker than those of water has no significance with respect to the relative stability of soils treated with the different materials.

Tests Used in Soil Stabilization

In practice, the suitability of graded mixtures is determined by means of sieve analysis of the aggregate retained on the No. 10 sieve, combined sieve and hydrometer analysis of the fraction passing the No. 10 sieve, and the Atterberg plasticity tests performed on the fraction of the material passing the No. 40 sieve, an effort being made to combine materials in such quantities that the resulting mixture will conform to the requirements of stabilized road surfaces. To this end, specifications have been prepared on the basis of grading and the plasticity index.

However, any chance combination of two materials that falls within the limits of the specifications may not be the best possible combination. As has been shown by Miller (32), the specifications may call for a plasticity index of between 4 and 12, but for two given materials (aggregate and binder), there is one combination having a definite plasticity index at which highest density may be obtained.

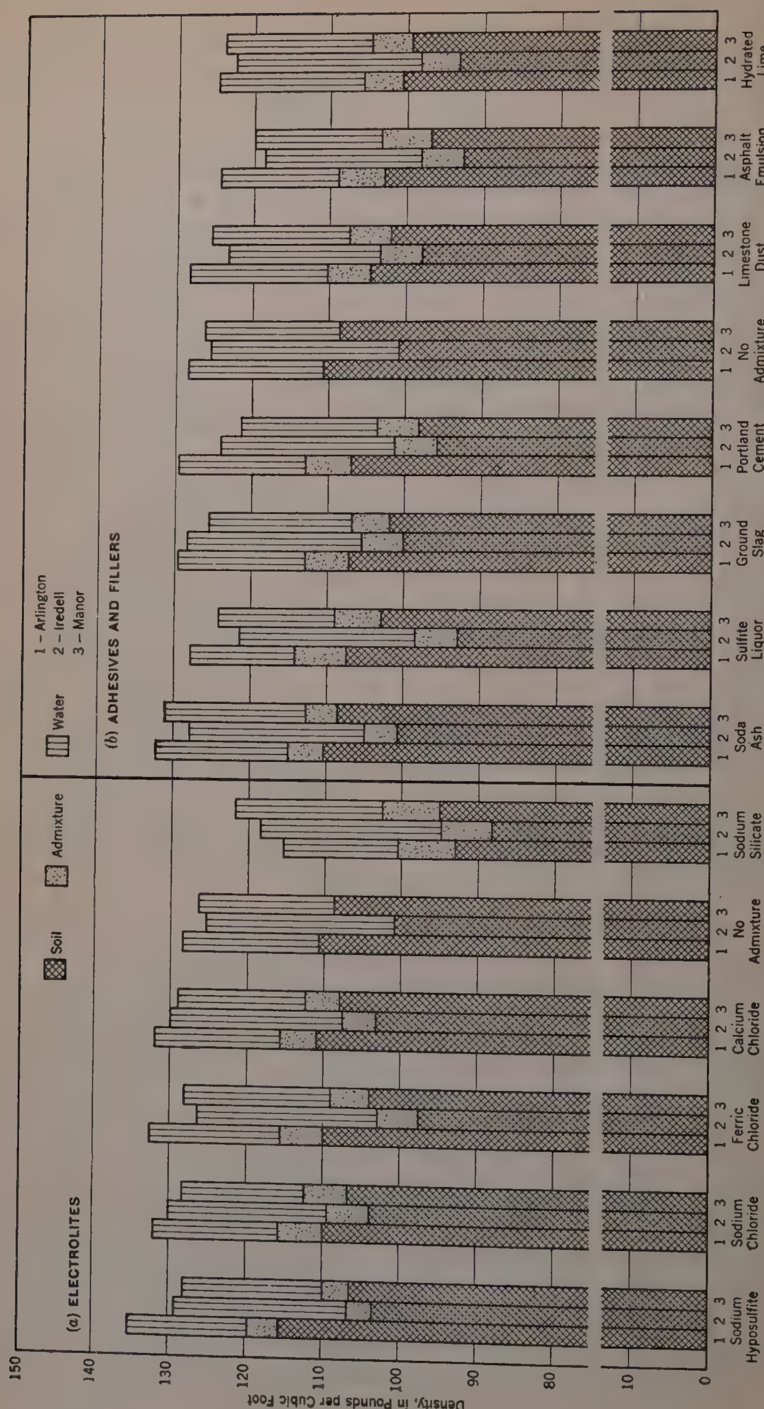

FIG. 5.—EFFECT OF VARIOUS ADMIXTURES ON THE DENSITIES OF COMPACTED SAMPLES.

The curve in Fig. 6 shows the maximum densities obtained in the ?roctor test on five mixtures of Potomac River sand and Arlington loam. :t indicates that the maximum density will be obtained by combining 30% of the Arlington loam with 70% of the sand. The plasticity index of this mixture is 6 and should be used for control in the field.

For the design of soil base ourses in which asphaltic emul-ions are to be used, C. L. Mc-Kesson, M. Am. Soc. C. E., advo-ates the use of special tests lescribed elsewhere (5). The mois-ure content and the quantity of inder required in soil base courses tabilized with Portland cement, ar, and certain asphalts are deter-nined by means of the Proctor ests.

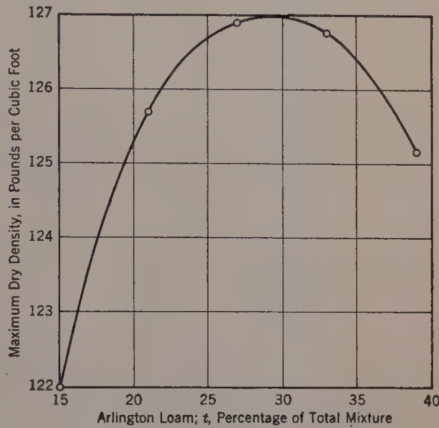

FIG. 6.—MAXIMUM DRY DENSITIES OF COMPACTED SAMPLES.

The Proctor tests are used ex-ensively in connection with the onstruction of earth dams and mbankments. However, it is be-ieved that a modification of these ests, which furnishes data, such as those shown in Fig. 2, allows more atitude of construction procedure. For the compactive pressure to be rovided by a given type of equipment, the corresponding optimum moisture ontent, resulting density, and plasticity needle readings are shown by the urves. Likewise, for any desired embankment density, the equivalent pres-ure that must be furnished by the rolling equipment, is disclosed.

To illustrate, let it be assumed that rolling equipment, exerting a com-active effort equivalent to a static pressure of 300 lb, is to be used. Then, the soil (Fig. 2) should be compacted at a moisture content of 2.9% of the weight of dried soil; the resulting density should be 104.4 ɔ per cu ft; and the plasticity needle reading, 117 lb per sq in. If, n the other hand, the embankment is to be constructed at an optimum noisture content equal to, say, the plastic limit (26%), the soil should e compacted at an equivalent static pressure of 130 lb per sq in. The esulting density is 99.2 lb per cu ft, and the plasticity needle reading, 9 lb per sq in.

The slopes of the density-moisture content curves may be quite different, s shown in Fig. 7. Assuming that the moisture content can be con-rolled in the field within a range of 5%, a variation of 2.5% above and elow the optimum would be expected if the control value was set at the ptimum moisture content. According to Fig. 7(*a*), densities from 98.4 ɔ per cu ft to 100.6 lb per cu ft would be expected from this particular oil. The density-moisture content relationship shown in Fig. 7(*b*) has steeper slope than the one shown in Fig. 7(*a*), although the indicated naximum densities are the same. A variation 2.5% above and below the

optimum moisture content would result in densities between 94.35 lb per cu ft and 100.6 lb per cu ft. By setting the control moisture somewhat above the optimum in the case of the soil with a density-moisture content relationship, such as that shown in Fig. 7(*b*), it is possible to reduce the variation in density. Thus, using moisture contents from 2.5% below to 2.5% above 26.2%, the resulting densities would be between 97.4 lb per cu ft and 100.6 lb per cu ft.

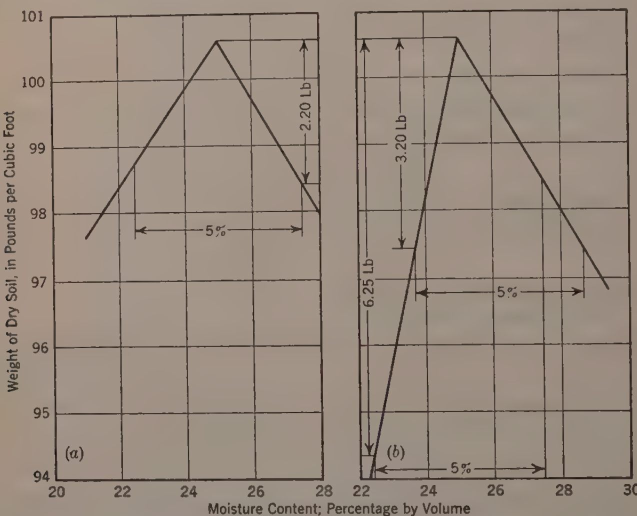

FIG. 7.—EFFECT OF MOISTURE CONTENT VARIATION ON DRY WEIGHT OF COMPACTED SOILS.

CONSTRUCTION VARIABLES

In the mixing of solid binders, such as clay and Portland cement, with aggregate, advantage is taken of the adsorption phenomenon that for thorough distribution it is only necessary for the materials to be brought in close enough association to permit the dry powders to become attached to and coat the aggregate.

Bituminous materials, in contrast, coat the soil more readily after the air films have been removed by wetting the particles. Therefore, water in various quantities is used to obtain the required distribution of tars and asphalts in soil mixtures.

The materials have been mixed by harrowing, blading, etc., by portable plants on the roads, and by means of plants set up at the source of supply of the soils or mineral aggregates.

Asphaltic materials used as stabilizers in Jackson County, Missouri (7), were introduced at desired depths below the surface of the loose road mix by means of a specially constructed "sub-oiler" (1*d*). It is essentially a tooth scarifier, having an oil line attached to the back of each tooth and running nearly to its point.

Graded mixtures have been compacted by rollers supplemented by traffic on roads and by rollers, trucks, etc., in fills. Generally, when the Proctor

method of control is used, the prongs of a sheep's-foot or tamping type roller, or equipment that produces a similar effect, are required to penetrate the unconsolidated material and compact the layer from bottom upward, in order that the layer of fine material may be consolidated uniformly throughout its thickness without stratification. Compaction by vibrating the soil is another method being investigated.

Summary

A review of the development of soil stabilization discloses the fact that the principal means followed in practice until several years ago involved the use of granular materials. With the increasing demand for farm-to-market roads the use of water-retentive chemicals was found to be a valuable supplement in the construction and stabilization of properly graded road mixtures. Attention was next directed toward the use of water-insoluble binders, including bituminous materials and Portland cement, for stabilizing poorly graded and fine-grained soils. Data now available indicate that:

1.—Soil stabilization involves phenomena in one or more of the following steps: (*a*) Removal of the air film by wetting the soil particles; (*b*) making the liquid films stronger and more lasting by use of deliquescent and water-retentive electrolytes; (*c*) use of waste aggregate materials to provide the proper neutrality of the mixtures; and (*d*) replacement of the liquid or air films with bituminous materials, Portland cement, and other insoluble binders in the stabilization of fine or poorly graded soils.

2.—Admixtures which have been found unsatisfactory for treating road surfaces subjected to abrasion by traffic and action of the climatic elements may provide benefit when used in base courses protected from these influences by impervious bituminous surfacings. The use of adhesives is not expected or intended to render the soil sufficiently hard or tough to resist the abrasive action of traffic, but simply to render it resistant to water from capillarity and thus to retain the same stability that the soil had at the density obtained during construction.

3.—Use of natural soil with the thinner films, or chemical solutions to reduce the thickness of the films and thus provide greater density, furnishes three distinct benefits to soil structures in service. The thinner the films the smaller is the quantity of free water that can be released as the soil becomes warmer. Consequently, soils with chemically thinned films retain greater uniformity of stability under changing temperatures.

Reduced film thickness causes soil to attain high density with less compactive effort. This means lower cost for compacting fills, earth dams, etc., and less time required under equal traffic conditions for road mixtures to become stable. The latter is especially important in base-course construction where the highest density is desired in the shortest possible time, in order to prevent additional and, perhaps non-uniform compaction of the base course, after the wearing surface is applied.

The freezing point lowers as the thickness of the film diminishes. Therefore, the denser the soil at relatively low moisture content, the less likelihood is there of the films freezing, even if they consist of water

alone. When the contained chemicals also depress the freezing point, a double provision against damage to stabilized soil due to frost is obtained.

Conclusion

In conclusion, it should be emphasized that soil stabilization is as yet in its infancy. Laboratory tests are helpful for disclosing the various possibilities of attaining stability of soils, but their value is limited and the constants furnished by them are truly constant only for the conditions of temperature, etc., at which the tests were made.

The plasticity tests disclose primarily only the binder properties of clay due to reversible phenomena. They do not throw light on the presence of colloidal cements that are likely to develop as the soils dry out during compaction under traffic.

The Proctor tests afford a simple and promising means of determining optimum binder contents and degrees of compaction required for the attainment of given densities. The optimum density to which a given soil should be compacted for greatest stability in embankments is not as yet known. It has been suggested that there is a theoretical optimum for a soil in dam construction beyond which tamping may become injurious (35). Meager data obtained in the laboratories of the U. S. Bureau of Public Roads seem to indicate the possibility of a drop in stability after the density of a sample at constant moisture content has been increased beyond a certain limit.

How much the different fine-grained soils will swell, over long periods of time, after being compacted to high density during the construction of embankments, is also a moot question. Generally, the pressure on the soil, at a given location, exerted by the weight of the embankment above it, is considerably less than the pressure at which the soil was compacted. Only colloidal glue action could prevent such soils from swelling. If swelling were likely to occur to considerable degree there would seem to be no reason for compacting the soil with a pressure much greater than that produced by the weight of the embankment the soil is to support.

The fact that some soils are particularly sensitive to small changes in moisture content indicates that from a practical standpoint they are undesirable for embankment construction. Furthermore, such soils when used as binders in stabilized road mixtures will re-act quickly to variations in moisture content with consequent changes in stability and cementing properties. The basic relationships between soil particles and surrounding moisture films are applicable to both embankments and stabilized soil roads, and any tests revealing these relationships are valuable aids in the design and construction of either type of structure.

Unfortunately, tests are not as yet available for disclosing the effects of climatic variables occurring during long periods of time on specially compacted and specially treated soils. As a result much of the desired information on the lasting effects of the base exchange, chemical actions, and especially produced film conditions must await the results of observations on stabilized soil roads and structures in service.

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PAPERS

SOIL REACTIONS IN RELATION TO FOUNDATIONS ON PILES

By R. M. Miller,[1] M. Am. Soc. C. E.

Synopsis

Technical literature dealing with foundation problems contains voluminous reports on the properties of clay, silt, and mud, and the reactions of these soils to loads transmitted through round or square plates of varying sizes. There has also been as much written on the properties and reactions of sand and other permeable soils under various conditions of loading. Seldom, however, does one find a clear analysis of the behavior of a combination of many soils, a condition that is encountered so frequently in the construction of foundations. Even less frequently does one discover a publication that is written for the practical use of the designer and the construction engineer who are seldom equipped for this highly specialized branch of the profession. It appears possible that if more attention were directed to the application of theory rather than to theory itself fewer foundation failures would result. The attempt has been made in this paper, therefore, to use the construction field as a source of information, and to correlate the collected data on pile foundations under varying soil conditions so that the reasons for success and failure become evident.

The data presented herein may tend to show that the "science" of pile foundations can be called a science only when the piles are driven to rock, gravel, hardpan, or other substantial sub-stratum. Very little is known of skin-friction values or of the distribution of loads to the subsoils and, therefore, not much is known about the correct spacing of piles. It will be evident that in soils of clayish characteristics, and even in saturated sands, the dynamic pile-driving formulas are only a poor indication of the carrying capacity of a single pile; and that the carrying capacity of a cluster or mat of piles seldom equals the value of a single pile multiplied by the number contained within the cluster.

Note.—Discussion on this paper will be closed in October, 1937, *Proceedings*.

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Average Practice

Records collated from 250 pile-driving projects, well distributed over the United States, form the basis of this paper. The analysis is interesting in that it reveals the variety of soil combinations extant and the average treatment, by practicing engineers, of this complicated problem. As recorded from the boring data the soils were of such an infinite variety of mixtures and in such an infinite variety of proportions that only three examples will be noted: (1) 5 ft of sawdust, old paving blocks, and general refuse, on 25 ft of very soft and slushy mud, changing gradually to 5 ft of firmer blue mud on sand; (2) 6 ft of sand and clay, on firm sand and silt, on coarse sand and quicksand, on medium coarse gravel; and (3) 20 ft of hydraulic sand-fill, on 25 ft of mud, on 5 ft of peat, on marl. During the construction of the foundations for the Laurel Homes Housing Project, Cincinnati, Ohio, in an area of 35 by 150 ft, loam top-soil was found on soils which may be described by the terms, clay, "bull liver," and quicksand. Within the same area were found twelve cesspools and one spring flowing strongly during the drought of the summer of 1936. Imagination cannot picture a mathematical treatment of such a problem. With no apologies, it should be stated that such inexact terms as quicksand, clay, the dirt movers' "bull liver", etc., are used throughout this paper as describing the soils more clearly to the minds of most engineers and contractors than the correct technical terms. No piles were driven for the Laurel Homes Project.

The driving of piles in the widely varying soils indicated by the foregoing, develop reactions as diverse as the soils themselves. In some instances a heave as much as 6 ft resulted, and, in others, settlement followed. Added resistance to driving after rest resulted in some cases and lessened resistance to driving after rest in others. Frequently, shorter piles were required as more were driven in a cluster, due to the consolidation of the surrounding soil, and at times longer piles were required as driving progressed.

The solution of this problem may be demonstrated by average data computed from the aforementioned records. If the assumption is made that the design load must be carried by the soil below the pile-points, and if the average design loads are divided by the area represented by the pile-spacing, comparative loads per square foot on the several types of subsoils can be ascertained. It is understood, of course, that whereas only average values, are being compared concentrations of loads may often occur beneath pile-points greatly in excess of the average.

From Table 1, it is evident that, in spite of errors in assumptions, most of the serious settlements of foundations on piles in America should occur on projects where the pile-points rest in clay or other plastic subsoils. It is also evident that average practice is at times wasteful where the pile-points rest on firmer sub-strata. It would appear, therefore, that where piles have been driven in plastic soils, the average design load per pile has been too much, or the spacing of the piles has been too small, for the

the average design load. It would also appear that where piles have been driven to rock or other hard strata advantage should be taken of the possibilities, such as increasing pile diameter, pile spacing, the design load per pile, and the unit load on the hard sub-strata. In addition, it must be apparent that end-bearing piles driven to solid sub-strata present comparatively little difficulty.

TABLE 1.—Bearing Values for Pile Foundations

Soil beneath pile-points *	Design Load, in Tons per Pile			Average load upon subsoil, in tons per square foot	Assumed safe load, in tons per square foot †
	From:	To:	Average		
(1)	(2)	(3)	(4)	(5)	(6)
Soft clay	12	35	25.0	2.8	1.0q
Sand	6	45	28.4	3.2	4.0‡
Gravel	14	40	28.7	3.2	6.0
Firm clay	20	39	29.5	3.3	3.0q
Rock	18	70	36.4	4.0	30–100.0§

* Average pile spacing equals 3 ft. † Building Code of Cincinnati, Ohio. ‡ Average for fine clean sand and coarse compact sand of Cincinnati Building Code. § Building Code of Boston, Mass.

Examination of the records indicates that the results are due in many instances to incorrect, incomplete, or a total lack of, subsoil data, to misinterpretation of the results of preliminary investigation, and often to too much dependence placed upon standards of pile loading and spacing.

Soils

Research and experience have brought to light soil characteristics that serve as a basis for the understanding of their reactions during the driving of piles. However, variations in the water content or imperceptible quantities of other soils in combination change the results so that at times predictions as to reactions are embarrassing.

In general, it may be said that: (1) Soils, to-day, are considered as solids definitely elastic in some degree; (2) compressibility varies from sand and gravel at the one extreme to gumbo and peat at the other; (3) sand under load compresses almost at once, whereas clay of low permeability from which water can only escape slowly under pressure, requires a long time for permanent deformation; and (4) undisturbed clay recovers considerable of the depth of its settlement when the load is removed, depending on the duration of application of the load.

The Behavior of Impervious Soils

The phenomenon of the compression and rebound of soils has been illustrated so many times by investigators, by means of compression and rebound curves, that additional curves are superfluous. It will be instructive, therefore, to examine a fairly common phenomenon that may serve to illustrate the behavior of the more impervious soils.

One of the railways in the East was built through red and yellow clay, with miles of clay cuts and fills. Large carloads of coal are hauled by heavy locomotives, producing heavy wheel loads. Observation of the track under traffic indicated that the clay sub-grade was originally very elastic, the deflection of the rails under the passing wheels being considerable, and the rebound being apparently equal to the deflection. For years after construction was completed, more and more ballast was added, and the track was lifted time and again, until, finally, for valuation purposes, the ballast was measured, and it was found that the road bed immediately under the ballast had assumed the shape of a ditch, 3 ft or more deep in places. This would mean that the repeated application of loads over a long period of time gradually compressed the clay until, after many load applications, permanent deformation was reached. Some evidence was discovered of side movement of the soil into cut ditches and of mixing of ballast and sub-grade.

The time required for driving of piles, however, is usually a matter of minutes and the application of comparatively few blows. If a clayish or impervious soil is considered as elastic, it should compress momentarily with the blow of the pile-hammer and rebound at once; or if the reaction is viewed as the displacement and flow of a plastic material under pressure the result in either case is movement upward or in the direction of the least resistance. This is known as "heave", and is less in soils containing larger proportions of sand or water. If the soil happens to be sand, the surface is more likely to subside than to heave. If the sub-soils are alternate beds of clay and sand, both heaving and subsidence may occur. No definite rules can be stated, but it is possible, from the examples that follow, to trace some similarity in the reactions of soils.

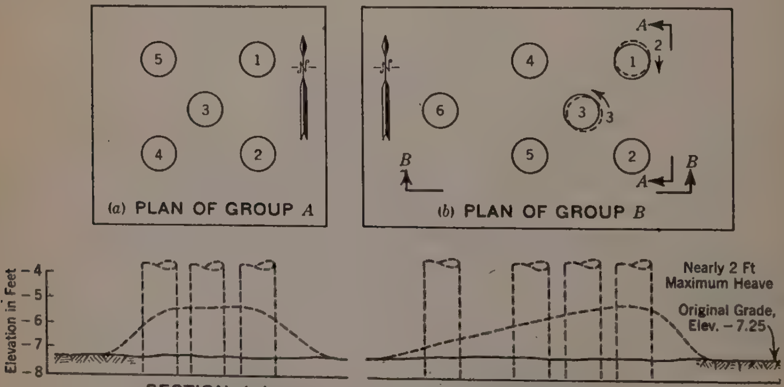


FIG. 1.—ARRANGEMENT OF PILES AND ORDER OF DIVISION.

Example 1.—During the spring of 1932, some interesting pile tests were made on the site of one of the larger Government Buildings in Washington, D. C. (3)[2] Two groups of piles were driven: Group A, of five piles, having an average length of 20 ft; and Group B, of six piles, having an average length

[2] The numerals in parentheses denote references to the Bibliography in the Appendix; thus: (2(*e*)) refers to Item No. 2, November, 1933, p. 1461.

of 13.6 ft. The piles in each group were spaced at 2.5-ft centers, and the two groups were separated by about 200 ft. All the piles were of straight shaft, uncased, compressed concrete, with a diameter of 15 in., and all but one (see Fig. 1, Pile 6, Group B), had a bulb or pedestal at the bottom.

From Table 2 it is evident that the soil had consolidated around the piles of Group A as the driving progressed, and the piles, therefore, had to be shortened. The ground was thrown vertically upward during the early part of the penetration of each pile; and yet, the maximum heave was only 0.5 ft. Fig. 2(*a*) shows that the strata penetrated consisted of a mixture of permeable and impermeable materials, the reactions of which were slightly on the side of the impermeable materials. Table 2(*b*) shows that, in Group B, the soil had not tightened up sufficiently to cause shortening of the piles as the driving progressed. The heave amounted to a maximum of 2 ft, equal to approximately 70% of the displacement of the piles in the clay and silt. It is evident, therefore, that the soil around Group B was of the clayish order (see Fig. 2(*b*)), as shown by its heaving reaction and small amount of consolidation. It is also evident that the soft concrete of the closely driven piles was more likely to become distorted in this group. This proved true; when the piles of Group B were excavated to the gravel stratum, 1 ft above final penetration, Piles Nos. 3, 4, and 5 had a tendency toward a

TABLE 2.—PILE-DRIVING DATA, EXAMPLE 1 (NO. 1 STEAM HAMMER; STROKE, 3 FEET)

Description	Pile Nos.					
	1	2	3	4	5	6
(*a*) Group A*						
Length, in feet	26	22	18	18	17	......
Blows per inch, in last foot of driving	5	5	7	5	7	......
(*b*) Group B						
Length, in feet	14	14	13	13	13	14
Blows per inch, in last foot of driving	7	6	6	6	7	6
Creep (in inches), in direction of arrow	2	......	3	......	......	......

* See Fig. 1.

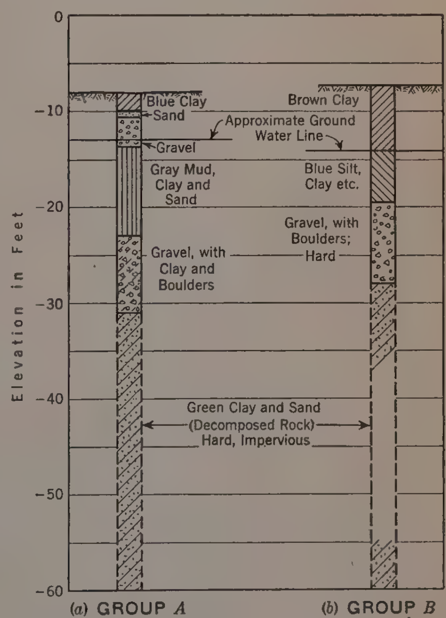


FIG. 2.—BORING DATA; EXAMPLE 1.

smaller sectional area in the clay just above the gravel and a slight lateral deflection, in the direction of least resistance, outward and away from the most highly compressed soil.

Example 2.—On the construction of the Union Terminal, at Cincinnati, Ohio, a large proportion of the pile foundations were in clays and combinations of clays, in which the heave at times amounted to as much as 3 ft. Many of the footings were supported by encased, cast-in-place, concrete piles, placed by driving a pipe or casing containing a core. On withdrawing the core, a corrugated iron pipe was placed in the casing and filled with concrete, after which the outer casing was withdrawn, leaving an annular space of 1 in. around the corrugated iron pipe. On one occasion the corrugated iron pipe was ordered removed after the casing had been withdrawn. Due to unavoidable delay, the orders were not followed until a few hours when it was found that the clay had gripped the corrugated iron pipe so tightly that it could not be removed without destroying it, illustrating perhaps the rebound of elastic material when the pressure is removed. For the same purpose Example 3 is given.

Example 3.—At Potomac Park, Washington, D. C., three test piles were driven in 1934. One was a composite pile with a steel-encased concrete action section, 16 ft long, and a lower wooden section, 30 ft long. The second, driven 15 ft away, was a steel-encased, cast-in-place, concrete pile, 53 feet long, and similar to those mentioned in Example 2. The third test pile was a standard, wooden, peeled pile, 52 ft long. They were each driven through 10 ft of hydraulic fill, 30 ft of Potomac River mud, and then, at 2 in. per blow for 3 ft and 1 in. per blow for 1 ft, tightening up to four blows per inch through the next 5 ft.

The steel-encased, concrete pile had been provided with a steel center bar, 1 in. square, so that the pile could be subjected to an upward pull. On removing the casing, the usual 1-in. annular space was left. Static load tests were made on the three piles, so that several days elapsed before a pull test could be undertaken. Hydraulic jacks were used in pulling, which applied an upward pull of 47 500 lb upon the 1-in. center bar before it parted. Deducting the weight of the pile, a net pull of 40 000 ft had been applied, which was equivalent to a frictional resistance of at least 243 lb per sq ft, giving an indication of the grip upon return of a very plastic soil.

Clayish or fine-grained soils usually heave in some degree during driving, depending upon the water content and mixture with other soils. After rest they regrip the pile with added resistances, at times amounting to as much as 2.5 times in 12 hr, and even 4 times in 24 hr. Whether the cause is attributed to the rebound of an elastic soil against the pile or to the readjustment of the internal structure of the soil after remoulding is not as important as the facts themselves. Either explanation may be true at different times.

Reactions of More Pervious Soils

The addition of sand or coarse-grained mud to soil combinations changes the results in a surprising fashion. Charles Terzaghi, M. Am. So. C. E.,

refers (1(*b*) to the driving of piles in fine-grained saturated sand, and notes that the surface of the sand subsided 1 ft due to the driving. Victor A. Endersby M. Am. Soc. C. E. (1(*c*)) describes the driving of piles in sand to satisfactory resistance on one of his projects. The piles were allowed to rest until the following day. When driving was resumed, it was discovered that they had lost 40% of their bearing power.

Example 4.—During the construction of the extension of Concrete Pier No. 314, Navy Yard, Charleston, S. C., under the supervision of the United States Bureau of Yards and Docks (5*a*), a number of 18-in., square, wooden, test piles were driven through 10 ft of silt and into stiff blue mud, apparently quite coarse-grained. At from 43 ft to 57 ft the test piles developed a resistance to driving of 40 tons, soon thereafter "fetching up" at virtual refusal. After a rest of from 3 hr to 2 days, driving was resumed, showing a marked decrease in resistance. The decreases in specific cases were from 43 to 21 tons, 45 to 32 tons, 47 to 39 tons, and 39 tons to 25 tons.

Example 5.—The following, to much the same purpose, is reported by Lieut. J. N. Laycock, U. S. Corps of Engineers (4*b*): Four timber test piles 18 in. square and 80 ft long, were driven at Balboa, Canal Zone, through soft silt to − 28 ft to − 43 ft; then through fairly compact and uniform blue clay, the blue clay extending well below the pile-points. With each of the four test piles, driving was suspended several times, the delays varying from a few minutes to overnight, and in every case of suspended driving, there occurred a loss of resistance. These losses varied from 3 000 lb for Pile No. 1, for a 15-min delay, to 48 000 lb out of 90 000 lb for Pile No. 4 for an overnight delay. Such losses follow so closely those of the test piles driven at the Charleston Navy Yard that both soil combinations must have been very coarse-grained, comparing in reaction to a saturated sponge placed suddenly under compression.

Example 6.—At the Naval Supply Depot, United States Naval Operating Base, San Diego, Calif. (5*b*), a number of concrete and wooden test piles were driven through 7 ft of soft plastic mud, carrying a small quantity of sand and shells. Beneath the mud lay strata of sandy clay and sand. A wooden test pile, 20 in. by 20 in. square, was driven to virtual refusal. The pile was allowed to rest 24 hr and driving was resumed. In the first few inches of re-driving there was a decrease in resistance, but the pile was again driven to refusal, after the driving had progressed 1 ft. It should be noted that Examples 4 and 5 referred to coarse-grained muds or more permeable materials. Example 6 refers to permeable sand and shells and impermeable mud, in which sufficient sand was present for the combination to act in a similar manner but in a lesser degree.

During the same tests at San Diego, an 18-in., precast, concrete pile was pulled, and brought up a natural shoe of compacted material on the point of the pile. The report states (5*b*) that "it consisted apparently of samples of the various materials through which the pile had passed, and was as hard as, and had the appearance of, soft rock." This phenomenon is not uncommon, as it has been noted many times in the driving and pulling of test piles in sand and shells. The possible explanation is that

a permeable saturated soil such as sand under pressure, loses its water content rapidly, and is consolidated quickly. Piles in saturated sand will drive to virtual refusal, quickly forcing out the water in the sand immediately surrounding the pile-points and forming a bulb of solid material. The water in the sand surrounding the consolidated bulb having a means of ready escape, the pressure will be quickly dissipated during rest. Following rest, the pile and its sand bulb having no longer high pressure to work against, will drive with lessened resistance. Continued driving, however, accompanied by further consolidation, at length, brings the piles to final refusal.

Example 7.—Resort has been had to the artificial consolidation of soils for the support of structures since the dawn of history. The Egyptian sand pile has been, and still is, used successfully for this purpose in many parts of the world. This pile consists in driving a hole in the ground, filling it with sand, and tamping the sand to refusal. This process provides a satisfactory foundation, provided the soil can be consolidated. For example, into the sub-soils beneath the Federal Legislative Palace, in the City of Mexico, Mexico (6), consisting of an extremely unreliable mixture of volcanic ejections and alluvial matter, were driven 150 000 sand piles, 8 in. in diameter on 20-in. centers. The displacement of the piles could have caused a heave of 5 ft, if no consolidation had occurred. As the heave averaged only 1 ft, however, consolidation in the amount of four-fifths of the displacement had taken place. However, when water cannot quickly be lost from fine-grained impermeable soils under quickly applied pressure, very little if any consolidation occurs. Increase in soil pressures, due to the driving of piles, may be temporary, whereas consolidation of the soil is of permanent value in increasing bearing power. It must be obvious, therefore, that the driving of piles in moist to wet soils that cannot be consolidated for a long time leads to uncertain results.

Skin Friction

Perhaps no factor in the design of pile foundations has been as fruitful of foundation troubles as skin-friction values and their relation to the problem. Offering no ready-made solution or convenient tabulation that will apply to all conditions, the attempt will be made to indicate by illustration the wide range of values that may be secured and the reasons why the assumptions are of such small use.

In the absence of better information, engineers have been forced to use the best data available. The friction between pile and soil has often been computed from the results of static load tests, deducting assumed point resistances with no allowance made for temporary soil pressures.

Much better than the average of published data are the values shown in Table 3. It can be seen that: Either the soil terms are not clear; the point-resistance deductions may have been in error; temporary soil pressures may have affected the result; or, perhaps, the surfaces of the piles themselves have been important factors; for example, compare Items Nos. 1 and 2 of Table 3, both of which are for soft blue clay.

Example 8.—Exhaustive tests were conducted by the Whangpoo Conservancy Board during the Shanghai Harbor investigation in China, in 1920–1921 (8). The friction value (in pounds per square foot) of blue clay at that time was reported variously, as follows:

Timber pile, 10 in. by 30 ft....................	340
Timber pile, 20 ft long.........................	360
Round Douglas fir pile, 45 ft long................	650
Rough-surfaced concrete pedestal pile, 33 ft long..	650

The report also indicates a slight decrease in skin friction for the longer piles and an increase of 10% in friction values during the subsequent twelve months.

TABLE 3.—VALUES OF SKIN FRICTION

Item No.	Soil	Friction value, in pounds per square foot	Point resistance deducted	Location
1	Soft blue clay.....................	1 850	No	Hull, England
2	Soft blue clay.....................	740	Yes	Portland, Me.
3	Soft muddy clay....................	370	Yes	Tunis, Algeria
4	Mud. sand, and clay................	370	Yes	Proctorville, La.
5	Soft clay..........................	295	No	Rhine Valley, Germany
6	Soft mud...........................	130	No	Rhine Valley, Germany
7	Mud................................	130	Yes	17th Street and North River, New York, N. Y.
8	Silty microscopic sand.............	130 to 900	No	Shanghai, China
9	Fluid mud..........................	90	Yes *	Aguia Creek, Va.

* 15 per cent.

Example 9.—In 1933, in connection with the Sewerage and Sewage Disposal Plant program in Columbus, Ohio (9*a*), it was decided to drive precast concrete piles and to test for uplift. Eight piles, 15 ft and 20 ft long, were driven through loam, clay, sand, and gravel. Ground-water level was quite near the surface. The eight piles were pulled with the following results:

Piles	Depth below the ground, in feet	Frictional resistance, in pounds per square foot
1	11	538
2, 3, and 4.............	14	557
5, 6, 7, and 8..........	19	613
Average of eight piles........	16	582

Evidently, the skin friction had been increased with depth in contrast to the results reported by the Whangpoo Conservancy Board (see Example 8).

Example 10.—Pull tests were also made for the Water Supply and Filtration Plant, at Fort Wayne, Ind. (4*c*); 4 600 conical concrete piles, averaging 16.8 ft in length being driven through dark yellow clay, fine sand, and coarse sand. Later, five piles were subjected to pull tests with hydraulic jacks. The friction value developed by these pull tests, averaged 755 lb per sq ft for steel on clay, fine sand, and coarse sand.

Example 11.—The sinking of caissons in the underpinning of the Naval Aircraft Building, at the League Island Navy Yard, Philadelphia, Pa., also

developed data contrary to that furnished by the tests at Shanghai as to the relation of friction values to length of piles. Records were kept by the U. S. Bureau of Yards and Docks of the pressures necessary to move the caissons at various depths. The caissons penetrated silt and sand to gravel and cobble at 50 ft to 60 ft. Making allowance for the upward pressure of the air, it was found that the pressure necessary to move the caissons plotted in nearly a straight line for the upper depths and then tended to a flattened parabola for the lower depths. The friction values increased from 100 lb per sq ft at a 15-ft depth to 305 lb per sq ft at a 60-ft depth.

Example 12.—To illustrate the relative skin-friction values of rough and smooth pile surfaces, it may be interesting to compare the results of the load test on Pile No. 6 of Group B, Example 1, with those made on the precast concrete piles that ultimately became a part of the building foundation. The precast piles were $13\frac{3}{4}$ in. in diameter and cylindrical in shape. Eleven of them scattered in the vicinity of Group B were test loaded. These precast piles (of 12.6-ft average length, for a settlement of $\frac{5}{64}$ in.) were supporting loads averaging 31 tons. An exact comparison cannot be made with the settlements of $\frac{3}{32}$ in. used in connection with the piles in Group B. However, $\frac{5}{64}$ in. is close enough for reasonable comparison. Since the point-bearing area of the precast pile was approximately equal to that of the bulbless Pile No. 6, Group B, it would appear that the friction values of the precast piles were equal to $\frac{31}{50}$ times that of the rough-surfaced bulbless pile in the same soil.

For the same purpose it should be added that a part of the Potomac Park tests (Example 3) consisted in test loading both the wood pile and the corrugated encased concrete pile described. The wood pile failed at a test load of 32 tons whereas the concrete pile stood successfully a test load of 45 tons, indicating that the surface and perhaps the shape of the pile are important factors.

Summary.—Skin friction cannot be assumed as a factor that in itself supports a structure, but rather as the means of spreading the load to the strata below the pile points. Knowledge of friction values, therefore, is of use in determining the bearing value of the pile and surrounding soil and whether or not the soil below the pile points is overloaded due to concentration.

Friction values may be affected by variation in the soils, combinations of several soils, variation in water content, the length and surface of the pile, and the time that has elapsed since driving. It must be evident, therefore, that any tabulation of friction values that is intended to cover all possible conditions must be accepted with reservations. Dependable results can be secured by the driving of test piles of the type selected and by pull testing, at a lapse of time. This delay will be discussed herein under the heading, "Load-Carrying Capacity and Static Load Tests." The results are equally valuable when, as occasionally happens, the shearing strength of the soil, and not the pile surface, is the vital factor because if the former is the case the pile surface must be sufficiently rough. If the pile surface is the vital factor a pile with a rougher surface can be selected.

Load-Carrying Capacity and Static Load Tests

There appears to be general agreement that the dynamic pile-driving formulas give only approximations of the carrying capacities of piles under static loads when driven in fine-grained impervious soils, but that the formulas are reasonably accurate when the piles are driven in sand or other pervious soils. As to the dependence generally placed upon the results of static load tests upon single piles, the results thus obtained are unreliable indications of what may be expected of the group or mat, of which the single pile forms a part. This applies, however, only to piles driven on close centers or in fine-grained impervious clayish soils. To illustrate how static load tests on single piles may mislead the engineer, a few examples will suffice.

Example 13.—The piles described as Group A in Example 1, were subjected to a final static load test of 250 tons, and Pile No. 5, of Group B, was loaded to 70 tons (see Fig. 1). Group A was loaded to 125 tons for 24 hr. The load was then increased to 250 tons, and allowed to remain for 5 days. The settlement was $\frac{5}{32}$ in. Pile No. 5, Group B, was test loaded to 25 tons for 24 hr, to 50 tons for another 24 hr, to 62 tons for 36 hr, and to 70 tons for 24 hr, with a total settlement of $\frac{1}{8}$ in. (see Fig 3). At a settlement of $\frac{3}{32}$ in., the piles of Group A were carrying an average of 38 tons each, and Pile No. 5, Group B, was carrying 62 tons.

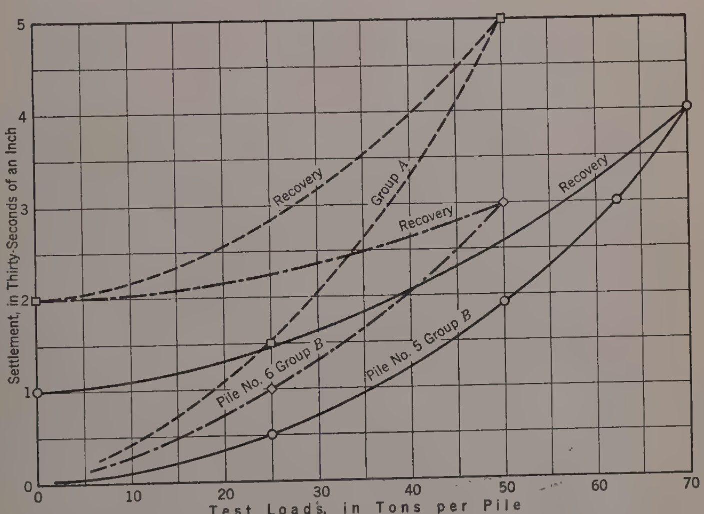


Fig. 3.—Comparative Load-Settlement Curves; Tests of Piles Shown on Fig. 1.

Comparative static load tests between single piles and groups of piles are very rare, due to the expense of making the tests and the speed usually demanded. In the absence of additional test results, the engineer must use the best information available bearing on the subject.

Example 14.—In 1918 and 1919, concrete piles to the number of 411, averaging 17.7 ft in length, of the compressed concrete pedestal type, were

driven for an industrial building of the Texas Oil Company, at Providence, R. I. There are no records of the borings taken. The driving record indicates that the piles should develop, according to the *Engineering News* formula, from 41 tons to 83 tons, the average being 53 tons each. The structure was designed for a final loading on the piles of 30 tons each. Static load tests of 45 tons were applied for 24 hr on one pile of a group and on an isolated pile. The test results showed practically no settlement. There seems to have been no trouble with the piles themselves, but check levels taken on the completed structure since 1925 showed a settlement of from 1.75 in. to 3.75 in.

Example 15.—For the grain elevator at Portland, Ore. (4*d*), fourteen wooden test piles were driven, well distributed over the construction area. These test piles as well as the permanent wooden piles were driven to a resistance of 25 tons (as determined by the *Engineering News* formula) through soil of considerably fluidity, to − 41 ft, the elevation of the pile-points. One test pile at the southeast corner of the Storage Annex was satisfactorily load tested to 40 tons. The foundation consisted of a 3-ft concrete mat cast upon the wooden piles, which were spaced 2.5 ft center to center. During construction, marked settlement was noted, and when the load per pile, under the Storage Annex, had reached 9 tons, the building had settled 1.96 ft at the northeast corner, 0.76 ft at the southwest corner, 1.4 ft at the southeast corner, and 1.72 ft at the northeast corner, in spite of the fact that the test pile near the southeast corner had supported a 40-ton test load satisfactorily.

One of the most important factors contributing to the misleading results often secured in the test loading of single friction piles is disregard of the time of testing. The common practice is to test-load single piles to the design load plus from 50 to 100%, the loads resting upon the piles from 24 to 48 hr, or perhaps longer, when the settlement is measured. If the settlement does not exceed the maximum (often specified as 0.01 in. per ton of load applied), the tests are accepted as satisfactory. Such results, however, may vary widely from the final value of the piles, as great changes often occur in the soils subsequent to the load tests.

If the piles are driven in a saturated pervious soil, where losses in resistance may reach 40% during the subsequent 24 hr, it follows that no static load tests should be made upon the piles until several days have elapsed. On the other hand if the piles have been driven in an impervious soil that heaves during driving, where increases in resistance may amount to several times the original value during the 30 days or more that follow driving, it seems useless to make static load tests upon the pile during that time. The problem, however, is not quite as simple as the foregoing implies, because piles often penetrate several types of soils, in which both "heave" and consolidation may occur. As boring data are not always sufficiently accurate guides, it would seem best when there is uncertainty to consider the soils as impervious and to test-load accordingly. In addition, if the test-loading of single piles were replaced by a group test loading, beneficial results would follow.

Spacing for Friction Piles

The correct spacing of piles is of importance for two reasons: (1) For cast-in-place concrete piles too close spacing in some soils is likely to cause distortion in the partly set concrete of contiguous piles if the driving is carried in rows continuously; and (2) for friction piles the correct spacing of a group or mat allows for the maximum load-carrying capacity of each pile. Under such conditions the pile and the surrounding soil are acting as a unit.

Numerous examples could be given of the distortion of cast-in-place concrete piles driven in different soils and with different spacing, if the information would be of any value. There appears to be no basis, however, for the derivation of formulas to cover the proper spacing of piles in relation to length, diameter, and soil conditions, in order to prevent distortion. However the simple device of driving alternate rows and reforming over the same ground to complete the unfinished rows is successful in preventing distortion and cracking if sufficient time has elapsed.

Reference has been made herein to the work of the Whangpoo Conservancy Board (8) in soil investigation and pile tests. Mr. H. F. Meyer, commenting on the Shanghai findings states (9):

"As soon as two piles are so near each other that the two volumes of influence cut into each other, the question becomes more complicated. The size of the affected prism of soil is so large that, in all ordinary cases under a footing or a foundation, they will overlap. The soil will thus be under greater pressure than when only a single pile is involved, the angle of friction will be smaller and the skin friction of the piles will decrease. By spacing the piles nearer than the spacing as given by the formula (developed by the Whangpoo Conservancy Board) no additional safety against settlement is gained, and the builder who has carefully test loaded every single pile and found that they would sink only $\frac{1}{4}$ inch for a load of 400 lb per square foot skin friction on their surface, will, to his astonishment, observe that his whole densely piled building settled more than a foot when completed, in spite of the fact that the piles are not nearly loaded up to their 'full capacity'. The distance from center to center of the friction piles driven next to each other, should never be less than half the circumference of one pile, if the whole friction area is to be utilized; the two piles, taken as one unit, will then present the same friction surface as would each of the two piles individually added together. If there are more than two piles, the circumference of the whole cluster of piles, should be bigger than or equal to the sum of the circumference of the individual piles in the cluster. The same should apply to any part of a cluster. Thus, in case of a rectangular foundation with equal distances between piles, and all piles square, and of the same size with sides 12″, the length of the entire foundation being 'A' and the width of same 'B', the distance 'Z' between piles will thus become: A and B $= 10'$; $Z = 4.5'$: A and $B = 100'$; $Z = 11.1'$; $A = 100'$; $B = 40'$; $Z = 8.7'$."

Such spacing is apparently intended for use only in the plastic clay to be found in Shanghai. Even in that soil, it appears to be sketchy indeed to engineers in the United States. On the other hand, the average spacing for friction piles used in the United States as discussed under the heading, "Average Practice", would appear to be the other extreme.

The transmittal of the load in part through friction tends to spread it upon the sub-soil and, therefore, has an important relation to pile-spacing and bearing power. If the spacing of the piles is such that the distribution of the load through skin friction to the sub-soil is uniform, the result is a "block load", and the piles and the soil surrounding them are acting as a unit. Obviously, the pile which (through side thrust or the roughness of the surface) transmits the largest percentage of the load to the surrounding sub-soil, will develop the least concentration of load under the point. Such piles, therefore, may be over-spaced and still act as a unit with the soil.

If the characteristics of the different soil strata that lie beneath the site are known, Boussinesq's analysis can be used with variations for the determination of the distribution of pressures to the sub-soils. Having solved for the pressure distribution on the crucial stratum, the proper spacing of the piles can be computed for uniform distribution of pressure. Although this practice would be a great improvement over some of the "hit-or-miss" methods, it is believed that in the hands of the inexperienced foundation engineer an actual test by means of driven and test-loaded groups of piles would be a more reliable method for securing this information.

Any group test intended for use in effecting uniform settlement in a pile foundation must cover such different conditions as changes of soil on the site and the difference in size and shape of the pile foundations.

Whether one or more group tests are made, the first step is a thorough exploration of the sub-soils by the use of core borings. From this information approximate pile lengths and the type of pile to be used can be selected. A test group can then be driven on centers preferably of two pile diameters and arranged so that the same loading platform can be used for the test loading of piles on centers of two diameters, four diameters, and for single piles.

To allow for the readjustment of the internal structure of the soil, the piles should rest after driving from a few to thirty days, depending upon the character of the soil. If, after the rest period, a single pile is test loaded by increments, followed by the test load of the piles on four diameters and, finally, by the test loading of the piles on two diameters with the settlements recorded as the increments of load are applied, data may be had that will indicate the carrying capacities of the piles of a mat, the diameters, and the spacing for an assumed settlement.

The fact that so many foundation settlements are dish-shaped indicates that the spacing between the piles toward the perimeter may be farther apart than the spacing toward the center of the structure. Without doubt an answer will be found to this part of the problem similar to that used in the design of the foundations (6) for the Federal Legislative Palace, in the City of Mexico (Example 7), in which Boussinesq's suggestions were followed in the plotting of an ellipse, the ordinates of which represent soil pressures at any given point.

Experience indicates that the expense of preliminary work for the analysis of soils, pile types, and economical spacing, will usually effect a sufficient saving in the first and later costs, to more than justify the expense. This is

particularly true of large foundation projects. It should be added, furthermore, that by using satisfactory load tests, the dynamic pile-driving formulas can be useful in selecting pile lengths and in comparing varying soil conditions. It need no longer be the "yardstick" for measuring bearing capacities, upon which so much has always depended.

Some Pile Foundation Settlements and the Causes

The intention in selecting examples of settlement of structures on pile foundations has been to present the case fairly from available records. It must be freely admitted, however, that all foundation settlements are not due to the use of friction piles in plastic soils, as might be inferred from the following examples.

End bearing piles driven to hard sub-strata act as columns and may be limited only to the bearing capacity of the sub-strata and the pile itself. However, at times the weakest link may be the column, because some of the piles may be overdriven, or cast-in-place concrete piles may, at times, be deformed or even pinched off. Local settlement on a pile or cluster may occur, but the records of such settlement are difficult to secure. Besides, injury to piles in placing is usually due to poor planning and supervision and is avoidable.

The following statement would appear to be a fair interpretation of the fundamental thought of the publications of Edmund A. Prentis and Lazarus White, Members, Am. Soc. C. E. (11): "It would seem elementary to state that each horizontal stratum has to support the total load above, including the overlying strata and the structures erected thereon." In the case of a pile foundation it is equally true that the soil strata below the pile-points must support the loads placed upon them, or settlement, and possibly failure, will follow. This principle is so fundamental and so clearly sensible that even the layman should be able to understand it. It would appear from the examples that follow, however, that this simple axiom is not generally recognized.

Most of the foundation settlements that will be described, fall into one main group, the causes and conditions of which are nearly identical. Perhaps the most common has followed the making of new land by filling in an old lake, swamp, or slough with a very soft bottom. Often the conditions that existed before filling, have been completely forgotten. When, years afterward, it is proposed to erect a structure upon the filled land, exploratory borings may or may not be taken. If piles are decided upon and are not driven through the compressible stratum at the bottom, or if spread-footings are used, settlement often results corresponding to the depth and compressibility of the soft stratum below the points of application of the loads. Later, when a complete survey of sub-surface conditions is made, the cause of the trouble is only too apparent. With this in mind, note the recurrence time after time of compressible sub-soils and particularly peat in the following examples.

Example 16.—For a great steel structure built not so many years ago, a creosoted timber bulkhead was driven in a bay. An hydraulic dredge

filled the area contained within the bulkhead with sand to a depth of approximately 20 ft. About 500 ft away, a structure somewhat similar to the one proposed had been in operation for a number of years, and no appreciable settlement had occurred. Apparently, soil conditions on the site of the proposed structure were identical with those of the older structure and, therefore, no preliminary soil explorations were made. As a part of the inshore foundations, pedestal footings were planned, each to rest on twenty-one wooden piles, 40 ft long, carrying 15 tons each. These piles were to be driven into the hydraulic sand fill mentioned.

The pile-driving began, and apparently satisfactory results were being secured, as measured by the *Engineering News* formula. The engineer in charge, however, felt something to be wrong and increased the number of piles and the lengths from 40 to 45 ft, then to 50 ft, and, finally, to 60 ft. The concrete for the pedestal foundations was poured, and shortly before the time for erecting the steel upon the pedestals, the tops were checked for elevation. It was seen at once that instead of bush-hammering, some of the pedestal tops were an inch or more below elevation, indicating settlement. Upon rechecking, it was found that during the following weeks, the same pedestals, even without load, had continued to settle at an alarming rate.

Test borings were taken, the results of which will be seen in Fig. 4. The cause of the trouble was obvious at once. The heavy sand fill had been placed upon sand, underlaid by very compressible soils. The piles were driven through the sand fill and the original sand, and developed the required resistance. However, inasmuch as the piles did not penetrate through the compressible soils and into the substantial strata beneath them, the added weight of the new fill caused settlement and the piles, of course, settled with the fill.

The steel superstructure was erected, although the pedestals continued to settle, compensation being made by means of driving steel wedges and placing plates under the column shoes. The settlements of the several foundations will be seen by referring to Fig. 5. However, after some months devoted to a close study of the problem, it became evident that the increments of settlement were decreasing and that, without corrective measures, settlement would finally cease.

If the plotting of the soil strata (Fig. 4) can be assumed as correct between the points at which test borings were taken, and if the settlement is assumed to be proportional to the thickness and compressibility of the strata beneath the pile-points, very little, if any, settlement might be anticipated in Foundations *S*5, *S*10, *S*11, *N*5, and *N*11. By the same reasoning the maximum settlement might be predicted in Foundation *N*10, diminishing in the following order: *S*7, *S*6, *N*9, *S*8, *S*9, *N*7, *N*6, and *N*8, which is very close to what occurred.

It should be added that any type of pile that did not reach the stratum described as "sandy marl", under the compressible peat, sand and peat, sand and mud, etc., would have proved as ineffective as did the wooden piles. Had the test borings been taken before construction, longer wooden piles would

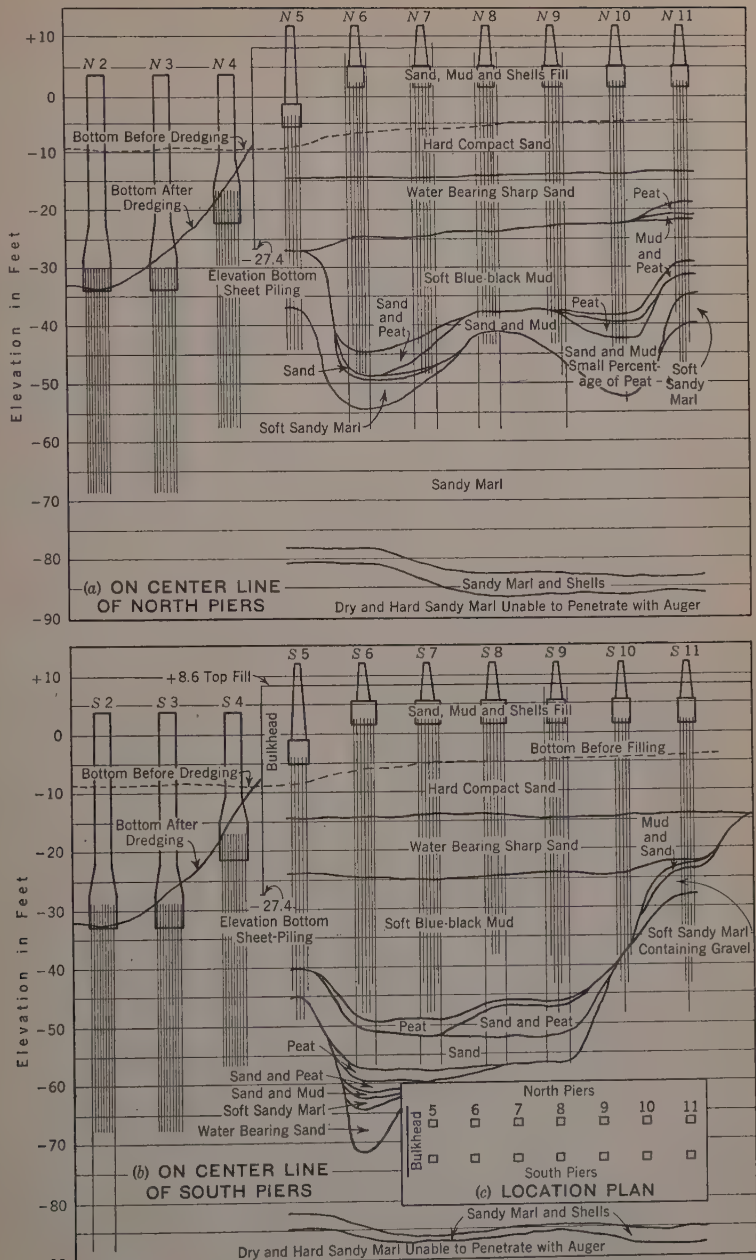


Fig. 4.—Boring Tests, Example 18.

have been used at an additional cost that would have been small compared to the several millions of dollars spent on the project.

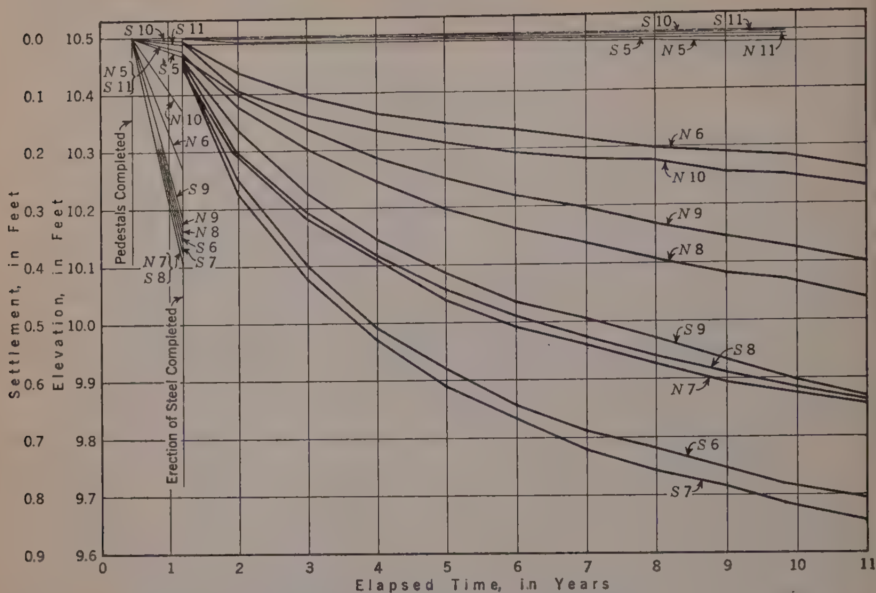


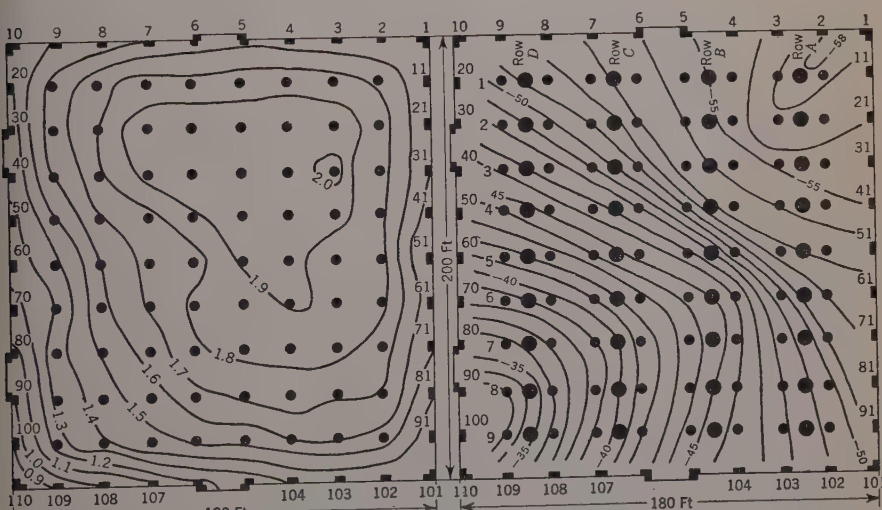
FIG. 5.—SETTLEMENT HISTORY, EXAMPLE 18.

Example 17.—During the rush construction period of 1917, an 11-story concrete warehouse was erected in the Brooklyn, N. Y., Navy Yard (5*c*). Due to the speed demanded, there was no time for sub-surface exploration. Piles were driven for the foundation from 10 ft to 34 ft in length into a fill over an old marsh. During the erection of the building, serious settlement occurred, which continued after completion. Subsequent investigation showed a stratum of peat 1 ft to 7 ft in thickness, at a depth of 30 ft to 40 ft below the surface, subjected to a load of approximately 35 lb per sq in. Tests on sample cubes of the peat gave a compression of 35% for this load. As such a condition is obviously disastrous, a part of the foundation was underpinned with steel pipe piles, filled with concrete. It is scarcely necessary to emphasize that had the times been normal, test borings would have been taken before construction, and piles would have been selected of sufficient length to reach good bearing below the compressible peat.

Example 18.—The original borings for the 12-story building of the Westinghouse Company, in Philadelphia, Pa. (4*e*) showed 8 ft to 10 ft of fill underlaid by uniform clay and sand to rock at 45 ft. Conical concrete piles, 25 to 30 ft long, were driven. Before the building was completed, settlement was noted. Investigation then disclosed that the soils were composed of 27 ft of loose fill, 4 ft of peat, 1 ft of gravel, 15 ft of silted peat, 8 ft of silt, and 2 ft of sand and gravel on hard mica schist. However, the strata were not uniform in thickness and the compressible peat lay mostly under the east side of the building, where 4 in. of settlement occurred. The west

side of the building did not settle, again demonstrating that the thickness of the compressible sub-soils corresponds to the settlement. There appears to be but one answer to such a problem—underpinning. It appears certain, however, that if dependable test borings had been taken before construction, and the piles driven to the same bearing reached by the underpinning, no settlement would have occurred, and considerable money would have been saved.

Example 19.—At the League Island Navy Yard, in Philadelphia (*5d*), during the rush of the World War, two projects were constructed on concrete piles under fairly similar conditions as to soil. In one case, conical, cast-in-place concrete piles were used, and, in the other, the straight shaft, compressed, uncased, cast-in-place concrete pile with a bulb at the bottom. This Navy Yard is situated at the west end of the Horse Shoe Bend of the Delaware River. The eastern part of the reservation has been built up by an alluvial deposit of river silt and organic matter overlying an earlier deposit of coarse sand, gravel, and cobble. The sub-soil formations under the two structures, a concrete storehouse and a 175-ft chimney for the Naval Aircraft Factory, are described as follows: For the storehouse six borings showed silt to a depth from 35 ft to 55 ft below mean low water. Sand and gravel were found under the silt, sloping from a high point at the northwest corner of the building to the low point at the southeast corner (see Fig. 6(*a*)). For the stack, the sub-soils correspond roughly, except that the compressible silt is deeper under the stack than under the storehouse.



(*a*) CONTOURS OF TOTAL SETTLEMENT, TO FEB. 1922 (CONTOUR INTERVAL 0.1 FT)

(*b*) CAISSON IDENTIFICATION AND ANTICIPATED BOTTOM CONTOURS

FIG. 6.—AIRCRAFT STOREHOUSE, U. S. NAVY YARD, PHILADELPHIA, PA.

The storehouse is a six-story concrete building, divided into ten bays, east and west, and nine bays north and south. Under the column footings, cast-in-place conical concrete piles, 30 ft long, were driven, designed to

carry 30 tons each. In driving these piles, only a small number were driven to a resistance corresponding to that required by the design. It was discovered, however, that the resistance increased considerably after rest, and as this factor has been the salvation of many projects under apparently similar conditions, construction was continued.

In June, 1918, load tests on one pier, equivalent to that required by the design, were made. The pier settled 3 in. in 45 days, but the increments of settlement decreased rapidly after the first week. Check levels were taken weekly on all the columns during construction, which showed decreasing increments of settlement, even though the column loads were increasing as construction progressed.

Several months after completion, settlement cracks appeared in each floor. In January, 1919, the total settlement varied from 6 in. to 16 in., corresponding to the depth of the compressible silt below the pile-points, modified by the characteristic cup-shaped settlement. Note the similarity of the contours of Fig. 6(*a*) relating to settlement and Fig. 6(*b*) referring to depth of compressible material.

The storehouse was subsequently underpinned successfully, but, of course, at considerable cost. Had war conditions not existed, and had there been an ample supply, wooden piles of sufficient length to reach into the sand and gravel undoubtedly would have been used.

Example 20.—During the same period the 175-ft chimney was constructed. The foundation rested upon 64 cast-in-place, uncased concrete piles, 36 ft long, with a bulb at the bottom. The penetration of the piles during driving was steady, and the resistances, according to the *Engineering News* formula, indicated an average safe load of 130 tons.

The foundation of the chimney was built in 1918. In October, 1919, settlement was detected through the leaning of the chimney toward the power house on the north. The angle of leaning was reported as 9 on 1. Records of the increments of settlement showed that they were decreasing and that the movement would cease in time, if the structure were left alone. However, corrective measures were undertaken, consisting of the construction of a concrete ring foundation on 71 wooden piles, designed to carry 15 tons each, but driven to solid bottom through the compressible silt.

Example 21.—The settlement of the new Navy Building, at Washington, D. C. (5*e*) is another example of the use of piles too short to reach a substratum of sufficient bearing power to carry the loads. Conical concrete piles were driven through a recent fill over an old marsh. Upon the piles a head-house was constructed 200 ft in length, with rear wings. It settled a maximum of 4 in. The loads on the piles at the face and under the interior piers are reported as conservative, but subsequent analysis showed that where the wings meet the head-house, the piles under the piers were overloaded. The piers did not settle most, however. The greatest settlement occurred in the area of the greatest depth of fill where the settling of the fill itself carried down with it, the piles and the structure.

Example 22.—In 1905 an armory was built in Minneapolis, Minn. (4*f*). The site originally had been swampy ground, reclaimed by filling with sand

and gravel, so that, at the time construction began, the soils from the surface downward consisted of 15 to 20 ft of sand and gravel fill, 6 in. to 41 ft of mud on sand and gravel, with ground-water level at 1.5 ft below the pile cut-offs. Wooden piles 30 to 40 ft long were driven for the building, 189 ft by 218 ft, and some of them passed through the sand and gravel fill and through the mud into the hard sand and gravel sub-stratum. This was the case at the southeast corner. At the northwest corner, however, although the piles were driven through the sand and gravel fill and into the mud stratum, there was perhaps 10 ft or more of mud between the pile-points and the solid sand and gravel. A fairly good picture of sub-soil conditions may be had from the settlements that occurred over a period of several years. The southeast corner settled not at all, the southwest corner, 3 in., the northeast corner, 0.5 in., and the northwest corner, 37.5 in. As such unequal settlements are disastrous, in 1913 the west side of the building was condemned. It is probable that if test borings had been taken before construction began, and some extra money spent for longer piles, nothing would ever have been heard of the settling of the Minneapolis Armory.

Example 23.—The factory buildings of the Jurgens Oil Mill Works between the River Maas and the Dike at Zwyndrecht, Holland, were constructed on made land, consisting of sand fill, hydraulically placed. In 1915, the Jurgens Margarine Company (12) decided to erect on the site an oil mill to contain heavy machinery, tanks, etc. Since it was evident that piles would be necessary, test piles, 20 m (65.6 ft) long, were driven. The piles were driven to a resistance of 30 blows for the last 30 cm (11.8 in.), or theoretically, to a static load equivalent to 50 tons per pile. Inasmuch as the piles were to be loaded to only 5 tons each, there was apparently a factor of safety of 10. Creosoted wooden piles, 20 m (65.6 ft) long were driven. In fact, it appears from the accounts that longer piles could not have been placed under continuous driving, as they crushed at the heads under the driving resistance of 30 blows for the last 30 cm (11.8 in.). The soils through which the piles were driven were saturated and consisted of 4.5 m (14.8 ft) of sand fill, 13 m. (42.7 ft) of peat and some clay, and 5 m (16.7 ft) of fine sand on coarse sand and gravel. From this, it may be seen that the pile-points reached into, but did not penetrate, the stratum of fine saturated sand, and fell short of the substantial stratum of coarse sand and gravel. Had the piles been allowed a period of rest and had driving been resumed, it appears probable that longer piles might have been driven farther and into the sand and gravel.

In 1916, the factory was completed, and by 1920 the Oil Hardening Building had settled so badly that collapse was threatened. Near the center, the structure had settled 70 cm (27.6 in.). For most of the building the load per pile did not exceed 3 tons although, under the tower and tank, the loads per pile were as great as 18 tons. The external loading on the piles, however, must have had little to do with the settlement, as the maximum did not occur where the piles carried the heaviest external loads. In fact, the piles under an extension of the floor outside the building, carrying almost no loads, settled in the same manner.

The engineering report contained the statement that the hydraulic sand fill, with its high friction value, was settling into the bed of 17 m (55.8 ft) of compressible peat and clay. By its grip upon the piles the fill was pushing them through the peat of comparatively low friction value, and thrusting the points farther into the bed of 5 m (16.4 ft) of saturated fine sand which was incapable of carrying the loads. The load of the hydraulic sand fill was estimated at 15 tons per pile, which still would have left an ample factor of safety, had the dynamic pile-driving formula been of any value.

Conclusion

In the past, the design of pile foundations in plastic soils has, in the main, followed standards that do not take into consideration all of the many factors involved in these complicated problems. Although most of the serious settlements of foundations on piles have taken place in plastic soils even there the results have, in the large majority of cases, been surprisingly successful. At times, however, foundation settlements occur that are very expensive to the owner and are no credit to either the architect or contractor.

Even when the best of core borings have been taken, classification and predictions as to reactions of soils on sight by the average engineer is extremely difficult, if not impossible. Soils into which piles are driven range from those that heave and do not consolidate easily to those that subside and consolidate almost at once. Within this broad range occur mixtures and combinations in infinite variety, the reactions of which are further complicated by variations in the water content. Physical analyses of course, can develop the nature of the soils and the proper classification can be made, from which experience can usually predict the result. For the average or inexperienced foundation engineer, however, a more reliable answer can be had from a combination of core borings and pile-driving and loading tests in the field.

The engineer who depends upon some convenient tabulation of friction values, who expects a static test load upon a single pile multiplied by the number of piles in his foundation to equal the total load-carrying capacity. who relies upon the results of static load tests made too soon after driving, or who places complete dependence upon the *Engineering News* formula under all conditions, may be not a little surprised, at times, at the results.

Standard pile spacing as accepted in the United States is usually too close for the loads applied upon them when the piles are driven in plastic soils. If the proper spacing is used corresponding to the length, size, shape, roughness of the pile surface, load, and the soil characteristics, greater loads can be placed upon the piles with less settlement. Mathematical analyses for pile spacing and the distribution of loads to sub-strata would undoubtedly be an improvement over the usual practices; but group test loadings approximating the conditions of construction and final soil consolidation should prove more dependable. When this is done, the dynamic formulas may be regulated to a position of secondary importance.

Had a sufficient number of borings been taken over the sites, and had the borings been accompanied by clear descriptions of the sub-soils, it is reasonably certain that only a few of the foundation settlements described in this paper would have occurred, since the piles in most cases would have been driven to virtual refusal upon satisfactory sub-strata. However, where the piles have been driven short of sub-strata on the theory that friction piles would support a structure in spite of overloaded compressible strata beneath, it can only be stated that such a saving in first cost is very expensive. If any of the compressible strata are suspected of being peat, or combinations of peat, and the peat bed is of appreciable thickness, disregard of such a warning is to invite disaster.

Reliable core borings should always be made in sufficient number to disclose the nature of the sub-soils beneath the entire site. When it is discovered that that strata upon which the pile points are to rest are perhaps insufficient to carry the loads to be placed upon them, thorough analyses and pile-driving and loading tests should be made. In a word, convenient tabulations of values or standards should be viewed with distrust, because each pile foundation presents a new problem. No two are exactly alike.

Acknowledgments

For the furnishing of a report of the Whangpoo Conservancy Board, the writer wishes to acknowledge thanks to Dr. Herbert Chatley, Acting Engineer-in-Chief; for data published in Public Works of the Navy and for suggestions, to Captain George A. McKay (C.E.C.), U. S. N., M. Am. Soc. C. E.; and for reports of tests, and of the construction, of projects, to The Western Foundation Company, The Raymond Concrete Pile Company, and The MacArthur Concrete Pile Corporation; and for suggestions and criticisms, to Major H. P. Burrel, of the Western Foundation Company, and to A. E. Cummings, M. Am. Soc. C. E., of the Raymond Concrete Pile Company.

APPENDIX

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

Discussion

By Philip L. Inch, Assoc. M. Am. Soc. C. E.

Philip L. Inch,[28] Assoc. M. Am. Soc. C. E. (by letter).[28a]—The intent and purpose of Table 1 was to enable an engineer to obtain a meridian, within a limit of 1′ of accuracy in azimuth, with the least inconvenience or preparatory study, ever bearing in mind that thousands of surveyors are not equipped educationally to cope with trigonometric formulas, logarithms, etc.; nor are computing machines generally available. Although the discussion has revealed little on the subject of the form of the table, the writer believes that the methods discussed have enriched the general knowledge of the subject.

The writer approves the suggestion to eliminate the use of Greek symbols, however familiar they may be; he agrees that less confusion will result.

As to star observations: To observe the upper or lower culmination of Polaris, or to determine a meridian by an hour-angle observation require accuracies in time readings that are too great for those who are working to an accuracy of 1′. Furthermore, both these methods and elongation require night work unless an instrument is specially equipped for daylight observations.

The determination of a meridian by the equal altitude method is orthodox; but, again, the necessary preparation and the time consumed fits it inadequately to the needs of the field engineer.

The computation of the fifth decimal place of the table has received some comment. It will be noted that, in the tables of natural sines and

Note.—The paper by Philip L. Inch, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Earl F. Church, Paul E. Wylie, James B. Goodwin, C. H. Swick, Philip Kissam, and George D. Whitmore; December, 1936, by Messrs. O. H. Chilton, Chalmers C. Schrontz, Frank M. Johnson, Walter H. Starkweather, and C. I. Day; January, 1937, by Messrs. F. L. McRee, F. J. Duarte, and Leonard C Jordan; February, 1937, by Messrs. J. C. Pinney, R. L. Vaughn and John C. Penn; and March, 1937, by Messrs. Robert H. Merrill, Crosby J. Wilkin, and C. S. Jarvis.

[28] U. S. Cadastral Engr., Public Survey Office, Reno. Nev.

[28a] Received by the Secretary April 3, 1937.

cosines, a 1′ variation in either appears as from 1 to 3 in the fourth decimal place. In the determination of the fifth place, an even number for a 5 in the sixth place was maintained.

The discussion has included the advisability of computing the table of 1′ differences in A and B upon the arc, instead of one-sixtieth of the difference between two arc-determined, even degrees. Intensive investigation has demonstrated that such a refinement would add little or no increase in the general accuracy of the table. Before any one of the basic calculations was accepted, from two to six observations on the arc were computed and comparisons made. An experimental table was also constructed and comparisons were made of solar observations in latitudes as high as 60° north; the declinations used ranged from the highest to the lowest. The computation by table was found to be well within the required limits. For reference, the entire table has been included herein as Table 9. This form includes values of A and B for vertical angles, h, from 15° to 54°, inclusive; and latitude, ϕ, from 31° N to 48° N.

In the use of Table 9, the slide-rule is commended. The necessary multiplications are made quickly, with an advantage both in the assurance of accuracy and in the time consumed.

The solar attachment has been introduced into the discussion. The writer is of the opinion that the initial cost is not so prohibitive as the continuing cost of adjustment and maintaining the adjustment.

The suggestion that sun altitudes between 10° and 49° be used has its inconsistencies. The writer agrees that 49° is high enough, considering the factor of safety. When it is at an altitude of 10° the sun is moving vertically, at too rapid a rate to insure a good observation. Moreover, the refraction is large and variable. Above 49°, although the refraction element is nearing its minimum, the sun is moving horizontally at a comparative rate as great as it is moving vertically at minimum altitudes. Furthermore, the least error in the horizontal reading appears directly in the bearing that is sought.

The ideal time for sun observations is about three hours either before or after apparent noon. Experiments, however, have demonstrated that everything depends upon the accuracy with which the observation is taken and how nearly the personal equation can be eliminated. At certain seasons of the year excellent observations can be obtained as early or as late as five hours before or after apparent noon. The freedom from personal inaccuracies is the paramount issue, not the time nor the computation by any particular accepted method.

A suggestion for solar observation is as follows: Reduce the use of the tangent screws to a minimum. Hold a limb of the sun tangent to one hair of the telescope with the tangent screw and allow the sun to drift into tangency with the other selected hair. Personally, the writer holds the faster moving limb of the sun tangent with the tangent screw. A little practice and one's judgment of the sun's movement can be placed

within a 2-sec interim. In an observation in which all four quadrants are used, the perfect co-ordination of two hands and an eye at eight different times is rare.

In the discussion, reference is made to the additional hairs that can be placed in a telescope to facilitate a solar observation. They enable the sun to be centered approximately at the intersection of the horizontal and vertical hairs. This arrangement gives excellent results; but few instruments are equipped with this accessory.

From the viewpoint of a field engineer, the most needed addition to the field procedure is some check on the accuracy of a solar observation in the field, and at the time of observation. Fig 4 will suggest a method of observation that will afford an excellent check upon the accuracy of each complete observation, by comparison with the others in any group. Observe the sun in this manner, recording the time, the horizontal angle, and the vertical angle. Combine the observations, respectively; thus: $\frac{1+8}{2}$, $\frac{2+7}{2}$, $\frac{3+6}{2}$, and $\frac{4+5}{2}$.

In theory, and in theory alone, the mean time, the mean horizontal angle, and the mean vertical angle of each complete set will be equal; one's judgment, together with the degree of accuracy required, should be one's best guide as to the rejection or acceptance of any set. One good observation is better than the mean of one good one and three poor ones. Observations taken 1 min apart, with 2 min between the series of direct and reversed readings, give very satisfactory results.

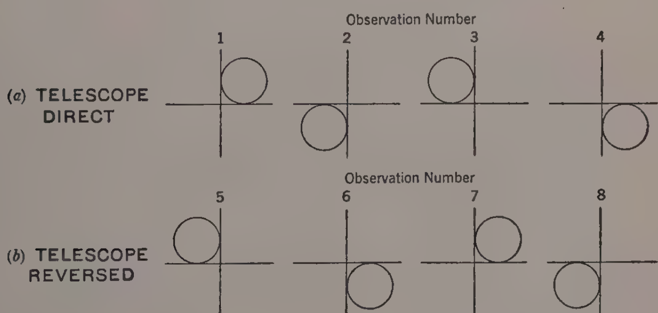

FIG. 4.

The ease of computing a solar observation by means of simple formulas, logarithms, and computing machines is apparent. The writer approves of their use, but only in skilled hands. The selection of a solar observation taken in 1910, as an example, has been commented upon; it was taken from the Manual of Instructions for the Survey of Public Lands of the United States, 1930 (page 112); it represents a criterion.

The writer wishes to acknowledge and to express his appreciation of the many interesting and valuable contributions to the discussion.

TABLE 9.—VALUES OF FACTORS A AND B, FOR A GIVEN ALTITUDE OF THE SUN, AT A GIVEN LATITUDE ON THE EARTH

(Including corrections for parallax and refraction)

Vertical angle, h	Symbol	Latitude, $\phi = 31°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ	Latitude, $\phi = 32°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ	Latitude, $\phi = 33°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ
15°.......	A	1.20746	9.8	21.7	1.22045	9.9	22.7	1.23409	10.0	23.9
	B	0.16036	18.9	10.7	0.16677	19.6	10.9	0.17331	20.4	11.2
16°.......	A	1.21331	10.3	21.8	1.22637	10.6	22.9	1.24008	10.7	24.0
	B	0.17168	19.1	11.4	0.17854	19.9	11.7	0.18556	20.6	12.0
17°.......	A	1.21961	11.2	21.9	1.23273	11.3	23.0	1.24651	11.5	24.1
	B	0.18313	19.3	12.2	0.19045	20.0	12.5	0.19793	20.8	13.1
18°.......	A	1.22634	12.0	22.0	1.23953	12.1	23.1	1.25338	12.3	24.3
	B	0.19468	19.5	13.0	0.20246	20.3	13.3	0.21042	21.0	13.6
19°.......	A	1.23352	12.8	22.1	1.24679	12.9	23.2	1.26073	13.0	24.4
	B	0.20637	19.7	13.7	0.21461	20.5	14.1	0.22304	21.3	14.4
20°.......	A	1.24118	13.5	22.3	1.25453	13.7	23.4	1.26855	13.8	24.6
	B	0.21820	20.0	14.5	0.22692	20.8	14.9	0.23583	21.6	15.0
21°.......	A	1.24929	14.4	22.4	1.26272	14.6	23.5	1.27684	14.7	24.7
	B	0.23017	20.2	15.3	0.23937	21.0	15.7	0.24877	21.9	16.0
22°.......	A	1.25792	15.2	22.6	1.27145	15.3	23.7	1.28566	15.6	24.9
	B	0.24231	20.5	16.1	0.25199	21.3	16.5	0.26189	22.2	16.9
23°.......	A	1.26705	16.1	22.7	1.28068	16.3	23.9	1.29500	16.4	25.1
	B	0.25461	20.8	17.0	0.26479	21.6	17.3	0.27519	22.5	17.7
24°.......	A	1.27670	17.1	22.9	1.29044	17.3	24.0	1.30486	17.5	25.1
	B	0.26709	21.1	17.8	0.27777	22.0	18.2	0.28868	22.8	18.6
25°.......	A	1.28694	17.9	23.1	1.30079	18.0	24.2	1.31533	18.2	25.5
	B	0.27977	21.5	18.6	0.29095	22.3	19.1	0.30238	23.2	19.5
26°.......	A	1.29765	18.9	23.3	1.31161	19.1	24.4	1.32627	19.4	25.7
	B	0.29265	21.9	19.5	0.30435	22.7	19.9	0.31630	23.6	20.4
27°.......	A	1.30901	19.9	23.5	1.32309	20.1	24.7	1.33788	20.4	25.9
	B	0.30577	22.2	20.4	0.31798	23.1	20.8	0.33047	24.0	21.3
28°.......	A	1.32095	21.0	23.7	1.33516	21.2	24.9	1.35009	21.4	26.1
	B	0.31910	22.7	21.3	0.33186	23.6	21.7	0.34489	24.5	22.2
29°.......	A	1.33352	22.0	23.9	1.34786	22.3	25.1	1.36293	22.4	26.4
	B	0.33270	23.1	22.2	0.34599	24.0	22.7	0.35958	25.0	23.2
30°.......	A	1.34676	23.2	24.2	1.36125	23.4	25.0	1.37646	23.7	26.7
	B	0.34655	23.6	23.1	0.36039	24.5	23.6	0.37455	25.5	24.1
31°.......	A	1.36068	24.4	24.4	1.37531	24.7	25.6	1.39069	24.9	26.9
	B	0.36068	24.1	24.0	0.37509	25.1	24.6	0.38982	26.0	25.1

Vertical angle, h	Symbol	Latitude, $\phi = 34°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ	Latitude, $\phi = 35°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ	Latitude, $\phi = 36°$ Factor	Differences for 1′ Vert. angle, h	Differences for 1′ ϕ
15°.......	A	1.24843	10.1	25.1	1.26351	10.2	26.4	1.27932	10.4	27.7
	B	0.18001	21.2	11.4	0.18687	22.0	11.7	0.19390	22.8	12.1
16°.......	A	1.25448	10.8	25.3	1.26964	11.0	26.5	1.28553	11.1	27.9
	B	0.19273	21.4	12.2	0.20007	22.2	12.5	0.20759	23.1	12.9
17°.......	A	1.26098	11.6	25.4	1.27622	11.7	26.6	1.29219	11.9	28.0
	B	0.20558	21.6	13.1	0.21341	22.5	13.4	0.22143	23.3	13.7
18°.......	A	1.26794	12.4	25.5	1.28326	12.5	26.8	1.29932	12.7	28.2
	B	0.21855	21.8	13.9	0.22688	22.7	14.2	0.23541	23.5	14.6
19°.......	A	1.27537	13.2	25.7	1.29078	13.4	26.9	1.30694	13.5	28.3
	B	0.23166	22.1	14.7	0.24049	23.0	15.1	0.24953	23.8	15.5
20°.......	A	1.28329	14.0	25.8	1.29879	14.1	27.1	1.31504	14.3	28.5
	B	0.24494	22.4	15.6	0.25427	23.3	16.0	0.26384	24.1	16.3
21°.......	A	1.29167	14.9	26.0	1.30727	15.1	27.3	1.32363	15.3	28.7
	B	0.25838	22.7	16.4	0.26823	23.6	16.8	0.27832	24.5	17.2
22°.......	A	1.30060	15.7	26.2	1.31631	15.9	27.5	1.33278	16.1	28.9
	B	0.27201	23.0	17.3	0.28237	23.9	17.7	0.29299	24.8	18.2
23°.......	A	1.31004	16.6	26.4	1.32586	16.8	27.7	1.34246	17.0	29.1
	B	0.28582	23.4	18.2	0.29671	24.3	18.6	0.30787	25.2	19.1
24°.......	A	1.32002	17.7	26.6	1.33596	17.9	27.9	1.35268	18.1	29.3
	B	0.29983	23.7	19.1	0.31126	24.6	19.5	0.32296	25.6	20.0
25°.......	A	1.33061	18.5	26.8	1.34668	18.7	28.1	1.36353	18.9	29.6
	B	0.31407	24.1	19.9	0.32603	25.0	20.4	0.33829	26.0	21.0
26°.......	A	1.34168	19.6	27.0	1.35789	19.8	28.3	1.37488	20.1	29.8
	B	0.32853	24.5	20.9	0.34104	25.6	21.4	0.35387	26.4	21.9
27°.......	A	1.35342	20.6	27.2	1.36976	20.8	28.6	1.38691	21.1	30.1
	B	0.34324	25.0	21.8	0.35632	25.9	22.3	0.36972	26.9	22.9
28°.......	A	1.36577	21.7	27.5	1.38226	21.9	28.9	1.39957	22.2	30.3
	B	0.35822	25.4	22.8	0.37187	26.4	23.3	0.38585	27.4	23.9
29°.......	A	1.37876	22.8	27.8	1.39541	23.1	29.1	1.41288	23.4	30.6
	B	0.37348	25.9	23.7	0.38771	26.9	24.3	0.40229	27.9	24.9
30°.......	A	1.39245	24.0	28.0	1.40927	24.3	29.4	1.42691	24.6	30.9
	B	0.38902	26.5	24.7	0.40385	27.5	25.3	0.41903	28.5	26.0
31°.......	A	1.40684	25.2	28.3	1.42383	25.5	29.7	1.44165	25.9	31.3
	B	0.40489	27.0	25.7	0.42032	28.0	26.3	0.43612	29.1	27.00

32°.......	A	1.37532	25.6	24.7	1.39011	25.9	25.9	1.40565	26.2	27.2	1.42198	26.5	28.6	1.43915	26.8	30.0	1.45717	27.1	31.6
	B	0.37511	24.6	25.0	0.39010	25.6	25.5	0.40542	26.6	26.1	0.42109	27.6	26.7	0.43713	28.6	27.4	0.45357	29.7	28.1
33°.......	A	1.39068	27.0	24.9	1.40564	27.2	26.0	1.42135	27.6	27.5	1.43786	27.9	29.0	1.45523	28.2	30.4	1.47345	28.6	31.9
	B	0.38986	25.2	26.0	0.40544	26.2	26.5	0.42136	27.2	27.2	0.43765	28.2	27.8	0.45432	29.3	28.5	0.47141	30.4	29.2
34°.......	A	1.40685	28.2	25.2	1.42198	28.5	26.5	1.43788	28.8	27.8	1.45458	29.1	29.3	1.47215	29.5	30.7	1.49058	29.9	32.3
	B	0.40496	25.7	27.0	0.42114	26.8	27.6	0.43768	27.8	28.2	0.45459	28.9	28.9	0.47191	30.0	29.6	0.48966	31.1	30.3
35°.......	A	1.42376	29.8	25.5	1.43907	30.2	26.8	1.45516	30.5	28.1	1.47206	30.9	29.6	1.48984	31.2	31.1	1.50849	31.6	32.7
	B	0.42040	26.4	28.0	0.43720	27.4	28.6	0.45437	28.5	29.3	0.47193	29.6	30.0	0.48991	30.7	30.7	0.50833	31.9	31.5
36°.......	A	1.44166	31.3	25.9	1.45717	31.6	27.2	1.47346	31.9	28.5	1.49057	32.3	30.0	1.50858	32.7	31.5	1.52746	33.1	33.1
	B	0.43622	27.1	29.1	0.45366	28.1	29.7	0.47147	29.3	30.4	0.48969	30.4	31.1	0.50835	31.5	31.9	0.52747	32.7	32.7
37°.......	A	1.46041	32.8	26.2	1.47612	33.2	27.5	1.49262	33.5	28.9	1.50996	33.9	30.4	1.52820	34.3	31.9	1.54732	34.8	33.6
	B	0.45246	27.8	30.1	0.47054	28.9	30.8	0.48902	30.0	31.5	0.50792	31.2	32.3	0.52727	32.4	33.1	0.54710	33.6	33.9
38°.......	A	1.48009	34.5	26.5	1.49601	34.9	27.9	1.51274	35.3	29.3	1.53031	35.7	30.8	1.54879	36.1	32.3	1.56818	36.5	34.0
	B	0.46913	28.5	31.2	0.48787	29.7	31.9	0.50703	30.9	32.7	0.52663	32.0	33.5	0.54670	33.3	34.3	0.56725	34.5	35.2
39°.......	A	1.50079	36.4	26.9	1.51693	36.8	28.3	1.53389	37.2	29.7	1.55170	37.6	31.3	1.57045	38.1	32.8	1.59010	38.6	34.5
	B	0.48625	29.4	32.4	0.50568	30.6	33.1	0.52554	31.8	33.9	0.54585	33.0	34.7	0.56665	34.2	35.5	0.58796	35.5	36.4
40°.......	A	1.52262	38.0	27.3	1.53899	38.4	28.7	1.55620	38.8	30.1	1.57427	39.3	31.7	1.59329	39.8	33.2	1.61323	40.3	35.0
	B	0.50388	30.2	33.6	0.52401	31.4	34.3	0.54459	32.7	35.1	0.56564	33.9	35.9	0.58719	35.2	36.8	0.60927	36.6	37.8
41°.......	A	1.54541	40.1	27.7	1.56204	40.5	29.1	1.57950	41.0	30.6	1.59784	41.4	32.2	1.61714	41.9	33.7	1.63738	42.5	35.5
	B	0.52202	31.3	34.8	0.54287	32.4	35.5	0.56419	33.7	36.4	0.58600	35.0	37.2	0.60833	36.3	38.1	0.63120	37.7	39.1
42°.......	A	1.56946	42.2	28.1	1.58634	42.7	29.6	1.60407	43.1	31.1	1.62270	43.6	32.7	1.64230	44.2	34.2	1.66286	44.7	36.1
	B	0.54071	32.2	36.0	0.56232	33.5	36.8	0.58440	34.8	37.7	0.60699	36.1	38.4	0.63012	37.5	39.5	0.65381	38.9	40.5
43°.......	A	1.59477	44.2	28.6	1.61193	44.9	30.0	1.62994	45.4	31.6	1.64888	45.9	33.2	1.66879	46.5	34.8	1.68968	47.0	36.6
	B	0.56001	33.2	37.3	0.58239	34.6	38.1	0.60526	35.9	39.0	0.62866	37.3	39.9	0.65261	38.7	40.9	0.67715	40.2	42.0
44°.......	A	1.62141	46.7	29.1	1.63885	47.3	30.5	1.65717	47.8	32.1	1.67641	48.3	33.8	1.69666	48.9	35.4	1.71790	49.5	37.3
	B	0.57995	34.4	38.6	0.60312	35.7	39.5	0.62681	37.1	40.4	0.65104	38.6	41.3	0.67584	40.0	42.4	0.70126	41.5	43.5
45°.......	A	1.64945	49.3	29.6	1.66720	49.8	31.1	1.68583	50.4	32.6	1.70541	51.0	34.3	1.72601	51.6	36.0	1.74762	52.2	37.9
	B	0.60056	36.6	40.0	0.62456	37.0	40.9	0.64909	38.5	41.8	0.67417	40.0	42.8	0.69986	41.5	43.9	0.72618	43.0	45.0
46°.......	A	1.67901	52.0	30.1	1.69708	52.5	31.6	1.71604	53.1	33.2	1.73598	53.7	35.0	1.75695	54.4	36.7	1.77894	55.1	38.6
	B	0.62191	36.9	41.4	0.64676	38.4	42.3	0.67216	39.9	43.3	0.69814	41.4	44.3	0.72474	43.0	45.4	0.75199	44.6	46.6
47°.......	A	1.71019	54.8	30.7	1.72858	55.4	32.2	1.74790	56.0	33.9	1.76821	56.7	35.6	1.78957	57.3	37.3	1.81197	58.1	39.3
	B	0.64404	38.3	42.7	0.66978	39.8	43.8	0.69608	41.4	44.9	0.72299	43.0	45.9	0.75053	44.6	47.1	0.77876	46.3	48.3
48°.......	A	1.74306	57.9	31.3	1.76181	58.5	32.8	1.78151	59.1	34.5	1.80220	59.9	36.3	1.82397	60.6	38.1	1.84680	61.3	40.0
	B	0.66701	39.7	44.4	0.69367	41.4	45.4	0.72091	43.0	46.4	0.74877	44.7	47.6	0.77730	46.4	48.7	0.80653	48.2	49.7
49°.......	A	1.77779	61.0	31.9	1.79692	61.9	33.5	1.81700	62.6	35.2	1.83811	63.3	37.0	1.86031	64.3	38.8	1.88359	64.8	40.9
	B	0.69090	41.5	46.0	0.71851	43.1	47.0	0.74673	44.8	48.1	0.77559	46.5	49.2	0.80514	48.3	50.5	0.83542	50.1	51.8
50°.......	A	1.81451	64.7	32.5	1.83403	65.4	34.2	1.85453	66.2	35.9	1.87607	68.6	37.8	1.89873	67.8	39.6	1.92249	68.6	41.7
	B	0.71577	43.2	47.7	0.74438	44.9	48.7	0.77361	46.7	49.8	0.80351	48.5	51.0	0.83413	50.3	52.3	0.86549	52.2	53.6
51°.......	A	1.85334	68.4	33.2	1.87328	69.3	34.9	1.89422	70.0	36.7	1.91622	70.9	38.6	1.93937	71.7	40.6	1.96364	72.6	42.6
	B	0.74170	45.1	49.4	0.77133	46.9	50.5	0.80163	48.7	51.6	0.83261	50.6	52.9	0.86433	52.6	54.2	0.89683	54.6	55.6
52°.......	A	1.89446	72.6	34.0	1.91484	73.4	35.7	1.93624	74.2	37.5	1.95873	75.1	39.4	1.98239	76.0	41.4	2.00720	77.0	43.5
	B	0.76876	47.2	51.2	0.79948	49.0	52.3	0.83087	51.0	53.5	0.86299	52.9	54.8	0.89587	55.0	56.2	0.92956	57.0	57.6
53°.......	A	1.93803	77.1	34.8	1.95888	77.9	36.5	1.98077	78.8	38.4	2.00378	79.7	40.3	2.02798	80.7	42.3	2.05337	81.7	44.5
	B	0.79705	49.3	53.1	0.82890	51.4	54.4	0.86146	53.4	55.5	0.89475	55.5	56.8	0.92884	57.6	58.2	0.96377	59.8	59.7
54°.......	A	1.98430	82.0	35.6	2.00564	82.9	39.0	2.02806	83.8	39.4	2.05162	84.8	41.6	2.07640	85.8	43.3	2.10239	86.9	45.6
	B	0.82670	51.9	55.1	0.85973	54.0	56.3	0.89350	56.1	57.6	0.92803	58.3	58.9	0.96339	60.5	60.4	0.99962	62.8	61.9

TABLE 9.—(*Continued.*)

Vertical angle, h	Symbol	Latitude, $\phi = 37°$			Latitude, $\phi = 38°$			Latitude, $\phi = 39°$			Latitude, $\phi = 40°$			Latitude, $\phi = 41°$			Latitude, $\phi = 42°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
			Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ
15°.......	A	1.29596	10.5	29.2	1.31344	10.6	30.6	1.33180	10.6	32.2	1.35110	10.9	33.8	1.37139	11.1	35.6	1.39273	11.3	37.4
	B	0.20111	23.7	12.3	0.20851	24.6	12.7	0.21611	25.5	13.1	0.22394	26.4	13.4	0.23200	27.3	13.8	0.24030	28.3	14.3
16°.......	A	1.30225	11.3	29.3	1.31981	11.4	30.8	1.33826	11.6	32.3	1.35765	11.7	34.0	1.37803	11.9	35.8	1.39948	12.1	37.6
	B	0.21531	23.9	13.2	0.22324	24.8	13.6	0.23138	25.7	14.0	0.23976	26.6	14.4	0.24838	27.6	14.8	0.25727	28.6	15.3
17°.......	A	1.30900	12.0	29.4	1.32665	12.2	30.9	1.34519	12.4	32.5	1.36469	12.6	34.2	1.38518	12.7	35.9	1.40674	12.9	37.8
	B	0.22967	24.2	14.1	0.23812	25.1	14.5	0.24680	26.0	14.9	0.25574	26.9	15.3	0.26494	27.9	15.8	0.27442	28.9	16.3
18°.......	A	1.31622	12.9	29.6	1.33397	13.0	31.1	1.35262	13.2	32.7	1.37222	13.4	34.3	1.39282	13.6	36.0	1.41450	13.8	38.0
	B	0.24416	24.4	15.0	0.25315	25.3	15.4	0.26238	26.2	15.8	0.27188	27.2	16.3	0.28166	28.2	16.8	0.29174	29.2	17.4
19°.......	A	1.32394	13.7	29.8	1.34179	13.9	31.3	1.36054	14.1	32.9	1.38026	14.3	34.6	1.40099	14.5	36.3	1.42279	14.7	38.2
	B	0.25881	24.7	15.9	0.26833	25.7	16.3	0.27812	26.6	16.8	0.28819	27.5	17.3	0.29856	28.5	17.8	0.30924	29.6	18.4
20°.......	A	1.33215	14.5	29.9	1.35011	14.5	31.5	1.36898	14.9	33.1	1.38883	15.1	34.8	1.40968	15.4	36.6	1.43162	15.6	38.5
	B	0.27364	25.0	16.8	0.28372	26.0	17.2	0.29406	26.9	17.8	0.30471	27.9	18.3	0.31567	28.9	18.6	0.32697	29.9	19.5
21°.......	A	1.34085	15.5	30.1	1.35893	15.7	31.7	1.37793	15.9	33.3	1.39790	16.1	35.0	1.41889	16.3	36.8	1.44097	16.6	38.7
	B	0.28866	25.3	17.7	0.29929	26.3	18.2	0.31020	27.3	18.7	0.32143	28.3	19.3	0.33300	29.3	19.9	0.34492	30.3	20.5
22°.......	A	1.35012	16.3	30.3	1.36832	16.6	31.9	1.38745	16.8	33.5	1.40756	17.0	35.2	1.42869	17.3	37.1	1.45093	17.6	39.0
	B	0.30388	25.7	18.7	0.31507	26.7	19.2	0.32656	27.6	19.7	0.33838	28.7	20.3	0.35056	29.7	20.9	0.36310	30.7	21.6
23°.......	A	1.35992	17.3	30.6	1.37826	17.5	32.1	1.39752	17.8	33.8	1.41778	18.0	35.5	1.43907	18.3	37.3	1.46146	18.6	39.3
	B	0 31932	26.1	19.6	0.33106	27.2	20.2	0.34314	28.0	20.7	0.35557	29.1	21.3	0.36836	30.1	22.0	0.38154	31.2	22.7
24°.......	A	1.37028	18.3	30.8	1.38875	18.6	32.4	1.40817	18.8	34.0	1.42858	19.1	35.7	1.45002	19.4	37.6	1.47259	19.7	39.6
	B	0.33497	26.5	20.6	0.34730	27.5	21.1	0.35996	28.5	21.7	0.37300	29.4	22.4	0.38642	30.6	23.1	0.40025	31.7	23.8
25°.......	A	1.38127	19.2	31.0	1.39989	19.4	32.6	1.41946	19.7	34.4	1.44004	20.0	36.0	1.46166	20.3	37.9	1.48440	20.6	39.9
	B	0.35087	26.9	21.5	0.36378	27.9	22.1	0.37705	28.9	22.8	0.39070	30.0	23.4	0.40476	31.1	24.1	0.41924	32.2	24.9
26°.......	A	1.39277	20.3	31.3	1.41154	20.6	32.9	1.43128	20.9	34.6	1.45202	21.2	36.3	1.47382	21.5	38.2	1.49676	21.8	40.2
	B	0.36702	27.4	22.5	0.38054	28.4	23.1	0.39441	29.5	23.8	0.40869	30.5	24.5	0.42340	31.6	25.3	0.43855	32.8	26.1
27°.......	A	1.40495	21.4	31.6	1.42389	21.7	33.2	1.44380	22.0	34.9	1.46472	22.3	36.7	1.48671	22.6	38.6	1.50985	23.0	40.6
	B	0.38347	27.9	23.5	0.39758	28.9	24.2	0.41208	30.0	24.9	0.42700	31.1	25.6	0.44236	32.2	26.4	0.45820	33.3	27.2
28°.......	A	1.41777	22.5	31.9	1.43689	22.6	33.5	1.45697	23.1	35.2	1.47809	23.4	37.0	1.50028	23.8	38.9	1.52363	24.2	41.0
	B	0.40020	28.4	24.6	0.41493	29.5	25.2	0.43006	30.5	26.0	0.44563	31.6	26.7	0.46166	32.8	27.5	0.47818	34.0	28.4
29°.......	A	1.43126	23.7	32.2	1.45055	24.0	33.8	1.47083	24.4	35.5	1.49215	24.7	37.3	1.51455	25.1	39.3	1.53812	25.5	41.4
	B	0.41724	29.0	25.6	0.43260	30.0	26.3	0.44838	31.1	27.1	0.46461	32.2	27.9	0.48133	33.4	28.7	0.49855	34.6	29.7
30°.......	A	1.44547	24.9	32.5	1.46496	25.2	34.1	1.48544	25.4	35.9	1.50697	26.0	37.7	1.52959	26.4	39.7	1.55340	26.8	41.8
	B	0.43461	29.5	26.7	0.45061	30.6	27.4	0.46704	31.8	28.1	0.48395	32.9	29.0	0.50136	34.1	29.9	0.51931	35.3	30.9
31°.......	A	1.46041	26.2	32.8	1.48010	26.5	34.5	1.50079	26.9	36.3	1.52254	27.3	38.1	1.54540	27.7	40.1	1.56945	28.2	42.2
	B	0.45233	30.1	27.8	0.46898	31.3	28.5	0.48609	32.4	29.3	0.50369	33.6	30.2	0.52181	34.8	31.1	0.54048	36.1	32.1

32°.......	A	1.47612	27.5	33.2	1.49602	27.5	34.9	1.51694	28.2	36.6	1.53892	28.7	38.5	1.56203	29.1	40.5	1.58634	29.5	42.6
	B	0.47043	30.8	28.9	0.48775	32.0	29.7	0.50554	33.1	30.5	0.52384	34.3	31.4	0.54269	35.6	32.4	0.56211	36.9	33.4
33°.......	A	1.49261	28.9	33.6	1.51274	29.3	35.2	1.53388	29.7	37.1	1.55611	30.2	39.0	1.57948	30.6	41.0	1.60406	31.1	43.1
	B	0.48893	31.2	30.0	0.50693	32.7	30.8	0.52542	33.9	31.7	0.54444	35.1	32.7	0.56403	36.4	33.7	0.58422	37.7	34.7
34°.......	A	1.50997	30.2	33.9	1.53032	30.7	35.7	1.55172	31.1	37.5	1.57421	31.5	39.4	1.59784	32.0	41.5	1.62271	32.5	43.6
	B	0.50786	32.3	31.2	0.52656	33.5	32.0	0.54576	34.7	32.9	0.56552	35.9	33.9	0.58587	37.2	34.9	0.60683	38.6	36.1
35°.......	A	1.52811	32.0	34.3	1.54871	32.5	36.1	1.57036	32.9	37.9	1.59312	33.4	39.9	1.61704	33.9	42.0	1.64221	34.4	44.1
	B	0.52723	33.1	32.4	0.54664	34.3	33.2	0.56657	35.6	34.2	0.58708	36.9	35.2	0.60821	38.2	36.3	0.62997	39.5	37.5
36°.......	A	1.54733	33.5	34.8	1.56819	34.0	36.5	1.59011	34.5	38.4	1.61316	34.9	40.4	1.63738	35.5	42.5	1.66286	36.1	44.7
	B	0.54708	33.9	33.6	0.56722	35.2	34.5	0.58790	36.5	35.5	0.60919	37.8	36.5	0.63110	39.2	37.7	0.65369	40.6	38.9
37°.......	A	1.56745	35.2	35.2	1.58858	35.7	37.0	1.61079	36.2	38.9	1.63414	36.7	40.9	1.65867	37.3	43.0	1.68449	37.8	45.3
	B	0.56744	34.5	34.8	0.58833	36.1	35.8	0.60978	37.4	36.8	0.63186	38.8	37.9	0.65459	40.2	39.1	0.67802	41.6	40.3
38°.......	A	1.58858	37.0	35.7	1.60999	37.5	37.5	1.63250	38.2	39.3	1.65616	38.6	41.5	1.68103	39.2	43.6	1.70719	39.7	45.9
	B	0.58834	35.8	36.1	0.61000	37.1	37.1	0.63224	38.5	38.2	0.65515	39.9	39.3	0.67871	41.3	40.5	0.70300	42.8	41.8
39°.......	A	1.61079	39.1	36.2	1.63251	39.6	38.0	1.65533	40.1	40.0	1.67932	40.7	42.0	1.70453	41.3	44.1	1.73101	42.1	46.6
	B	0.60982	36.8	37.4	0.63227	38.2	38.4	0.65532	39.6	39.6	0.67905	41.0	40.7	0.70348	42.5	42.0	0.72866	44.1	43.3
40°.......	A	1.63422	40.8	36.7	1.65625	41.3	38.6	1.67940	41.9	40.4	1.70374	42.5	42.6	1.72932	43.2	44.9	1.75624	43.8	47.2
	B	0.63192	37.9	38.8	0.65518	39.3	39.8	0.67907	40.8	41.0	0.70366	42.2	42.2	0.72898	43.7	43.5	0.75507	45.3	44.9
41°.......	A	1.65868	43.1	37.3	1.68105	43.6	39.2	1.70455	44.2	41.2	1.72925	44.9	43.3	1.75521	45.5	45.5	1.78253	46.2	47.9
	B	0.65467	39.1	40.2	0.67877	40.5	41.3	0.70352	42.0	42.5	0.72899	43.5	43.7	0.75522	45.0	45.1	0.78225	46.7	46.7
42°.......	A	1.68449	45.3	38.0	1.70720	45.9	40.0	1.73107	46.2	41.8	1.75616	47.2	44.1	1.78252	47.9	46.2	1.81026	48.7	48.5
	B	0.67812	40.3	41.3	0.70308	41.8	42.7	0.72872	43.4	44.0	0.75511	44.9	45.3	0.78227	46.6	46.7	0.81027	48.2	48.2
43°.......	A	1.71166	47.7	38.5	1.73474	48.3	40.4	1.75899	49.0	42.5	1.78448	49.7	44.7	1.81128	50.4	47.0	1.83946	51.2	49.5
	B	0.70232	41.7	43.1	0.72818	43.2	44.3	0.75473	44.8	45.6	0.78206	46.4	46.9	0.81020	48.1	48.3	0.83919	49.8	49.9
44°.......	A	1.74025	50.2	39.1	1.76371	50.7	41.1	1.78836	51.6	43.2	1.81428	52.3	45.6	1.84153	53.1	47.8	1.87018	53.9	50.3
	B	0.72733	43.1	44.6	0.75410	44.7	45.8	0.78160	46.3	47.2	0.80990	48.0	48.6	0.83904	49.7	50.1	0.86907	51.5	51.7
45°.......	A	1.77035	52.9	39.8	1.79422	53.6	41.8	1.81930	54.3	44.0	1.84567	55.1	46.2	1.87338	56.0	48.6	1.90253	56.8	51.1
	B	0.75317	44.6	46.2	0.78090	46.3	47.5	0.80938	48.0	48.8	0.83868	49.7	50.3	0.86886	51.5	51.8	0.89995	53.3	53.5
46°.......	A	1.80208	55.8	40.5	1.82637	56.5	42.6	1.85190	57.3	44.8	1.87875	58.1	47.0	1.90695	59.0	49.5	1.93663	59.9	52.1
	B	0.77995	46.3	47.9	0.80866	48.0	49.2	0.83815	49.7	50.6	0.86849	51.5	52.1	0.89974	53.4	53.7	0.93194	55.3	55.4
47°.......	A	1.83554	58.8	41.2	1.86028	59.6	43.4	1.88629	60.4	45.6	1.91363	61.3	47.9	1.94236	62.2	50.4	1.97259	63.2	53.0
	B	0.80771	48.0	49.6	0.83744	49.8	50.9	0.86798	51.6	52.4	0.89941	53.5	53.9	0.93177	55.4	55.6	0.96511	57.4	57.4
48°.......	A	1.87082	62.1	42.0	1.89604	63.1	44.2	1.92255	63.8	46.4	1.95041	64.8	48.8	1.97970	65.7	51.4	2.01051	66.8	54.0
	B	0.83652	49.9	51.2	0.86731	51.8	52.7	0.89894	53.7	54.2	0.93148	55.6	55.9	0.96500	57.6	57.6	0.99953	59.7	59.4
49°.......	A	1.90810	65.7	42.9	1.93382	64.9	45.1	1.96085	67.5	47.4	1.98928	68.5	49.8	2.01914	69.5	52.4	2.05057	70.6	55.1
	B	0.86648	52.0	53.2	0.89837	53.9	54.6	0.93113	55.9	56.2	0.96485	57.9	57.9	0.99956	60.0	59.6	1.03533	62.1	61.6
50°.......	A	1.94750	69.5	43.8	1.97376	70.4	46.0	2.00135	71.4	48.3	2.03036	72.4	50.8	2.06084	73.5	53.5	2.09291	74.7	56.3
	B	0.89767	54.2	55.1	0.93071	56.2	56.6	0.96465	58.2	58.2	0.99958	60.3	59.9	1.03554	62.5	61.8	1.07260	64.7	63.8
51°.......	A	1.98919	73.5	44.7	2.01600	74.5	47.0	2.04419	75.6	49.4	2.07381	76.7	51.9	2.10495	77.8	54.6	2.13771	79.0	57.5
	B	0.93017	56.6	57.1	0.96442	58.7	58.6	0.99958	60.8	60.3	1.03578	63.0	62.1	1.07304	65.3	64.0	1.11144	67.6	66.1
52°.......	A	2.03331	78.0	45.7	2.06072	79.0	48.0	2.08953	80.1	50.5	2.11982	81.3	53.0	2.15164	82.5	55.8	2.18513	83.8	58.7
	B	0.96411	59.2	59.2	0.99961	61.3	60.8	1.03606	63.6	62.5	1.07357	65.9	64.4	1.11220	68.2	66.3	1.15200	70.7	68.5
53°.......	A	2.08008	82.8	46.7	2.10812	83.9	49.1	2.13759	85.1	51.6	2.16857	86.3	54.3	2.20113	87.6	57.1	2.23539	89.0	60.1
	B	0.99960	62.6	61.3	1.03640	64.3	62.9	1.07419	66.6	64.8	1.11308	69.0	66.8	1.15313	71.5	68.8	1.19440	74.0	71.0
54°.......	A	2.12974	87.9	47.9	2.15845	89.1	50.3	2.18862	90.4	52.9	2.22035	91.9	55.6	2.25368	93.1	58.5	2.28876	94.6	61.5
	B	1.03678	65.0	63.6	1.07495	67.1	65.3	1.11414	69.9	67.2	1.15448	72.4	69.2	1.19602	75.0	71.3	1.23882	77.8	73.7

TABLE 9.—(*Continued*)

Vertical angle, h	Symbol	Latitude, $\phi = 43°$			Latitude, $\phi = 44°$			Latitude, $\phi = 45°$			Latitude, $\phi = 46°$			Latitude, $\phi = 47°$			Latitude, $\phi = 48°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
			Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ
15°.......	*A*	1.41519	11.4	39.4	1.43882	11.6	41.6	1.46371	11.9	43.7	1.48994	12.1	46.1	1.51760	12.3	48.6	1.54679	12.5	51.4
	B	0.24887	29.3	14.8	0.25772	30.3	15.2	0.26688	31.4	15.8	0.27637	32.4	16.4	0.28620	33.7	17.0	0.29640	34.9	17.7
16°.......	*A*	1.42205	12.3	39.6	1.44579	12.5	41.8	1.47084	12.7	43.9	1.49717	13.0	46.3	1.52496	13.2	48.9	1.55429	13.4	51.6
	B	0.26645	29.6	15.8	0.27593	30.6	16.3	0.28573	31.7	16.8	0.29582	33.0	17.6	0.30641	34.0	18.2	0.31734	35.2	18.9
17°.......	*A*	1.42942	13.2	39.8	1.45329	13.4	41.9	1.47844	13.6	44.3	1.50500	13.7	46.5	1.53287	14.1	49.1	1.56235	14.4	51.9
	B	0.28421	29.9	16.9	0.29432	31.0	17.4	0.30478	32.0	18.0	0.31561	33.2	18.7	0.32683	34.4	19.4	0.33849	35.6	20.2
18°.......	*A*	1.43731	14.0	40.0	1.46131	14.3	42.2	1.48660	14.5	44.4	1.51324	17.8	46.8	1.54133	15.0	49.5	1.57100	15.3	52.1
	B	0.30215	30.2	17.9	0.31289	31.3	18.5	0.32401	32.4	19.2	0.33552	33.6	19.9	0.34746	34.8	20.6	0.35985	36.0	21.5
19°.......	*A*	1.44573	15.0	40.2	1.46987	15.2	42.4	1.49530	15.4	47.7	1.52391	12.7	47.1	1.55035	16.0	49.7	1.58017	16.4	52.5
	B	0.32027	30.6	19.0	0.33167	31.7	19.6	0.34346	32.8	22.0	0.35566	34.0	21.1	0.36832	35.2	21.9	0.38145	36.4	22.8
20°.......	*A*	1.45470	15.9	40.5	1.47899	16.1	42.6	1.50457	16.4	44.9	1.53154	16.7	47.4	1.55996	17.0	50.0	1.58999	17.3	52.7
	B	0.33864	30.9	20.1	0.35068	32.1	20.8	0.36314	33.2	21.5	0.37604	34.4	22.3	0.38942	35.6	23.3	0.40331	36.9	24.0
21°.......	*A*	1.46421	16.8	40.7	1.48865	17.1	43.0	1.51443	17.8	45.2	1.54157	17.7	47.7	1.57018	18.0	50.3	1.60038	18.4	53.1
	B	0.35722	31.4	21.2	0.36993	32.5	21.7	0.38307	33.7	22.7	0.39668	34.9	23.5	0.41079	36.1	24.4	0.42544	36.9	25.5
22°.......	*A*	1.47432	17.9	41.0	1.49894	18.1	43.2	1.52487	18.4	45.6	1.55220	17.7	48.0	1.58100	19.1	50.7	1.61142	19.5	53.5
	B	0.37606	31.8	22.3	0.38943	33.0	23.0	0.40326	34.1	23.9	0.41759	34.9	24.8	0.43245	36.6	25.7	0.44787	37.9	26.7
23°.......	*A*	1.48503	18.9	41.3	1.50982	19.2	43.5	1.53594	19.5	45.9	1.56347	19.8	48.4	1.59249	20.2	51.2	1.62311	20.6	53.9
	B	0.39516	32.3	23.4	0.40921	33.4	24.2	0.42374	34.5	25.1	0.43880	35.7	26.0	0.45441	36.9	27.0	0.47061	38.3	28.1
24°.......	*A*	1.49634	20.0	41.6	1.52132	20.3	43.8	1.54764	20.6	46.2	1.57537	21.0	48.8	1.60462	21.4	51.4	1.63548	21.8	54.3
	B	0.41452	32.8	24.6	0.42927	34.0	25.2	0.44442	35.3	26.3	0.46021	36.6	27.3	0.47658	37.9	28.3	0.49358	39.3	29.4
25°.......	*A*	1.50834	20.9	42.0	1.53353	21.3	44.1	1.56000	21.7	46.7	1.58800	22.1	49.1	1.61744	22.5	51.8	1.64854	23.0	54.8
	B	0.43420	33.3	25.7	0.44964	34.6	26.6	0.46562	35.8	27.6	0.48217	37.0	28.6	0.49932	38.3	29.7	0.51713	39.7	30.8
26°.......	*A*	1.52089	22.2	42.3	1.54629	22.6	44.6	1.57304	22.9	47.0	1.60124	23.3	49.5	1.63096	23.8	52.3	1.66232	24.2	55.2
	B	0.45419	33.9	26.9	0.47035	35.1	27.9	0.48707	36.4	28.8	0.50437	37.6	29.9	0.52231	39.0	31.1	0.54094	40.4	32.3
27°.......	*A*	1.53420	23.3	42.7	1.55982	23.7	45.0	1.58679	24.1	47.4	1.61523	24.6	50.0	1.64521	25.0	52.7	1.67685	25.5	55.7
	B	0.47454	34.5	28.1	0.49142	35.7	29.1	0.50888	37.0	30.1	0.52696	38.3	31.2	0.54570	39.7	32.5	0.56517	41.1	33.7
28°.......	*A*	1.54820	24.6	43.1	1.57405	25.0	45.4	1.60128	25.4	47.9	1.63000	25.8	50.4	1.66023	26.4	53.2	1.69216	26.9	56.2
	B	0.49524	35.2	29.4	0.51286	36.4	30.4	0.53108	37.7	30.5	0.54995	39.0	32.6	0.56952	40.4	33.8	0.58983	41.8	35.2
29°.......	*A*	1.56293	25.9	43.5	1.58902	26.3	45.9	1.61653	26.7	48.3	1.64550	27.2	50.9	1.67604	27.7	53.7	1.70827	28.2	56.7
	B	0.51634	35.8	30.6	0.53470	37.1	31.7	0.55370	38.4	32.8	0.57337	39.8	34.0	0.59376	41.2	35.3	0.61494	42.7	36.7
30°.......	*A*	1.57845	27.0	43.9	1.60480	27.7	46.3	1.63257	28.1	48.8	1.66182	28.6	51.4	1.69267	29.2	54.3	1.72522	29.7	57.3
	B	0.53783	36.6	31.9	0.55696	37.8	33.0	0.57675	39.2	34.2	0.59724	40.6	35.4	0.61848	42.1	36.8	0.64055	43.5	38.2
31°.......	*A*	1.59476	28.6	44.4	1.62139	29.0	46.8	1.64944	29.6	49.3	1.67900	30.1	52.0	1.71017	30.7	54.8	1.74306	31.2	57.9
	B	0.55976	37.3	33.2	0.57967	38.6	34.3	0.60027	40.0	35.6	0.62160	41.5	36.9	0.64371	42.9	38.3	0.66667	44.5	39.8

32°.......	*A*	1.61192	30.4	44.9	1.63883	30.5	47.3	1.66718	31.1	49.8	1.69706	31.6	52.5	1.72856	32.1	55.4	1.76180	32.8	58.3
	B	0.58216	38.2	34.5	0.60287	39.5	35.7	0.62430	40.9	37.0	0.64648	42.4	38.3	0.66948	43.9	39.8	0.69336	45.4	41.4
33°.......	*A*	1.62993	31.6	45.4	1.65714	32.1	47.8	1.68582	32.7	50.4	1.71604	33.2	53.1	1.74789	33.9	56.0	1.78150	34.5	59.2
	B	0.60506	39.0	35.9	0.62658	40.4	37.1	0.64884	41.9	38.4	0.67190	43.3	39.8	0.69580	44.9	41.4	0.72061	46.8	43.0
34°.......	*A*	1.64888	33.0	45.9	1.67641	33.6	48.4	1.70542	34.3	50.9	1.73598	34.9	53.7	1.76820	35.6	56.7	1.80220	36.3	59.9
	B	0.62848	40.0	37.3	0.65084	41.3	38.5	0.67396	42.8	39.9	0.69790	44.4	41.4	0.72273	46.0	43.0	0.74851	47.6	44.7
35°.......	*A*	1.66869	35.0	46.4	1.69655	35.6	49.1	1.72600	35.0	51.6	1.75693	36.7	54.4	1.78954	37.4	57.4	1.82395	38.1	60.6
	B	0.65245	40.9	38.7	0.67565	42.5	40.0	0.69967	43.9	41.4	0.72453	45.5	43.0	0.75030	47.1	44.6	0.77706	48.8	46.4
36°.......	*A*	1.68968	36.6	47.0	1.71789	37.2	49.6	1.74762	37.2	52.2	1.77893	37.9	55.1	1.81196	38.6	58.1	1.84680	39.3	61.3
	B	0.67701	42.0	40.1	0.70109	43.5	41.5	0.72600	45.0	43.0	0.75180	46.6	44.6	0.77854	48.3	46.3	0.80631	50.0	48.1
37°.......	*A*	1.71165	38.5	47.6	1.74023	39.1	49.8	1.76993	40.4	52.9	1.80164	41.2	55.8	1.83509	41.9	58.8	1.87038	42.6	62.1
	B	0.70221	43.1	41.6	0.72718	44.7	43.1	0.75302	46.2	44.6	0.77978	47.9	46.2	0.80752	49.6	48.0	0.83632	51.3	49.9
38°.......	*A*	1.73472	40.5	48.3	1.76368	41.1	50.9	1.79420	41.8	53.6	1.82637	42.5	56.5	1.86026	43.2	59.6	1.89603	44.2	63.0
	B	0.72807	44.3	43.2	0.75397	45.9	44.7	0.78075	47.4	46.2	0.80849	49.5	48.0	0.83726	51.0	49.8	0.86712	52.8	51.7
39°.......	*A*	1.75897	42.7	49.0	1.78834	43.4	51.6	1.81929	44.0	54.3	1.85189	44.8	57.3	1.88627	45.8	60.5	1.92254	46.5	63.9
	B	0.75465	45.6	44.7	0.78149	47.2	46.3	0.80924	48.9	48.0	0.83821	50.3	49.4	0.86783	52.3	51.6	0.89878	54.3	53.6
40°.......	*A*	1.78456	44.5	49.7	1.81435	43.5	52.2	1.84566	46.2	55.1	1.87874	47.0	58.3	1.91374	47.7	61.1	1.95041	48.8	64.8
	B	0.78200	46.9	46.4	0.80982	48.6	48.0	0.83859	50.3	49.7	0.86838	52.1	51.4	0.89921	54.1	53.6	0.93135	55.9	55.6
41°.......	*A*	1.81127	47.0	50.1	1.84132	47.8	53.1	1.87338	48.6	56.0	1.90695	49.5	59.0	1.94235	50.4	62.3	1.97970	51.4	65.7
	B	0.81015	48.7	48.0	0.83897	50.1	49.7	0.86878	51.9	51.4	0.89964	53.7	53.4	0.93165	55.6	55.4	0.96488	57.6	57.6
42°.......	*A*	1.83946	49.5	51.2	1.87017	50.3	53.9	1.90248	51.1	56.8	1.93659	52.1	59.9	1.97255	53.0	63.2	2.01048	54.0	66.8
	B	0.83917	49.9	49.8	0.86902	51.7	51.4	0.89990	53.5	53.3	0.93186	55.4	55.2	0.96500	57.4	57.4	0.99942	59.4	59.6
43°.......	*A*	1.86913	52.1	52.0	1.90033	52.9	54.7	1.93317	53.6	58.5	1.96783	54.2	60.9	2.00438	55.8	64.2	2.04290	56.8	67.9
	B	0.86912	51.6	51.5	0.90004	53.4	53.3	0.93200	55.3	55.6	0.96511	56.5	57.2	0.99945	59.3	59.4	1.03507	61.5	61.8
44°.......	*A*	1.90034	54.8	52.7	1.93207	55.7	55.6	1.96545	56.8	58.7	2.00068	57.8	62.0	2.03786	58.8	65.2	2.07700	60.0	69.9
	B	0.90007	53.3	53.4	0.93208	55.2	55.3	0.96518	57.2	57.2	0.99948	59.4	59.2	1.03502	61.3	61.6	1.07195	63.5	64.0
45°.......	*A*	1.93321	57.8	53.8	1.96549	58.7	56.7	1.99950	59.7	58.8	2.03536	60.8	63.0	2.07314	61.9	66.4	2.11300	63.1	70.2
	B	0.93205	55.2	55.3	0.96521	57.2	57.2	0.99950	59.2	59.2	1.03500	61.3	61.4	1.07182	63.5	63.7	1.11005	65.8	66.2
46°.......	*A*	1.96786	60.1	54.8	2.00072	61.9	57.7	2.03532	62.9	60.8	2.07181	64.0	64.1	2.11029	65.2	67.6	2.15085	66.3	71.4
	B	0.96519	57.3	57.2	0.99952	59.3	59.2	1.03502	61.4	61.3	1.07180	63.6	63.5	1.10992	65.8	66.0	1.14950	67.8	68.6
47°.......	*A*	2.00440	64.2	55.8	2.03786	65.3	58.7	2.07305	66.5	62.0	2.11024	67.7	65.3	2.14940	69.0	68.9	2.19075	70.2	72.8
	B	0.99954	59.4	59.3	1.03509	61.5	61.3	1.07187	63.7	63.5	1.10995	65.6	65.8	1.14942	68.4	68.0	1.19042	70.8	71.1
48°.......	*A*	2.04293	67.8	56.9	2.07704	69.0	58.8	2.11295	70.1	63.2	1.15085	71.3	66.6	2.19080	72.7	70.2	2.23290	74.1	74.2
	B	1.03519	61.8	61.4	1.07201	64.0	63.5	1.11010	66.2	65.7	1.14953	68.6	68.2	1.19042	71.1	70.8	1.23289	73.6	73.6
49°.......	*A*	2.08363	71.7	58.0	2.11842	72.9	61.0	2.15500	74.1	64.4	2.19365	75.5	67.9	2.23440	76.8	71.6	2.27737	78.3	75.6
	B	1.07227	64.3	63.6	1.11041	66.6	65.7	1.14984	69.0	68.1	1.19069	71.4	70.6	1.23306	74.0	73.3	1.27703	76.6	76.2
50°.......	*A*	2.12666	75.9	59.2	2.16217	77.1	62.2	2.19947	78.4	65.8	2.23895	79.7	69.2	2.28050	81.3	73.1	2.32433	82.9	77.2
	B	1.11086	67.1	65.9	1.15038	69.4	68.1	1.19124	71.9	70.5	1.23356	74.5	73.1	1.27744	77.1	75.9	1.32300	79.8	79.0
51°.......	*A*	2.17218	80.3	60.5	2.20845	81.7	63.5	2.24653	83.1	67.1	2.28679	84.6	70.8	2.32926	82.2	74.7	2.37406	87.8	78.9
	B	1.15109	70.0	68.3	1.19204	72.5	70.6	1.23437	75.1	73.1	1.27824	77.7	75.8	1.32370	80.5	78.7	1.37091	83.4	81.8
52°.......	*A*	2.22037	85.1	61.6	2.25744	86.5	64.9	2.29637	88.0	68.6	2.33756	89.5	72.4	2.38100	91.9	76.2	2.42672	93.0	80.6
	B	1.19309	73.2	70.7	1.23553	75.8	73.2	1.27944	78.5	75.7	1.32488	81.2	78.3	1.37200	84.2	81.6	1.42097	87.1	84.8
53°.......	*A*	2.27144	90.4	63.2	2.30936	91.9	66.3	2.34916	93.4	70.2	2.39128	95.2	74.0	2.43567	97.0	78.0	2.48250	98.8	82.5
	B	1.23701	76.7	73.3	1.28101	79.4	75.9	1.32652	82.2	78.5	1.37362	85.2	81.1	1.42250	88.2	84.6	1.47323	91.3	87.4
54°.......	*A*	2.32566	96.1	64.7	2.36450	97.7	67.9	2.40522	99.3	72.0	2.44839	101.0	75.8	2.49384	106.8	79.9	2.54178	104.9	84.5
	B	1.28302	80.5	76.1	1.32865	83.4	78.9	1.37584	86.3	81.3	1.42473	89.3	84.5	1.47540	92.5	88.4	1.52803	95.8	91.2

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ECONOMIC DIAMETER OF STEEL PENSTOCKS

Discussion

BY H. K. BARROWS, M. AM. SOC. C. E.

H. K. BARROWS,[40] M. AM. SOC. C. E. (by letter).[40a]—The formulas developed by the authors are of interest, and are novel in certain respects. The use of the Scobey formula for friction head loss, involving, as it does, special exponential values for V and D other than the square and first powers, respectively, causes Equation (11) to make the economic diameter vary as the $\sqrt[6.9]{}$, whereas if the simpler common formula for friction head losses,

$$h_f = \frac{K'}{D} \times \frac{V}{2\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

is used, this becomes $\sqrt[7]{}$. It seems questionable whether the use of the Scobey formula is preferable under the circumstances.

Furthermore, the introduction of entrance losses, bend losses, etc., commonly of minor effect, could well be avoided, for the sake of simplicity, without appreciable effect upon results. The use of a loss factor to take account of load factor is logical and warrants further study and determinations of such factors. The writer has found the formula,[41]

$$D = 0.215 \sqrt[7]{\frac{K'\,b'\,Q_a^3\,s}{a\,r\,H}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

for steel pipes, convenient for obtaining a quick approximation of the best penstock size as a basis for further detailed study made by assuming different sizes, determining annual cost (of pipe plus value of power lost), and plotting total yearly cost against diameter.

In Equation (67), in addition to the notation of the paper: b' = the value of power, in dollars per theoretical horse-power per year; Q_a = aver-

NOTE.—The paper by the late Charles Voetsch, M. Am. Soc. C. E., and M. H. Fresen, Assoc. M. Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: March, 1937, by Messrs. R. A. Monroe, William E. Rudolph, and Peter Bier; April, 1937, by Adolpho Santos, Jr., Assoc. M. Am. Soc. C. E.; and May, 1937, by Messrs. Joseph D. Lewin, F. Knapp, and Ralph W. Powell.

[40] Prof., Hydr. Eng., Mass. Inst. Tech., and Cons. Engr., Boston, Mass.

[40a] Received by the Secretary April 22, 1937.

[41] "Water Power Engineering", by H. K. Barrows, 1934 Edition, p. 357.

age discharge, or maximum discharge, times a capacity factor; s = unit stress value of steel; and K' = friction factor in the formula,

$$h_f = \frac{K' L V^2}{2 D g} \qquad \text{(68)}$$

The use of the average discharge applies fairly closely where individual penstock lines are considered, if wheel units are operated by number and gate for best results. For a long single penstock the correct discharge would be more nearly the maximum discharge times the square of the capacity factor.

Similar formulas have been deduced for concrete and wood-stave penstocks, the one for concrete being as follows:[42]

$$D = 0.215 \sqrt[7]{\frac{K' b' Q^3_a}{r\left(0.0000290\, a_c + 0.384\, p\, a_s + \frac{a_s H}{s_s}\right)}} \qquad \text{(69)}$$

in which, in addition to the notation of the paper: a_c = cost of concrete, in dollars per cubic yard; p = ratio of cross-sectional steel area of longitudinal steel to that of concrete in the same plane; a_s = cost of steel reinforcement, in dollars per pound; and, s_s = allowable stress in hoop steel, in pounds per square inch. The quantity in parenthesis includes the effect of concrete cost, longitudinal steel, and hoop steel respectively.

For wood-stave penstocks:

$$D = 0.215 \sqrt[7]{\frac{K' b' Q^3_a\, s}{r H (a_b + 0.000016\, a_w\, s)}} \qquad \text{(70)}$$

in which, a_b = cost of steel bands, in dollars per pound; and a_w = cost of wood staves, in dollars per board-foot.

Equation (67) for steel pipes is simple enough to be useful; but Equations (69) and (70) are somewhat long and involved in use, and the writer prefers, in general, to determine the best size of penstock by curves rather than by formula, as this enables all special items of cost, other other than of pipe alone, to be included.

[42] "Water Power Engineering", by H. K. Barrows, 1934 Edition, p. 358.

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DISCUSSIONS

STRESSES AROUND CIRCULAR HOLES IN DAMS AND BUTTRESSES

Discussion

BY FRED L. PLUMMER, M. AM. SOC. C. E.

FRED L. PLUMMER,[37] M. AM. SOC. C. E. (by letter).[37a]—Based on the assumptions defined by Equations (9) the author has developed formulas giving the stresses around circular openings in mass structures. The derivation is presented in a clear and straightforward manner, and illustrates a very skillful use of mathematical theorems. The writer wishes to call attention to the fact that the aforementioned assumptions cannot always be justified, and to question whether, in any case, the designing engineer is warranted in using such complicated formulas when they are developed from assumptions which, at best, can give only a crude approximation of the stresses that will exist in the actual structure.

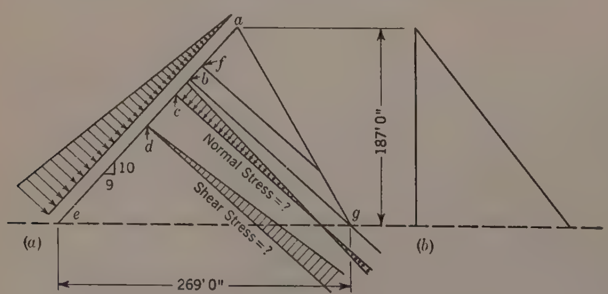


FIG. 12.

Fig. 12(a) shows a buttress similar to the one used by the author in his illustrative example. Obviously, any plane section cut perpendicular to the up-stream face of the buttress, except for the short section, a-b, would extend indefinitely into the foundation. The assumptions that the normal

NOTE.—The paper by I. K. Silverman, Jun. Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1937, by Messrs. R. D. Mindlin, and Chesley J. Posey; April, 1937, by Messrs. J. H. A. Brahtz, V. L. Fedorov, and F. W. Hanna; and May, 1937, by C. P. Vetter, M. Am. Soc. C. E.

[37] Cons. Engr.; Associate Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

[37a] Received by the Secretary April 8, 1937.

stresses on such planes (c) are distributed linearly in accordance with Equation ($9a$), and that the shear stresses along such planes (d) increase uniformly with the distance from the loaded face in accordance with Equation ($9c$), are obviously far from even a reasonable approximation of the probable stress distribution. On the other hand, the assumption that the direct compression parallel to such planes is constant in accordance with Equation ($9b$) may be more true for planes through c and d, Fig. 12, than for a plane through f.

It would seem, therefore, that no formula based on these assumptions could be of any value in studying stresses at any point in the section, b-e-g, of this buttress. Since stress distributions in Nature cannot be altered abruptly, it is probable that the stresses along a plane through f are very similar to those on a plane through b. In consequence, it does not seem likely that the stress distribution assumptions defined by Equations (9), nor any formulas based upon these assumptions, could be properly applied to any part of the buttress shown in Fig. 12(a), except possibly a small segment at the top of the buttress.

As the slope of the up-stream face of the section is made steeper and the section more nearly approaches that shown in Fig. 12(b), the aforementioned assumptions seem to lead to closer approximations of the actual stress distributions. Even in the case shown in Fig. 12(b) however, the actual stress distribution may vary quite materially from that indicated by these assumptions. Such variation may be illustrated by considering the very simple case shown in Fig. 13(a), which is drawn from a photoelastic picture showing the variation of maximum shearing stresses in

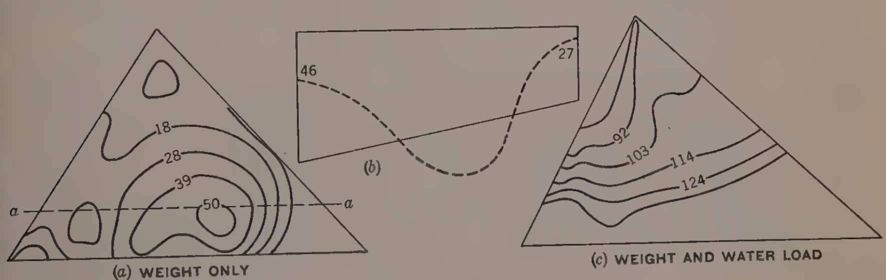


FIG. 13.

the buttress of Fig. 12(a) due to the weight of the buttress itself. The usual assumptions, analogous to those based on Equations (9), would indicate maximum shearing stresses along the plane, a-a, varying uniformly from 27 lb per sq in. at the up-stream face to 46 lb per sq in. at the down-stream face as shown by Fig. 13(b). The distribution of shearing stresses given by the model is shown by the dotted line. This would indicate that the direct compression stresses caused by the triangular mass of concrete above this section, are not distributed uniformly, but that the intensities are much greater near the middle of the section. This effect has been recognized in many other types of problems.

Fig. 13(*c*) shows the variation of shearing stresses in the same buttress due to the weight of the buttress and, in addition, a water load applied to an area on the up-stream face twelve times as wide as the buttress. In each case, the lines represent contours joining those points where the maximum shearing stresses in the analogous concrete buttress equal the values indicated on the diagrams, the unit being pounds per square inch. In neither case, nor along any plane section, does the stress distribution agree closely with that given by the assumptions defined by Equations (9). On the other hand, Fig. 14 shows a section for which the stress distribution given by the model and that based on these same assumptions agree rather closely on several typical sections and for both dead load, and combined dead and water loads.

The writer wishes to emphasize the fact that the preceding statements imply no criticism of the author's mathematical derivation, nor of the resulting formulas. Granting the underlying assumptions, the formulas correctly represent the stress conditions around a circular opening. Unfortunately, many designers of engineering structures have been trained to rely too implicitly on printed formulas and, in consequence, often use such aids without understanding the underlying assumptions and, more important, the resulting limitations of every such formula. The formulas under discussion were originally called to the writer's attention by a young engineer who was applying them to a structure for which they could not possibly give results of any value.

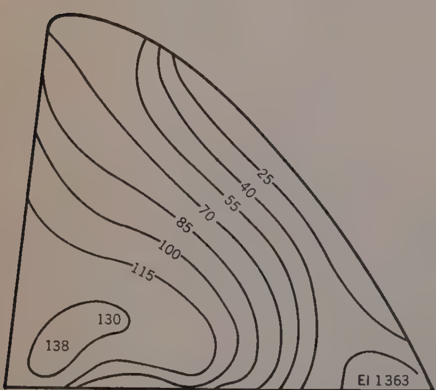


Fig. 14.

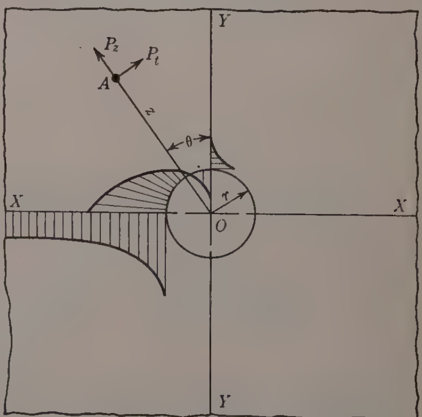


Fig. 15.

It is important that engineers recognize the fact that openings in massive structures in which the normal principal stresses are all compressive may result in tangential tension stresses near the boundaries of such openings. Obviously, in concrete structures, it is wise to provide steel reinforcement in sufficient quantity, placed so as to resist such stresses properly. Equation (1) indicates how much the tension stress in a bar may be increased due to the presence of a small circular hole. Fig. 15

shows the variation of stresses near a circular opening in an area, the least dimension of which is at least four times the diameter of the hole. In Fig. 15 a uniform compressive stress is assumed to act in the direction of the Y-Y axis. Due to the presence of the hole, tension stresses are created, and the compression stresses are increased as indicated. Formulas developed by A. and L. Föppl[38] give the radial, tangential, and maximum shearing stress at any point, A, as:

$$s_z = \frac{s}{2}\left[1 = \frac{r^2}{z^2} + \left(1 - \frac{4\,r^2}{z^2} + \frac{3\,r^4}{z^4}\right)\cos 2\,\theta\right] \ldots\ldots (108a)$$

$$s_t = \frac{s}{2}\left[1 + \frac{r^2}{z^2} - \left(1 + \frac{3\,r^4}{z^4}\right)\cos 2\,\theta\right] \ldots\ldots\ldots\ldots (108b)$$

and,

$$s_Q = \frac{s}{2}\left[-1 - \frac{2\,r^2}{z^2} + \frac{3\,r^4}{z^4}\right]\sin 2\,\theta \ldots\ldots\ldots\ldots (108c)$$

in which s = the principal stress at A if no hole is present (s can act in any direction); r = the radius of the hole; z = the distance from the center of the hole to Point A; s_Q = maximum unit shearing stress at Point A; and, θ = the angle as shown referred to the direction in which s acts.

If θ is made equal to 90°, Equation (108a) reduces to Equation (1) giving the stress distribution along the X-X axis. With θ equal to zero, the tangential tension stresses along the Y-Y axis may be found. The greatest tension stress occurs at the boundary of the hole and equals in intensity the applied compression stress, s. This tension stress decreased along the axis, Y-Y, to zero at a point, $r\sqrt{3}$, from the center of the hole. By integration the total tension for unit width may be found to equal $\frac{-s\,r}{3\sqrt{3}}$, with this resultant force acting at a distance approximately equal to 1.173 r from the center of the hole. If z is made equal to r the resulting formulas define the variation of stress around the boundary of the hole, indicating tangential tension decreasing in intensity from s to zero as θ is increased from zero to 30°, and then tangential compression increasing from zero to $3\,s$ as θ increases from 30 to 90 degrees.

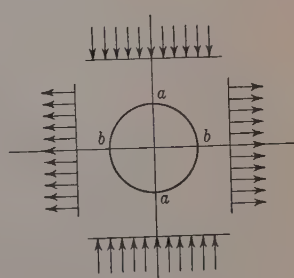


FIG. 16.

To illustrate how simply these relations may be utilized consider that the principal stresses in a given region of a mass structure have been determined, assuming no hole to be present. The values are shown in Fig. 16. The greatest tangential tension stress around the boundary would occur at Point a, and would equal (445 + 3 × 15 = 490 lb per sq in.). Similarly, the great-

[38] "Drang und Zwang," p. 314.

est compression stress would occur at Point b and would equal ($3 \times 445 + 15 = 1\,350$ lb per sq in.). The stresses at other points on the boundary of the hole can be found in a similar manner by the use of Equation (108b), with $z = r$, the effects of the two principal stresses being computed separately and then combined algebraically. The stress distribution used in this illustration duplicates approximately that which the author found to occur around the hole in the buttress analyzed in his illustrative example.

Fig. 17.

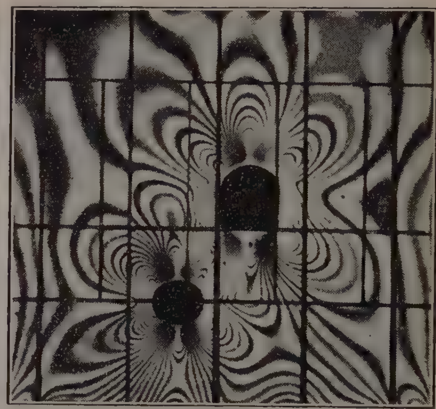
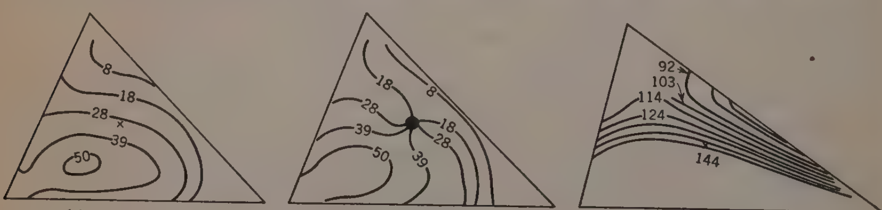


Fig. 18.

Using the data of the paper, the writer has computed the principal stresses that would act in a solid buttress at a point corresponding to the center of the hole specified by the author. Duplicate computations based on three separate sets of generally accepted assumptions result in values of the major principal stress varying in compression from 300 to 570 lb per sq. in. As indicated previously the value obtained by the author was about 445 lb per sq in. in compression. In the writer's opinion it is this part of the problem which needs serious study. Moreover, it would seem that designers must look to experimental, rather than analytical, methods.



(a) WEIGHT ONLY (b) WEIGHT ONLY (c) WEIGHT AND WATER LOAD

Fig. 19.

Figs. 17 and 18 show photo-elastic studies of stress distribution in the region of two openings in both a gelatin and a marblette model of a section of a concrete dam. The writer has found this method of study to be very useful and, in general, more reliable than any purely analytical method which of necessity must be based on simplifying assumptions. Obviously, it would be impractical to derive formulas which would be applicable to

the complicated situation illustrated by Figs. 17 and 18. Fig. 19 shows a model study of the buttress analyzed by the author. The scale of the model is too small to determine accurately the stress variation around the circular openings. A large scale model of a part of the buttress would be used for this purpose. However, the models show clearly that the stresses in the region of the hole are quite different from those given by computations based on the assumptions used by the author.

The curves in Figs. 14 and 17 and the photo-elastic views in Fig. 18, are taken from studies made in co-operation with the office of the U. S. Engineer Corps, at Huntington, W. Va., and under the direction of Lt. Col. John F. Conklin, District Engineer, and A. L. Alin, M. Am. Soc. C. E., Chief of Design on the Bluestone Project.

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DISCUSSIONS

GRAPHICAL DISTRIBUTION OF VERTICAL PRESSURE BENEATH FOUNDATIONS

Discussion

BY MESSRS. NATHAN M. NEWMARK, A. E. CUMMINGS, AND D. P. KRYNINE

NATHAN M. NEWMARK,[13] JUN. AM. SOC. C. E. (by letter).[13a]—The method presented by Mr. Burmister for finding vertical pressures beneath foundations graphically is interesting and simple to use. The author's procedure is more rapid than the method of dividing the loaded area into elements of load which may be replaced by concentrated forces. The writer wishes to supplement Mr. Burmister's method by describing for comparison two other procedures, one analytical and one graphical, which seems to have similar advantages of speed and simplicity.

An analytical procedure for computing the intensity of vertical stress beneath a loaded area was developed by the writer at the University of Illinois.[14] This procedure makes use of a table of values of vertical pressure at unit depth beneath the corner of a uniformly loaded rectangle of dimensions, m by n. The table is applicable to all depths and to all dimensions of loaded area. The formula by which the numerical values were computed was obtained by integration over a rectangular area of Boussinesq's formula for the pressure due to a concentrated load. The tabulated coefficients of pressure are given in terms of the vertical load per unit of area on the rectangular area and are dimensionless.

The pressure at points other than under the corner of a rectangular area is found[14] by combining upward and downward loads on not more than four rectangles. For example, the vertical pressure at a depth of 10 ft beneath the center of the footing of dimensions, 6 ft by 8 ft, shown in Fig. 5, is found as the sum of the vertical pressures 10 ft beneath the common corner of four equal rectangles, 3 ft by 4 ft. The "relative dimensions" (m and n) for these rectangles are 0.3 by 0.4. With these values

NOTE.—The paper by Donald M. Burmister, Assoc. M. Am. Soc. C. E., was published in January, 1937, *Proceedings*. Discussion on this paper has been published in *Proceedings*, as follows: May 1937, by Messrs. William B. Kimball, I. M. Nelidov, George Paaswell, and Jacob Feld.

[13] Research Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[13a] Received by the Secretary March 29, 1937.

[14] "Simplified Computation of Vertical Pressures in Elastic Foundations", by Nathan M. Newmark, Univ. of Illinois, Eng. Experiment Station, *Circular No. 24* (1935).

of m and n, the table[14] referred to lists a value of influence pressure of 0.04742. Then the influence pressure for the entire footing is four times this value, or 0.18968.

TABLE 6.—INFLUENCE PRESSURES

Depth under center of Footing No. 1, in feet	Influence Pressure Due to:	
	Footing No. 1	Footing No. 2
10	0.1897	0.00097
20	0.0542	0.00372
40	0.0141	0.00520
80	0.0035	0.00265

For the problem shown in Fig. 5 of the paper the results given in Table 6 were obtained by this procedure. The writer was unable to make comparisons with the results obtained for the problem of Fig. 4, since no dimensions are given for that problem. Except for pressures near the surface, such a foundation plan can be considered to be replaced by a rectangular area uniformly loaded with the same total load as is applied to the footings. In view of the fact that the computation of pressure in

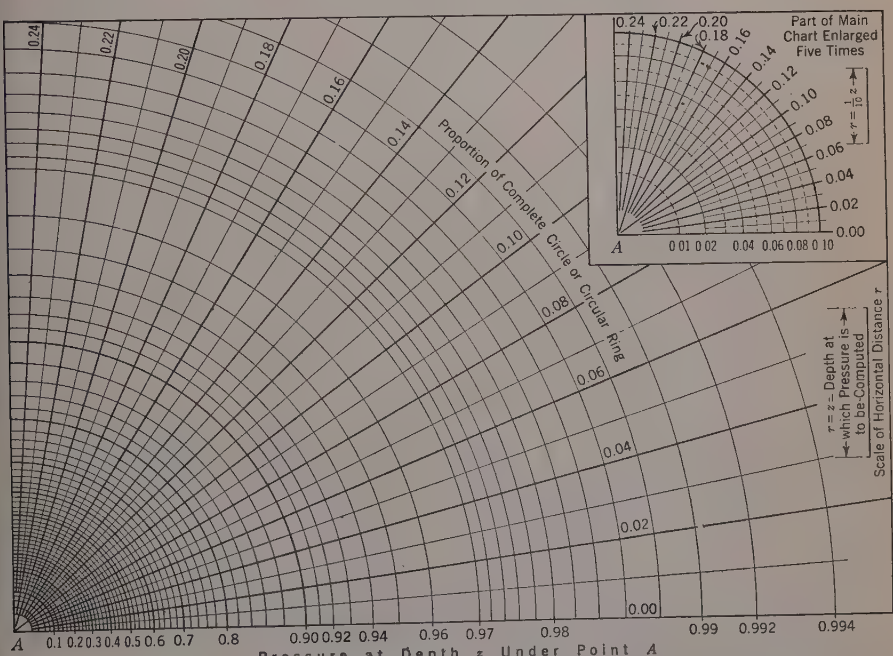

FIG. 11.—FOR UNIFORM LOAD OF UNIT INTENSITY ON AREA OF COMPLETE CIRCLE BOUNDED BY RING DESIGNATED.

soil is based on assumptions that are only approximately true, such a replacement is permissible. Near the surface, the pressures are affected by the distribution of contact pressure between the footings and the soil, and this distribution may vary between rather wide limits.

It seems that the author's graphical procedure is limited to computations at depths for which charts are available. Only one chart is necessary for all depths if the scale of the foundation plan is changed properly with the depth. For example, Fig. 3, the author's chart for a 10-ft depth, may be used for, say, a 20-ft depth if the scale of distance to the center of each loaded ring is multiplied by 2, so that the rings become 2 ft wide.

If a graphical procedure is to be used, the writer prefers a pressure chart such as Fig. 11 in which each loaded ring accounts for the same influence pressure. Subdivisions of the rings may then be made by radial lines. The chart can be made independent of depth by drawing the foundation plan to scale such that the depth at which the pressure is to be computed is taken as the unit of length.

Values of $\frac{r}{z}$ for given values of $\frac{p_z}{p}$ may be computed by re-arranging Equation (3). The writer has made such computations, the results of which are given in Table 7. With these data Fig. 11 may be constructed for the graphical computation of pressure. For the part within the ring marked

TABLE 7.—VALUES OF RELATIVE RADIUS OF CIRCLE, $\frac{r}{z}$, FOR GIVEN VALUES OF $\frac{p_z}{p}$

$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)
0	0	0.36	0.5887	0.70	1.1097	0.970	3.0590
0.02	0.1164	0.38	0.6126	0.72	1.1561	0.972	3.1377
0.04	0.1661			0.74	1.2062	0.974	3.2240
0.06	0.2052	0.40	0.6370	0.76	1.2607		
0.08	0.2391	0.42	0.6617	0.78	1.3206	0.976	3.3194
		0.44	0.6869			0.978	3.4259
0.10	0.2698	0.46	0.7127	0.80	1.3871	0.980	3.5457
0.12	0.2983	0.48	0.7392	0.82	1.4618	0.982	3.6823
0.14	0.3252			0.84	1.5469	0.984	3.8404
0.16	0.3511	0.50	0.7664	0.86	1.6459		
0.18	0.3761	0.52	0.7945	0.88	1.7636	0.986	4.0268
		0.54	0.8235			0.988	4.2519
0.20	0.4005	0.56	0.8536	0.90	1.9083		
0.22	0.4244	0.58	0.8849	0.91	1.9948	0.990	4.5326
0.24	0.4481			0.92	2.0943	0.992	4.8990
0.26	0.4715	0.60	0.9176	0.93	2.2108	0.994	5.4116
0.28	0.4948	0.62	0.9519	0.94	2.3505	0.996	6.2197
		0.64	0.9880	0.95	2.5235	0.998	7.8738
0.30	0.5181	0.66	1.0261				
0.32	0.5415	0.68	1.0665	0.96	2.7477	1.0	∞
0.34	0.5650						

0.90, the area between successive circles accounts for an influence pressure of 0.02. Each of the elements of area bounded by the heavy solid lines, outside the ring marked 0.10, accounts for an influence pressure of 0.002, and each of the elements bounded by light lines accounts for an influence pressure of 0.0002. In order to subdivide the area more uniformly, intermediate circles are drawn for the area outside the circle marked 0.90.

The method of using the chart[14a] is, as follows: Suppose the depth at which the pressure is desired is 35 ft. Then draw the foundation plan on

[14a] Copies of the original graph (size 11 in. by 16 in.) are available for free distribution. Send request to Department of Civil Engineering, University of Illinois, Urbana, Ill.

thin paper or tracing cloth to such a scale that the distance, z, in Fig. 11 represents 35 ft. Set the point at which the pressure is desired at Point A. Add the loaded rings and segments, each multiplied by the value of p on the corresponding area, to obtain p_z; or count the number of blocks or elements of area loaded and multiply by the value of a single block to get the total vertical pressure. A different drawing of the foundation plan is required for each depth investigated, but only the single chart is needed.

A. E. CUMMINGS,[15] M. AM. SOC. C. E. (by letter).[15a]—For the calculation of the stress distribution in the ground under a foundation, the author recommends the use of a simple algebraic expression which is often referred to as "the Boussinesq equation." According to Professor Burmister, this equation represents the only means available for the solution of such problems. The writer is of the opinion that the derivation of the equation is based on assumptions which are rarely fulfilled by an actual foundation problem. Accordingly, he proposes to outline briefly the original derivation[16] of the equation and to discuss the assumptions on which it is based.

Boussinesq assumes a rectangular co-ordinate system (Fig. 12), with the XY-plane as the plane horizontal boundary of the semi-infinite solid

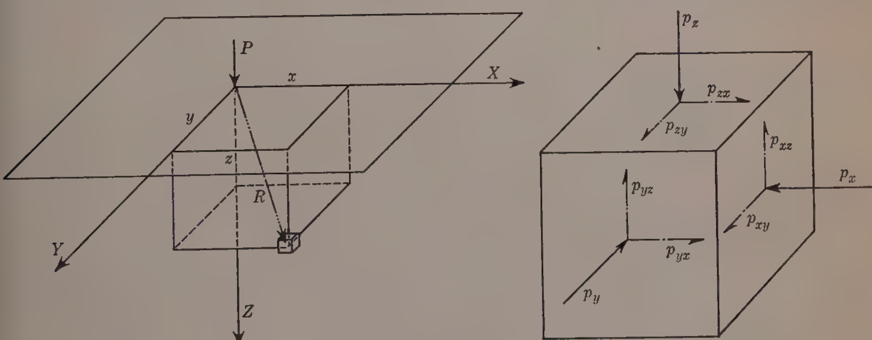

FIG. 12.

and with the positive Z-axis directed downward into the solid. He begins with the general differential equations of elastic equilibrium in the form commonly known as the body-shift equations. These are:

$$(\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u = 0 \qquad (14a)$$

$$(\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v = 0 \qquad (14b)$$

and,

$$(\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w = 0 \qquad (14c)$$

[15] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[15a] Received by the Secretary April 21, 1937.

[16] "Application des Potentiels a l'Etude de l'Equilibre et du Mouvement des Solides Elastiques", by J. Boussinesq, Gautier-Villars, Paris, 1885.

In Equations (14), u, v, and w are the displacements in the X, Y, and Z directions, respectively; and λ and μ are the elastic constants of Lamé, μ being the shear modulus. The cubical dilatation is:

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad (15)$$

and ∇^2 is the Laplacian operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (16)$$

It is necessary to find expressions for u, v, and w that will satisfy the differential Equations (14) and for this purpose Boussinesq uses certain potential functions. He expresses the displacements in terms of the following:

The inverse or Newtonian potential is:

$$U = \int \frac{dm}{R} \qquad (17a)$$

the direct potential is:

$$V = \int R\, dm \qquad (17b)$$

the first logarithmic potential of three variables is:

$$\psi = \int \log (z + R)\, dm \qquad (17c)$$

and the second logarithmic potential of three variables is:

$$\Psi = \int [- R + z \log (z + R)]\, dm \qquad (17d)$$

In Equations (17), dm is the infinitesimal element of the mass producing the field of force and $R^2 = x^2 + y^2 + z^2$. The integration extends over the mass, m.

By expressing the displacements as the space derivatives of one or the other of these potential functions, Boussinesq finds solutions of the differential Equations (14). These solutions he calls "simple integrals" but, in general, they are not satisfactory for the semi-infinite solid as they do not satisfy the required boundary conditions at the free horizontal surface of the solid. In order to satisfy these boundary conditions, Boussinesq uses combinations of several potentials. For the semi-infinite solid subjected to a purely vertical load on its plane horizontal boundary, he takes the displacements in the form:

$$u = - \frac{1}{4\, \pi\, \mu} \frac{\partial^2}{\partial x\, \partial z} \int R\, dm - \frac{1}{4\, \pi\, (\lambda + \mu)} \frac{\partial}{\partial x} \int \log (z + R)\, dm \qquad (18a)$$

$$v = -\frac{1}{4\,\pi\,\mu}\,\frac{\partial^2}{\partial y\,\partial z}\int R\,dm - \frac{1}{4\,\pi\,(\lambda + \mu)}\,\frac{\partial}{\partial y}\int \log\,(z + R)\,dm \quad \text{..(18b)}$$

and,

$$w = -\frac{1}{4\,\pi\,\mu}\,\frac{\partial^2}{\partial z^2}\int R\,dm + \frac{2\,\lambda + 3\,\mu}{4\,\pi\,\mu\,(\lambda + \mu)}\,\frac{\partial}{\partial z}\int \log\,(z + R)\,dm \quad \text{..(18c)}$$

Equations (18) are seen to be combinations of the direct potential and the first logarithmic potential, and it is easily shown by differentiation that they are solutions of the differential Equations (14).

To obtain the stresses from the displacements, Boussinesq makes use of the equations of the mathematical theory of elasticity in which the stresses are expressed in terms of the derivatives of the displacements. These are:

$$p_x = -\lambda\,\Delta - 2\,\mu\,\frac{\partial u}{\partial x} \quad \text{........................(19a)}$$

$$p_y = -\lambda\,\Delta - 2\,\mu\,\frac{\partial v}{\partial y} \quad \text{........................(19b)}$$

$$p_z = -\lambda\,\Delta - 2\,\mu\,\frac{\partial w}{\partial z} \quad \text{........................(19c)}$$

$$p_{zx} = p_{xz} = -\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad \text{......................(19d)}$$

$$p_{zy} = p_{yz} = -\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \quad \text{......................(19e)}$$

and,

$$p_{xy} = p_{yx} = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \text{......................(19f)}$$

In Equations (19), the values of p with the single subscript are the normal stresses and those with the double subscripts are the shears.

If it is assumed that a point load, P, is applied normally to the surface of the solid so that $\int dm = P$, Equations (18) and (19) give the stresses at any point within the solid, as follows:

$$p_x = \frac{P}{2\,\pi}\left\{\frac{3\,z\,x^2}{R^5} + \frac{\mu}{\lambda + \mu}\left[\frac{R^2\,(z + R) - x^2\,(z + 2\,R)}{R^3\,(z + R)^2} - \frac{z}{R^3}\right]\right\} \quad \text{..(20a)}$$

$$p_y = \frac{P}{2\,\pi}\left\{\frac{3\,z\,y^2}{R^5} + \frac{\mu}{+\,\mu}\left[\frac{R^2\,(z + R) - y^2\,(z + 2\,R)}{R^3\,(z + R)^2} - \frac{z}{R^3}\right]\right\} \quad \text{..(20b)}$$

$$p_z = \frac{3\,P\,z^3}{2\,\pi\,R^5} \quad \text{..............................(20c)}$$

$$p_{zx} = \frac{3\,P\,z^2\,x}{2\,\pi\,R^5} \qquad (20d)$$

$$p_{zy} = \frac{3\,P\,z^2\,y}{2\,\pi\,R^5} \qquad (20e)$$

and,

$$p_{xy} = \frac{P}{2\,\pi}\left[\frac{3\,x\,y\,z}{R^5} - \frac{\mu}{\lambda + \mu}\,\frac{x\,y\,(z + 2\,R)}{R^3\,(z + R)^2}\right] \qquad (20f)$$

In these stress equations the plus sign represents compression and the minus sign, tension. Equation (20*c*) is easily recognized as "the Boussinesq equation."

Whether or not Equation (20*c*) is applicable to a given foundation problem depends on how well the foundation problem conforms with the assumptions on which the derivation of the equation is based. As an illustrative example, the author has used a structure built on isolated spread-footings (see Fig. 4). However, there is nothing in the paper to indicate that the charts should not be used for large mat foundations. The assumptions on which Equation(20*c*) is based are more readily understood by consideration of a mat foundation. Furthermore, the author has made several statements which are of con-

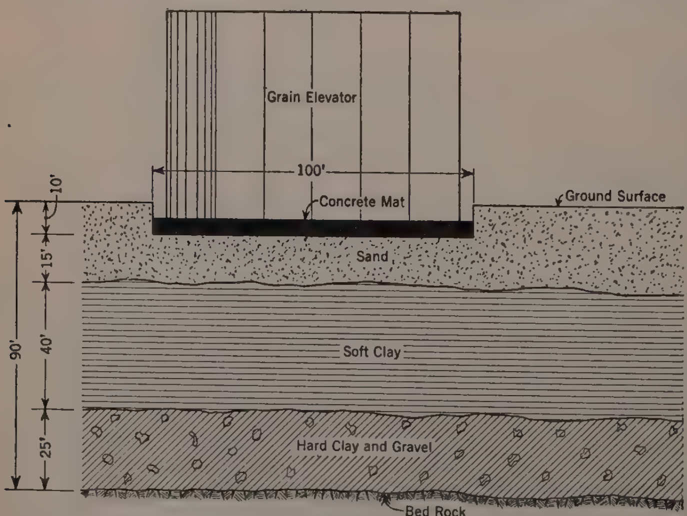

FIG. 13.

siderable interest when applied to a large mat. The writer proposes, therefore, to consider not only Fig. 4, but also the foundation problem illustrated in Fig. 13. This is not an imaginary soil condition. It is more or less typical of certain parts of Chicago, Ill., Detroit, Mich., Toledo, Ohio, and other cities on the Great Lakes.

One important assumption on which Equation (20*c*) is based is that the solid must be homogeneous to an infinite depth. This is required by

Equations (18) since the space derivatives of the potential functions in these equations are continuous functions and do not vanish except at infinity. Discontinuities in the soil structure are not compatible with Equations (18). In Fig. 13, the surface of the bed-rock is at a depth of 90 ft below the ground surface. This rock may be considered as practically rigid so that the vertical displacements must vanish at the plane of the rock surface. The horizontal displacements may or may not vanish, depending on the friction between the rock and the soil stratum just above it. Equation (20*c*) does not apply to this case.

A method of taking into account the discontinuity at the rock surface has been published by Professor M. A. Biot.[17] According to Professor Biot's analysis, the vertical normal stress on the center line of the load at the rock surface is considerably greater than the stress calculated with Equation (20*c*). The plane of contact between the sand and the soft clay represents another type of discontinuity. Arthur Casagrande, Assoc. M. Am. Soc. C. E., has studied this condition[18] and has found that a dense sand bed apparently spreads the load so that the vertical normal stresses in the clay are much less than the values given by Equation (20*c*). This question of discontinuities in the soil structure applies to Fig. 4 as well as to Fig. 13. In Fig. 4, it is the width of the entire building that must be considered and not the diameter of the individual footings. Whenever there is bed-rock or some other abrupt change in soil stratification at a depth no greater than one or two times the diameter of the building, Equation (20*c*) does not provide the proper answer.

A second assumption on which Equation (20*c*) is based is that the load must be at the surface of the solid and not below the surface. This requirement is also due to Equations (18). Fig. 13 shows the bottom of the mat at a depth of 10 ft below the surface. This is not an uncommon condition and, in some cases, buildings have several sub-basements so that their foundations are 25 or 30 ft below the surface. Equation (20*c*) is not suitable for such conditions, and it is necessary to use an analysis of the type published in 1936[19] by R. D. Mindlin, Jun. Am. Soc. C. E. Stresses obtained from Mr. Mindlin's equations may vary considerably from those given by Equation (20*c*), depending on the size of the loaded area, its depth below the surface, and the depth of the plane for which the stresses are being calculated.

A third assumption involved in the derivation of Equation (20*c*) is that the load must be normal to the surface of the solid. The displacements represented by Equations (18) are for normal loads only. When the point load, P, is changed to a distributed load, $p\,dx\,dy$, which is to be integrated over a finite area, this normal load requirement does not change, and the load on the finite area must also be purely normal to the

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[17] "Effect of Certain Discontinuities on the Pressure Distribution in a Loaded Soil", *Physics*, December, 1935, Vol. 6, p. 367.

[18] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1122.

[19] "Force at a Point in the Interior of a Semi-Infinite Solid", *Physics*, May, 1936, Vol. 7, p. 195.

surface of the solid. In some cases there is a tendency for the soil to attempt to escape laterally from under the foundation. The condition is similar to that of a compression test on a concrete cylinder in which there are tangential friction forces in the planes of contact between the test blocks and the ends of the cylinder. If the soil attempts to move outward, the friction forces on the soil surface are directed radially inward.

The resultant of the normal load and the friction force at any point makes an angle with the normal, the magnitude of the angle depending on the amount of friction. This oblique resultant force represents a more general case than the purely normal force. For the oblique force the stress distribution within the solid is quite different from that produced by the normal force alone. Boussinesq[20] has solved the problem for the oblique force with the aid of the second logarithmic potential given by Equation (17*d*). Professor O. K. Froehlich[21] has derived equations which show the effect of tangential forces in the contact plane. The existence of this phenomenon has been noticed and discussed[22] by M. L. Enger, M. Am. Soc. C. E. When there are tangential friction forces in the contact plane, the stress distribution in the soil is not that of Equations (20).

The fourth assumption on which Equation (20*c*) is based is that the solid must satisfy Hooke's law and must be a truly elastic isotropic solid in the sense required by the mathematical theory of elasticity. The fact that no elastic constants appear in Equation (20*c*) does not nullify this requirement. The derivation of this formula is based on the differential Equations (14) and these apply only to elastic isotropic solids which satisfy Hooke's law. In order to fulfill these requirements the soil would have to be capable of resisting all tensile, compressive, and shearing stresses represented by Equations (20). Granular soils, for instance, lack the capability of resisting tensile forces except in so far as they may already be under compression due to body forces. At a point on the X-axis at the surface of the solid (in Fig. 12, $z = 0$; $y = 0$; and $x = R$), the shears, p_{zx}, p_{zy}, p_{xy}, and the normal stress, p_z, all vanish. The normal stress p_x, is a radial tension and p_y is a tangential compression. Concentric circles drawn on the surface of the solid, with the origin as a common center, are contracted.

If the physical properties of the soil are such that it cannot resist this tensile stress, p_x, the stress field cannot be represented by Equations (20). Isotropy requires that the elastic properties of the soil would have to be the same in all directions. Stratified silts and varved clays are not isotropic in this sense. Hooke's law requires a linear relationship between stress and strain both for loading and for unloading. In some soils the stress-strain loading curve is a straight line for small strains but in many soils it is not and, in most soils, the process is not reversible. The result is that, in general, it is not valid to consider the soil under a

[20] "Application des Potentiels", p. 182.

[21] "Druckverteilung im Baugrunde", Julius Springer, Vienna, 1934, p. 55.

[22] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1580.

foundation as an elastic isotropic solid. A method of determining the stress distribution in certain types of anisotropic solids has been published by K. Wolf.[23]

The author mentions the "disturbed zone" defined by Kögler and Scheidig and remarks that this region of disturbance extends to a depth of two or three times the greatest width of the footing. Apparently, the author is thinking only of the relatively small spread-footings shown in Fig. 4. The question arises as to how this idea may be applied to Fig. 13. Two or three times the width of this footing means 200 or 300 ft in the ground. It seems scarcely likely that a region of disturbance would extend to any such depth as this. In their original description[24] of this phenomenon, Kögler and Scheidig did not publish any mathematical analysis by means of which the extent of this region could be calculated. However, they called attention to the fact that the depth of the disturbance depended on the physical properties of the soil and the intensity of the surface load. Actually this region of disturbance is a region of plastic flow.

Professor O. K. Froehlich has published[25] equations which define the boundaries between the elastic and the plastic regions. Froehlich's analysis is based on Mohr's theory of plastic flow and his equations include the size of the loaded area, the intensity of the surface load, and the physical characteristics of the soil. In either Fig. 4, or Fig. 13, the extent of this region of plastic flow would depend on these various factors. The region is not merely a function of the diameter of the footing. In addition to this "region of disturbance," Kögler and Scheidig described a "region of no stress" which lies near the surface and surrounds the loaded area. It is in this "region of no stress" that the theory of elasticity, as represented by Equations (20), includes a tensile stress. The inability of sand to resist this tensile stress is part of the cause of the existence of the "region of disturbance."

The author also discusses the distribution of pressure in the contact plane and states that this becomes unimportant at depths several times the width of the footing. Again, he appears to be thinking only of the small footings of Fig. 4. It is interesting to consider this statement in connection with Fig. 13. As Professor Burmister states, this surface pressure distribution depends on the relative elastic properties of the soil and the structure. It can vary from a distribution which is a maximum at the center and a minimum at the edges to one which is a minimum at the center and a maximum at the edges. In Fig. 13 the entire depth of soil between the bottom of the footing and the surface of the rock is only eight-tenths of the diameter of the footing. It is in this region that the non-uniformity of the surface loading is most important. Even if the rock were not there, and if the soil were a homogeneous, elastic, isotropic solid, the vertical normal stresses on horizontal planes in the

[23] "Ausbreitung der Kraft in der Halbebene und im Halbraum bei Anisotropen Material", *Zeitschrift für ang. Math. und Mech.*, 1935, Vol. 15, No. 5, p. 249.

[24] *Die Bautechnik*, April 6, 1928, p. 206.

[25] "Druckverteilung im Baugrunde", p. 72.

region down to an 80-ft depth would be seriously affected by the distribution of pressure in the contact plane. The author's remark about the unimportance of the distribution of contact pressure is apt to be misleading. There are many kinds of foundation problems in which the distribution of pressure in the contact plane cannot be ignored.

The calculation of stress distributions in the ground under a foundation represents the first half of the problem of settlement analysis. The other half is concerned with the laboratory soil tests and the application of the test results to full-sized structures. Many records of observations on the settlements of structures have been published during the past few years. In a few cases there is agreement between prediction and observation, but in many cases there are wide variations. These differences between prediction and observation could result either from the laboratory tests or from the assumed stress distribution in the soil. As far as the stress distribution is concerned, it is the writer's opinion that this so-called "Boussinesq equation" is being somewhat overworked. Foundation problems seldom conform with the assumptions which are the bases of its derivation. Furthermore, the equation is not the only means available for the analysis of stress distributions in the ground under a foundation.

The extent of actual knowledge available at the present time on stress distributions under full-sized structures is extremely limited. The writer, therefore, is inclined to question the desirability of putting into general use such charts as those proposed by the author. These charts are based on an equation taken from the mathematical theory of elasticity, and the equation itself is subject to several definite restrictions. If the assumption is granted that the soil is an elastic, isotropic solid there still remain such questions as: Is the load at the surface or below the surface? Is the load normal to the surface or are there horizontal forces in the contact plane which produce, with the vertical force, an oblique resultant? Is there an unyielding rock stratum or other discontinuity in the soil at a depth no greater than one or two times the diameter of the structure? The author's charts do not take any of these conditions into account

If it is admitted that the soil is not an elastic, isotropic solid, it becomes necessary to abandon the use of the theory of elasticity altogether, or at least to modify the equations of that theory before they are used to calculate stress distributions in soils. Apparently, it is true that, at considerable depths below the surface, the materials of the earth behave approximately as elastic, isotropic solids. Much of the science of seismology is based on this assumption.[26] However, it is the writer's opinion that most foundation problems are concerned with surface phenomena. In general, the entire depth of soil involved in the problem is of the same order of magnitude as the diameter of the structure, and there are usually a number of other special conditions to be considered. It would seem better,

[26] "Physics of the Earth", Vol. VI, The National Academy of Sciences, Washington, D. C., 1933.

therefore, to leave the door open to a consideration of these special conditions rather than to advocate the use of charts which do not take them into account at all. In any case, the charts and tables should be supplemented by a complete statement of the assumptions on which they are based so that the user may be fully aware of their limitations.

D. P. KRYNINE,[27] M. AM. SOC C. E. (by letter).[27a]—The object of this discussion is: First, to emphasize the outstanding place which the chart method advanced by Professor Burmister occupies among other methods of determining the vertical pressure within a loaded earth mass; and, second, to suggest some further simplifications of the method in question.

Existing Methods of Determining Vertical Pressure Within a Loaded Earth Mass.—The problem of determining the vertical pressure, p_z, at a point within an earth mass as caused by a structure at the top (horizontal boundary plane) of that mass, has been solved, to date, in four different ways: (1) Subdividing the loaded area at the base of the structure into small rectangles and replacing each of them with an equivalent concentrated load acting at the center of the rectangle in question; (2) subdividing the loaded area by arcs of circles and their radii into small elements, and replacing the latter with equivalent concentrated loads; (3) developing theoretical (so-called "strict") formulas to express the pressure, p_z, caused by a rectangular or circular loaded area; and (4) compiling tables and tracing graphs. In practically all existing methods the validity of the Boussinesq formula is an assumption; in other words, the earth mass is considered to be an elastically isotropic body. Furthermore, only uniform load distribution through the loaded area is considered, with a few exceptions, mentioned subsequently. The writer has also proposed a method of determining the vertical pressure, p_z, within any earth mass and under uniformly or non-uniformly loaded structures of an arbitrary shape.[28] For the purposes of this discussion, however, the writer's method will be left entirely out of consideration.

A characteristic representative of the methods corresponding to Item (1) is that proposed by Glennon Gilboy, Assoc. M. Am. Soc. C. E.,[29] in 1933. This method is the one most used by soil mechanics investigators in the United States. It yields quite satisfactory results in the case of uniformly loaded areas composed of one or more rectangles. A semi-graphical method elaborated by Messrs. Kögler and Scheidig,[30] of Germany, corresponds to Item (2). This method may be applied to both uniformly and non-uniformly loaded areas. Owing to certain assumptions the method furnishes approximate results. The group under Item (3) is the most numerous. The stressed condition beneath circular and rectangular uniformly and non-uniformly loaded areas has been studied by various investigators, and

[27] Research Associate in Soil Mechanics, School of Eng., Yale Univ., New Haven, Conn.

[27a] Received by the Secretary May 8, 1937.

[28] *Proceedings*, Am. Soc. C. E., April, 1937, p. 669.

[29] Progress Report of the Committee of the Society on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 784.

[30] *Die Bautechnik*, Vol. 6, p. 231 (1928).

pressures have been found mostly at points along the vertical center line of a circle or of a rectangle, as well as under a corner of a rectangle. In this respect, works of Boussinesq, Love,[31] and other theoreticians in the field of elasticity, are to be mentioned (also those of Professor F. Schleicher,[32] of Germany, Dr. O. K. Fröhlich,[33] of Holland, and Professor F. E. Relton,[34] of Egypt). A. E. Cummings,[35] M. Am. Soc. C. E., has studied, in a comprehensive manner, a considerable number of cases of uniformly and non-uniformly loaded circular areas and has developed formulas to compute pressures at points along the vertical center line. Since the methods corresponding to Item (3) are applicable only in the case of regular, circular, or rectangular areas, they have, as a rule, only a restricted value.

So far as the writer is aware, there are two works to be classified under Item (4): That of Nathan M. Newmark, Jun. Am. Soc. C. E.,[36] and that of Dr. Wilhelm Steinbrenner,[37] of Austria. Both these investigators studied the case of uniformly loaded rectangles and computed pressures at the corner; Mr. Newmark has compiled comprehensive tables and Dr. Steinbrenner has traced ingenious graphics which permit the computation not only of the pressure, p_z, but also of the value of the elastic settlement at the corner of a uniformly loaded rectangle. Although the methods of this group refer to rectangular loaded areas only, their particular value consists in their tendency to facilitate the work of the designer, placing in his hands such time-saving devices as tables and graphs.

Chart Method Discussed.—The work of Professor Burmister belongs to Group (4); but it is more applicable to actual engineering computations than the work of Dr. Steinbrenner which is the closest to it as far as the method of attack is concerned. This is because Professor Burmister's method can be used in the case of a uniformly loaded structure of an arbitrary shape. His graphs permit the taking of readings at every foot, or, if desired, more closely, thus approaching the continuity of load distribution. The writer cannot qualify the work of Professor Burmister otherwise than as a correct step in the right direction; but, at the same time, he wishes to emphasize a few features of this method which should be improved, at least partly.

The fact that, in this method, the earth mass is considered only as an elastically isotropic body, cannot be considered as a weakness of the method since the theory of the concentration factor has not been generally adopted as yet by all soil mechanics investigators; and yet it has been felt that at greater depths the Boussinesq formula holds. The method in question,

[31] *Philosophical Transactions,* Royal Soc., Lond., Series A, Vol. 228, p. 377 (1928–29); also "Theory of Elasticity", by S. Timoshenko, p. 335 (1934); see, also, Bibliography given by Love, *Philosophical Transactions,* Royal Soc., Lond., Series A, Vol. 228, p. 377 (1928–29).

[32] *Der Bauingenieur,* Vol. 7, pp. 931, 949 (1926); also, Vol. 14, p. 242 (1933).

[33] "Druckverteilung im Baugrunde", 1934.

[34] *Proceedings,* International Conference on Soil Mechanics and Foundation Engineering, Vol. I, p. 57 (1936).

[35] *Transactions,* Am. Soc. C. E., Vol. 101 (1936), p. 1072.

[36] *Circular 24,* Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1935.

[37] *Die Strasse,* October, 1934; also a special publication by that journal entitled, "Bodenmechanik und neuzeitlicher Strassenbau", p. 75 (1936).

however, involves some less important disadvantages of a purely practical character; (*a*) The given structure or set of footings must be re-drawn on transparent paper to the scale of the diagram (Fig. 3), and this means a certain loss of time; and (*b*), the pressures can be determined only for the depths, for which charts have been prepared. It is true that for intermediate values the pressures can be determined by interpolation; but to use the interpolation method, at least three pressures at three different depths must be found first, before it is possible to trace a curve. Two pressures would give only a straight line and a rough interpolation.

A Simplification Suggested.—Furthermore, Professor Burmister suggests that the re-drawn plan of the structure be pinned down at Point *A*, Fig. 3, and that the plan be revolved for making readings. This procedure may be substantially simplified as follows: (*a*) Charts are prepared on tracing cloth; and (*b*), the plan of the structure is drawn on ordinary paper to the scale of the chart. Fig. 14 represents the first quarter of the foundation layout in Fig. 4. If the vertical pressure is to be determined at some depth under Point *A*, all the footings should be referred to an arbitrary line, *AB*, passing through Point *A*, as follows: With Point *A* as a center, arcs are drawn through all breaks in the plans of the footings. Distances of each arc intersecting footings are added, using dividers to measure chords; for this purpose, these distances are plotted along each corresponding arc, starting from the point of intersection of that arc with Line *AB*. For instance, Segments *pq* (Footing No. 4) and *rs* (Footing No. 2), when plotted upward from Line *AB*, Fig. 14, are represented in Segment *mn*, the length of the latter being equal to the sum, $\overline{pq} + \overline{rs}$. Joining all points such as *m*, Areas *atb* and *cmd* would be obtained bounded at the bottom by the straight line, *AB*, and at the top by the irregular curves shown in Fig. 14. The vertical pressure, p_z, produced by the loaded areas, *atb* and *cmd*, at a particular point of the earth mass located at the vertical

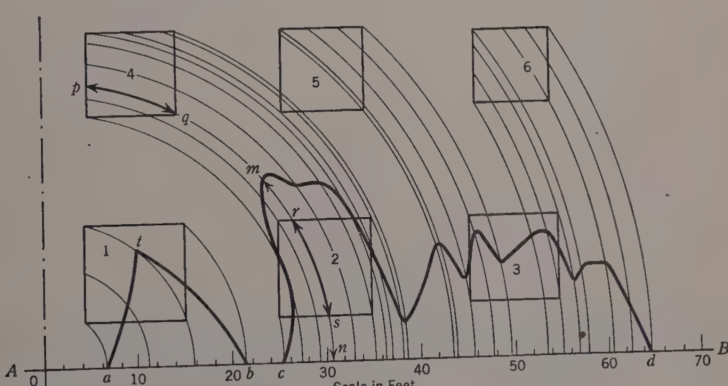

FIG. 14.

passing through Point *A*, equals that produced at that particular point by Footings Nos. 1, 2, 3, 4, 5, and 6. To measure this pressure, the transparent chart is simply placed over the layout of the foundation, making

Points A of both drawings coincide (see Figs. 3 and 14); and Line AB of the layout (Fig. 14) is made to coincide with the basic horizontal line at the bottom of the chart (Fig. 3). Then readings are made along the upper boundary curve, of Areas *atb* and *cmd*.

The writer has made a comparative computation using both Professor Burmister's chart method and the suggested simplification. The saving in time obtained in the latter case can be estimated at least at 25 per cent. The more numerous the footings (or the buildings in the case of a built-up area), the more considerable will be the time saved. Such a feature of the chart method as Table 2 should be abandoned if this simplification is used. Moreover, it would be no longer necessary to pin the graph at Point A and rotate the plans.

Theoretical Basis of the Simplification.—The theory underlying the suggested simplification, may be deduced from Fig. 15. In both Fig. 14 and Fig. 15, Point A is the projection on the horizontal boundary plane of Point O, at which the vertical pressure, p_z, is to be determined. Loads $P_1 = P_2 = P_3$, acting at different points of a circumference for which Point A is the center, produce equal vertical pressures, p_z, at Point O (Fig. 15(a)). This is because the values, r and z, influencing the vertical pressure, p_z, at Point O, are constant for all points along that circumference.

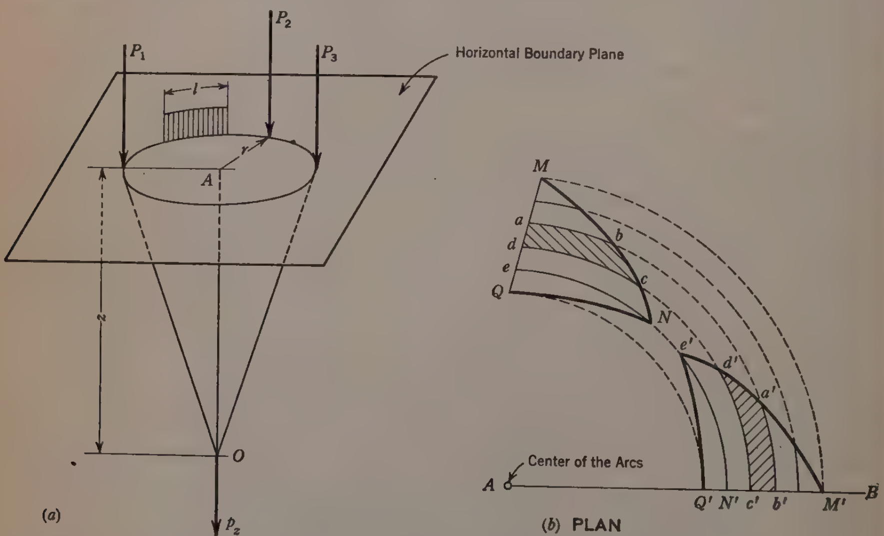

FIG. 15.

In an analogous manner, a line load of a certain intensity and of a length, l (Fig. 15(a)), acting along a part of the circumference under consideration, may be moved to any other place on the same circumference without influencing the vertical pressure, p_z, at Point O. Furthermore, a loaded strip, *abcd* (Fig. 15(b)), may be transferred to Line AB, in the form of the strip, $a'b'c'd'$. The entire area, MNQ, if referred to Line AB, would be represented by the area, $Q'e'M'$. The construction used in Fig. 14 is based on the same idea.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STANDARD PRACTICE IN SEPARATE SLUDGE DIGESTION

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SLUDGE DIGESTION

Discussion

By Messrs. E. P. Leclercq, and H. E. Schlenz

E. P. Leclercq,[14] Assoc. M. Am. Soc. C. E. (by letter).[14a]—The wealth of material and data collected in this report will be of useful service in the design, operation, and maintenance of sewage treatment plants using separate sludge digestion. Although the subject of sludge digestion is not new, relative data have never been compiled and published so thoroughly that, with the rapid increase within the last decade in the number of sewage treatment plants using separate sludge digestion, the publication of these data is most timely.

A subject of prime importance to the municipality and one to be carefully considered by consulting and designing engineers, involves various questions of engineering economics: Is the proposed method feasible? Are the additional structures justified? And is the additional cost of operation and maintenance justified? The quantity and quality of sludge are functions of the quantity and type of sewage, and these, in turn, are directly related to the population served by the plant, to the sewerage system (whether it is separate or combined), to the presence of trade wastes, etc. The production of gas, therefore, is dependent on the quantity and quality of the sludge. In some cases it has been found that the gas collected has been sufficient not only to operate the hot-water boilers for the heating necessary around the plant, but also to operate gas engines for the generation of power in the plant; and sometimes

Note.—This progress report of the Committee of the Sanitary Engineering Division on Sludge Digestion on Standard Practice in Separate Sludge Digestion, was published in January, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the Report.

[14] With The Dorr Co., Inc., New York, N. Y.

[14a] Received by the Secretary March 18, 1937.

there remains a surplus of power to be marketed. There is also the other case in which the small plant produces only sufficient gas to operate a hot-water boiler for heating the sludge digestion tank. It is apparent, therefore, that a certain limit exists beyond which a plant of limited size does not justify the additional expense for separate sludge digestion.

Another problem that affects separate sludge digestion concerns the practice of adding garbage that has been ground into the sewage. There are already many plants where this method is used, and it has been found that this process increases the quantity of sludge formed in the sedimentation tanks. Where domestic sewage is so affected by trade wastes as to reduce the volume of sludge formed in relation to the total quantity of sewage, the addition of garbage which has been ground no doubt will assist in maintaining an increased volume of sludge.

In the operation of the plant, it is important to give careful and constant attention to the gas production, and to pressure losses through the pipe lines, meters, engines, etc., especially when the gas is to be used for power purposes. It is quite necessary to use means of protection against explosion and fire by keeping a careful check on losses along pressure lines and using suitable devices to prevent the entrance of any flames or the admixture of air in explosive proportions. It seems, therefore, that the operation of the plant must be maintained by such methods that a maximum output may be obtained with a minimum of operating expense and the elimination of hazards.

Corrections to be made in this report, as published in January, 1937, *Proceedings,* are as follows: On page 40, line 32, change "308 gal" to "306 gal"; in line 33, change "ten cities" to "nine cities"; in line 34, change "in the other nineteen" to "in the others"; in Table 1, replace all values in Column (16) by the following: "92, 50, 42, (—), 47, (—), (—), 47, 84, 100, 55, 53, 52, 57, 86, (—), (—), (—), (—), 35, 58, 110, 43, 178, 42, 45, 70, (—), 97, 35"; on page 42, line 9 from the bottom, change "nineteen cities" to "twenty-two cities"; in Table 4, captions to Columns (15), (16) and (17), in the numerator change "Column (16)" to "Column (14)" in each case; on page 57, line 25, change "Figs. 2 and 3" to "Fig. 16 and Table 15"; in Table 9*a* move index reference to Footnotes *h* and *i* down one line; and in Fig. 11, change the caption to read "A 300-Horse Power Engine Operated by Sludge Gas, Peoria, Ill."

H. E. SCHLENZ,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—One who carefully studies the data reported by the Committee will appreciate the wealth of information which has been gathered. The members of the Committee have gone a long way in what they term as a "progress report" to cover all phases of the subject. One can not help but feel, however, that the

[15] Vice-Pres. and Sales Mgr., Pacific Flush-Tank Co., Chicago, Ill.

[15a] Received by the Secretary March 26, 1937.

report should be dated as of 1931 or 1932 since, in the main, it covers the subject prior to that time. It will be agreed that a goodly proportion of the data is applicable at present, regardless of the reporting date; for instance, the data on the "Characteristics of Sewage of Typical Cities in the United States" is quite satisfactory for use in present-day problems.

On the other hand tables of installations of types of digestion tanks as reported in Part III portray a very different picture when presented to include only installations made prior to the middle of 1931, as against the inclusion of those from that time until the actual publication of the report. A careful checking of the installations of digestion tanks of the separate sludge digestion type reveals the following interesting comparisons.

(*a*) Table 7 lists 29 installations of open sludge tanks, with stirring equipment of one type or another, installed prior to 1930. Since that time only 3 additional installations of that type have been reported.

(*b*) Table 8 lists 46 installations of covered digestion tanks with rotary stirring equipment designed for gas collection, in the United States, and 6 in foreign countries, by including installations only to about the middle of 1931. It appears that about 10 additional installations of another type in the United States might have been included in this table. However, since the foregoing date there have been installations of the same type made at 106 cities in the United States and more than 21 in foreign countries, including projects under construction.

(*c*) The report mentions only 3 cities using tanks equipped with floating covers. Since that time there have been installations of digesters equipped with floating covers in 149 cities in the United States, including projects under construction.

(*d*) The report projects itself into some of the more recent digestion installations by merely mentioning that there are about 30 cities in the United States that have multi-stage digestion tanks under construction.

A rough summary indicates that since the middle of 1931 there have been installations or projects being constructed with modern digestion equipment, arranged for gas collection, in at least 285 cities in the United States, as compared with the total of about 56 which might have been listed prior to the middle of 1931.

Therefore, since the time of the data on installations presented in the report there has been a fivefold increase in the number of this modern type of digestion installations; therefore, one would expect that certain phases of separate sludge digestion have changed materially in the past five years rendering the report somewhat out of date as far as practice in the United States is concerned.

It is believed that mention might reasonably have been made of some of the large plants constructed since the compilation of the tables of installations included in the report. A few of the larger installations include Cleveland, Ohio, with twelve 95-ft hexagonal digesters (the largest heated digestion plant in the United States) and six 51-ft circular digesters;

Baltimore, Md., with two 100-ft circular units; Atlanta, Ga., with four plants with a total of six tanks 90 ft in diameter, two tanks 55 ft in diameter, and two tanks 30 ft in diameter; Appleton, Wis., with four tanks 70 ft in diameter; Ann Arbor, Mich., with two tanks 65 ft in diameter; Monroe, Mich., with two tanks 60 ft in diameter; and Denver, Colo., being constructed with four tanks 85 ft in diameter.

Under the discussion of gas production from sewage sludge the average heat value of gas from the installations reported in Table 3 is given as 730 Btu per cu ft. This value is quite misleading since the average is taken from a number of unrelated sources. A more complete analysis of Table 3 indicates that (*a*) the average for gas from the eight reported separate sludge digestion plants in the United States is 652 Btu per cu ft; (*b*) the average for gas from the three reported Imhoff plants in the United States is 792 Btu per cu ft; and (*c*) the average for gas from the seventeen reported separate sludge digestion plants in Europe is 738 Btu per cu ft.

It will be noted that the gas from the Imhoff tank plants and European separate digestion plants largely influenced the reported value. A study of a large number of separate digestion plants in the United States indicates the production of gas with heat values of between 600 and 700 Btu per cu ft, with a general average of about 650 Btu per cu ft.

Gas from Imhoff tanks is higher in heat value, due largely to the absorption of carbon dioxide from the gas by the flowing sewage, giving a resulting higher methane percentage.

It is equally true that the data in Table 4 should be segregated to group: (*a*) Separate digestion plants of the United States; (*b*) separate digestion plants of Europe; (*c*) Imhoff tank plants of the United States; and (*d*) Imhoff tank plants of Europe—so that true averages of the various values might be drawn.

A recent study made by the writer, of data from a large number of plants, indicated that one might reasonably expect a production of from 9 to 12 cu ft of gas per lb of volatile solids added to a separate sludge digestion tank, with an average of about 10 cu ft per lb of volatile solids added, or from 17 to 19 cu ft of gas per lb of volatile solids destroyed.

It is believed that a study of the more recent digestion plants will show gas productions from heated digestion plants to average more than the value of 0.61 cu ft per capita per day, as summarized from Table 4. The value will range more nearly between 0.7 and 1.0 cu ft per capita per day. It is further believed that gas production data of the five years since 1932 are more reliable, due to experience which has been gained in preventing gas leakage from digesters, more widespread use of meters built especially for measuring sewage gas, and the provision of more complete digester operating control and compilation of records.

Comparisons of a number of separate sludge digestion plants indicate that it is desirable to consider the loading of volatile solids per cubic foot of sludge digestion capacity per period of time, at the same time that the quantity of gas produced per pound of volatile solids added is discussed. The combination of the two factors gives a value for the gas production

in cubic feet per cubic foot of sludge digestion capacity per period of time, thereby giving a true index to the effectiveness of the digestion in the installation being considered.

Under the discussion of temperature, it is believed that a very important consideration which should be taken into account in selecting an operating temperature is the greater sensitiveness of the process when digestion is carried on at the higher temperature range in the neighborhood of 130° F, as against the normal temperature range at 80° to 90° F.

Fig. 7, showing the cross-section of a digester, indicates a bottom slope of almost 1 vertical in 2 horizontal. Although such steep slopes might have been common in 1931 due to the influence of Imhoff tank design, it was soon found that, with heated digestion tanks, very much flatter slopes might be used. Tanks have operated successfully with bottom slopes of 1 vertical in 12 horizontal. Bottom slopes of 1 on 5 or 6 are most commonly used at present for heated tanks due to the nature of the digested sludge and the activity in the tank which promotes a definite circulation of the contents.

An analyses of the utilization and sale of gas at sewage treatment works at the present time (1937) would make it possible to enhance the usefulness of Table 9 greatly. It is possible now to refer to a considerable number of plants in the United States utilizing gas for power purposes—making it unnecessary to rely for data upon so great a proportion of foreign installations. For example:

Place	Horse-power	Place	Horse-power
Springfield, Ill.[16]	157	Litchfield, Ill.	30
Los Angeles, Calif.	200	Dixon, Ill.	30
Cedar Rapids, Iowa	300	La Crosse, Wis.	180
Janesville, Wis.	30	Chicago Heights, Ill.	125
Peoria, Ill.	835	Atlanta, Ga. (Intrenchment Creek	300
Topeka, Kans.	360	Atlanta, Ga. (Clayton Plant, approximate)	600
Coney Island, New York	900	Durham, N. C.	300
Ann Arbor, Mich.	120	Fort Atkinson, Wis.	40
Richmond, Ind.	120	Rockville Center, N. Y.	...
Edwardsville, Ill.	15	Grand Rapids, Mich.	50+
Sheyboygan, Wis.	240	Aurora, Ill.[16]	140
Green Bay, Wis.	210	District of Columbia	1 200
Elmhurst, Ill.	30	Madison, Wis.	230
Kokomo, Ind.	45	Monroe, Wis.	30
Hutchinson, Kans.	90	Escanaba, Mich.	15
Kewanee, Ill.	30		
Wichita, Kans.	70		

Considerably more might be said regarding explosion hazards in connection with digestion tanks and gas-burning appliances. The need of flame traps or protective devices should be mentioned and emphasized. The arrangement of control chambers to give the utmost protection to the

[16] Listed incorrectly in Table 9.

operator, so as to eliminate the retention of suffocating and toxic gases, should be given careful consideration by the designer and by those charged with the approval of such designs. Ground-level entrances rather than tunnels are to be preferred. Fig. 17 shows a popular and proper arrangement of a control chamber and entrance.

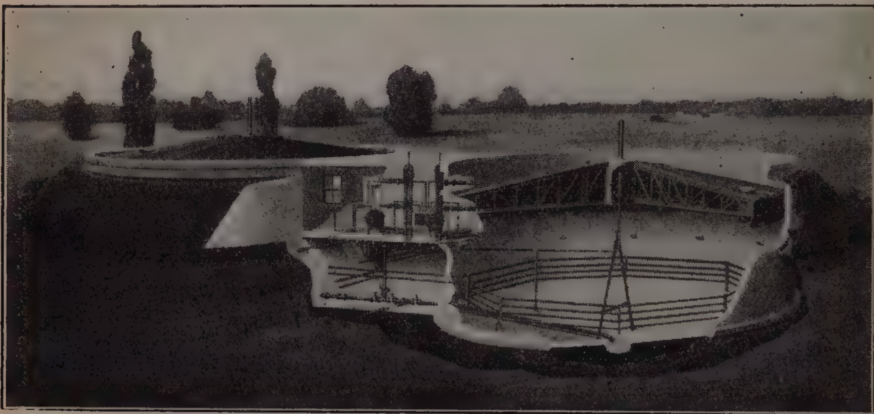

FIG. 17.—ARRANGEMENT OF CONTROL CHAMBER AND ENTRANCE TO GIVE GREATEST PROTECTION TO OPERATOR FROM TRAPPED GASES.

In connection with the computation for tank capacity as given in Part IV of the report great reliance is placed in certain assumptions. For instance, the value, t_o, used to denote the time required for substantially complete digestion is arbitrarily defined as the time required to reduce the volatile solids by 75 per cent. A compilation of calculations made of the percentage reduction of volatile solids by digestion at a number of existing plants indicates that the original content of volatile solids greatly influences the percentage reduction of such solids to obtain a well-digested sludge. The curve shown in Fig. 18 indicates this relationship (see, also, Table 25). A sludge with 60% of volatile solids, when added to a digester, need only have its volatile solids reduced approximately 46% to produce a resultant digested sludge as stable as a sludge of original content of 75% volatile solids, which has its volatile solids reduced 75% by digestion. The curve in Fig. 18 was produced from actual plant data using values based upon each operator's judgment as to the time for the drawing of what he determined to be a sufficiently well-digested sludge.

Many values have been given regarding the limits of the daily additions of raw solids to a digester. One may refer to actual large-scale operating plants to arrive at some indication of loadings of raw solids that may be successfully handled. At the Cleveland (Ohio) Westerly Plant it was found possible to load the digestion tanks to the extent of 5.6 lb of total solids (dry basis) per cu ft of tank contents per 30-day period, or 4.5 lb of volatile solids per cu ft of tank contents per 30-day period. These values are equivalent to allowing 5.36 cu ft of tank

volume per lb of total dry solids added per day, or 6.67 cu ft of tank volume per lb of volatile solids added per day.

TABLE 25.—SOURCE OF DATA; REDUCTION OF SOLIDS BY DIGESTION

Item No.	City (1)	Time of test (2)	Final volatile content of sludge; percentages (3)
1	Birmingham, England....	....	44
2	Philadelphia, Pa.........	....	45
3	Hurlock, Md...........	....	38
4	Grand Rapids, Mich.....	1931	47
5	Aurora, Ill...............	August, 1931	45
6	Elyria, Ohio.............	August, 1931	46
7	Springfield, Ill...........	August, 1930	41
8	Aurora, Ill...............	September, 1931	44
9	Plainfield, N. J..........	August, 1930	50
10	Elyria, Ohio.............	1931†	43
11	Cleveland, Ohio*........	1933	49
12	Durham, N. C...........	February, 1936	42
13	Janesville, Wis..........	July, 1936	47
14	Janesville, Wis..........	1935-1936‡	50

* Westerly plant.
† High.
‡ Average, 1935-1936.

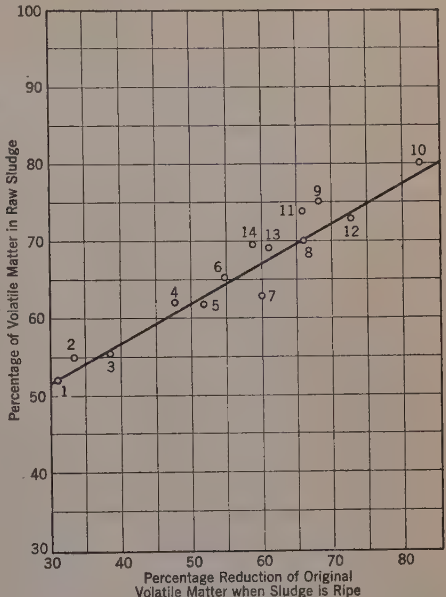

FIG. 18.—REDUCTION OF SOLIDS BY DIGESTION, EXPRESSED IN TERMS OF VOLATILE SOLIDS.

Loadings for shorter periods of time may be considerably greater. The curve in Fig. 19 shows the loadings of raw solids at the Cedar Rapids (Iowa) plant for various periods of time based both on total solids and volatile solids. The values taken from Fig. 19 show an allowance for daily additions of raw solids differing greatly from that which might be obtained from an arbitrary value as reported, such as the "ratio of volatile solids added daily, not exceeding 3% to 5% of the volatile solids remaining in the tank." However, one must assume that proper digester control to allow maximum loadings requires frequent withdrawals of small quantities of digested solids from digestion chambers.

In connection with the "Operating Routine of Sludge Digestion," as reported in Part VI, a number of observations are in order, as follows: In placing separate digestion tanks in operation it has been found that the heating of the tank contents close to the optimum digestion temperature (using outside sources of heat) results in accelerated starting of the digestion and eliminates an overloading of solids and a "foaming" period. This initial heating can generally be accomplished economically and more easily than obtaining material for seeding.

The report indicates that there is considerable difference of opinion regarding the stirring of the contents of a digester; but then it discusses only one side of the question. On the other hand, it should be realized that in the modern heated digestion tank quite a violent action

takes place with resultant natural and selective circulation of the tank contents. The main reason for considering the separation of digestion into two stages is to be able to obtain a better supernatant liquor from the less active second stage, showing that the activity in a digester is more than sufficient without additional stirring to distribute raw solids throughout the tank and maintain a favorable reaction. As solids approach complete digestion and do not produce excessive "gas-lifting activity" they gradually segregate to the lower part of the digester where they are ready for withdrawal. Stirring by external means tends to disturb such solids, which normally should remain at the draw-off point, substituting some partly digested material that is withdrawn prematurely.

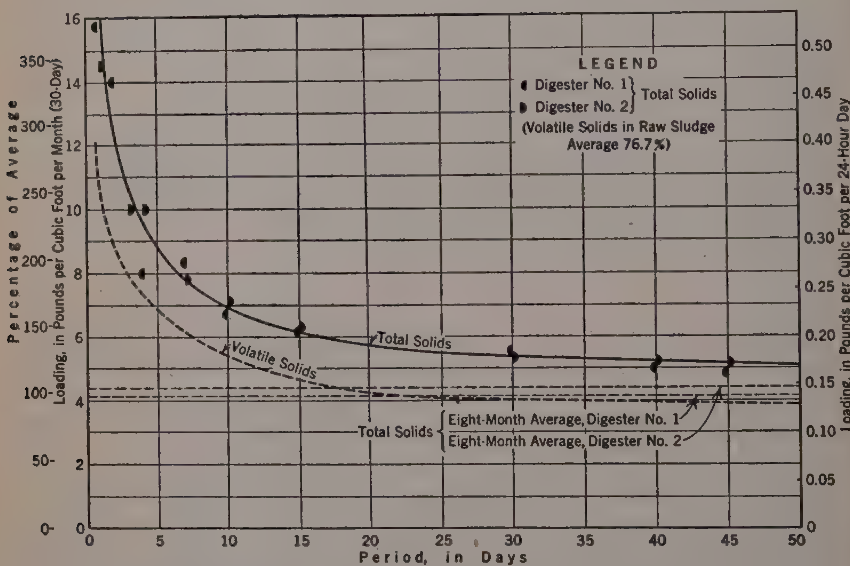

FIG. 19.—CURVES FOR CEDAR RAPIDS, IOWA, SHOWING ACTUAL LOADING OF RAW SLUDGE SOLIDS, EXPRESSED AS POUNDS DRY WEIGHT ADDED PER CUBIC FOOT PER MONTH.

There appears to be a decided tendency toward the practice of providing quiescent settling for supernatant liquor taken from digestion tanks, returning the solids to the digester and the clarified liquor to the raw sewage of the plant. This is especially true where digesters are operated with high solids loadings per cubic foot of capacity.

In addition to the statements made in the report, it has been indicated that "foaming" may generally be the outward appearance of a digester undergoing active digestion following a period of building up of solids during retarded digestion. Many times, the addition of lime will aggravate the condition by increasing the activity. For such a condition the removal of sludge from the tank to start the tank anew is not the logical solution. It is most desirable to lighten the loading of additional raw solids, draw sludge in small quantities at frequent intervals, and keep the gas-vent areas free for the easy escape of the gas.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RAINFALL INTENSITIES AND FREQUENCIES

Discussion

BY MESSRS. J. O. JONES, CHARLES W. SHERMAN, GLEN N. COX, GARRETT B. DRUMMOND, EUGENE L. GRANT, ADOLPH F, MEYER, AND CLINTON L. BOGERT

J. O. JONES,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—A method of increasing the size of the sample of excessive precipitation data for storms of short duration (not exceeding 120 min) is described in this paper. The writer agrees that there is no single station record long enough to give what may be regarded as a fair sample of the total "population" of such data. In any statistical study of chance events the methods of statistics assume that an unbiased sample is available. The authors are to be congratulated for a fine contribution to a difficult and important subject.

It may be a debatable question whether it is permissible to combine the records of stations widely separated geographically, unless the rainfall characteristics are similar in other respects. As an extreme example, one would scarcely feel justified in combining the record of a station in Assam,

TABLE 11.—RAINFALL CHARACTERISTICS, STATE OF KANSAS

Division	Average coefficient of variation	Average ratio of the minimum to the mean	Average ratio of the maximum to the mean
Eastern	0.21	0.60	1.50
Middle	0.25	0.55	1.63
Western	0.28	0.49	1.72

(where the warm southwest monsoons from the Indian Ocean coming in contact with the Himalayas produce precipitation of from 400 in. to 800 in. per yr) with that of Boise, Idaho.

NOTE.—The paper by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, Esq., was published in February, 1937, *Proceedings*. Discussion on this paper has been published in *Proceedings*, as follows: April, 1937, by Messrs. Victor L. Cochrane, and L. K. Sherman.

[14] Prof. of Hydr., Univ. of Kansas, Lawrence, Kans.

[14a] Received by the Secretary, March 20, 1937.

TABLE 12.—FREQUENCY OF EXCESSIVE PRECIPITATION

Inches of rainfall	Number of Occurrences						
	Kansas City	Topeka	Iola	Wichita	Dodge City	Concordia	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(a) Five-Minute Duration							
0.125	21	17	23	18	13	7	99
0.175	54	60	54	60	37	28	293
0.225	59	70	60	50	43	40	322
0.275	47	61	36	38	22	19	223
0.325	26	24	34	23	12	21	140
0.375	14	21	9	10	8	6	68
0.425	4	8	10	7	5	7	41
0.475	4	6	4	3	6	2	25
0.525	5	1	3	2	3	3	17
0.575	2	1	0	3	1	1	8
0.625	1	..	1	..	0	0	2
0.675	..	..	1	..	0	0	1
0.725	..	..	0	..	1	1	2
0.775	..	..	0	..	..	..	0
0.825	..	..	1	..	..	..	1
0.875	..	..	0	..	..	..	0
0.925	..	..	0	..	..	..	0
0.975	..	..	1	..	..	..	1
	..	..	..	..	..	..	1 243
(b) Ten-Minute Duration							
0.175	..	1	..	1	1	1	4
0.225	12	13	15	16	7	5	68
0.275	42	28	27	23	15	14	149
0.325	39	59	46	51	37	27	259
0.375	36	47	39	31	22	27	202
0.425	25	31	32	19	24	13	144
0.475	16	19	17	17	12	8	91
0.525	33	30	12	16	5	7	103
0.575	9	10	16	5	4	5	49
0.625	6	7	7	6	3	8	37
0.675	2	7	5	8	8	6	36
0.725	2	6	5	5	3	5	26
0.775	2	3	1	4	2	1	13
0.825	2	3	4	3	1	3	16
0.875	3	2	1	1	0	1	8
0.925	1	..	1	3	2	1	8
0.975	2	..	2	..	3	0	7
1.025	1	..	0	..	2	0	3
1.075	1	..	1	..	..	0	2
1.125	1	..	0	..	..	0	1
1.175	..	..	0	..	..	1	1
1.225	..	..	0	..	..	..	0
1.275	..	..	0	..	..	..	0
1.325	..	..	0	..	..	..	0
1.375	..	..	1	..	..	..	1
	..	..	..	..	..	..	1 228
(c) Fifteen-Minute Duration							
0.275	9	5	12	4	1	1	32
0.325	17	18	13	15	10	5	78
0.375	34	37	27	34	16	26	174
0.425	37	29	28	30	24	21	169
0.475	16	30	28	17	14	13	118
0.525	23	30	30	18	15	9	125
0.575	13	25	13	16	11	8	86
0.625	20	10	14	10	9	5	68
0.675	19	15	12	8	6	5	65
0.725	13	7	7	10	3	5	45
(c) Fifteen-Minute Duration (Continued)							
0.775	3	10	6	4	6	6	35
0.825	1	4	5	3	5	6	24
0.875	4	3	4	6	2	5	24
0.925	3	5	5	5	2	3	23
0.975	3	4	4	5	4	1	21
1.025	2	5	2	2	2	2	15
1.075	0	1	1	2	0	0	4
1.125	3	1	2	2	2	1	11
1.175	1	..	0	0	2	0	3
1.225	0	..	1	1	0	0	2
1.275	0	..	0	1	1	2	4
1.325	0	..	1	1	1	0	3
1.375	0	..	0	..	..	1	1
1.425	0	..	0	..	..	0	0
1.475	1	..	0	..	..	0	1
1.525	0	..	0	..	..	0	0
1.575	1	..	0	..	..	1	2
1.625	0	..	1	..	..	..	1
1.675	1	..	..	..	..	..	1
	..	..	..	..	..	..	1 136
(d) Twenty-Minute Duration							
0.35	14	8	14	9	6	6	57
0.45	50	42	31	44	19	24	210
0.55	36	46	45	27	27	20	201
0.65	26	33	29	31	16	10	145
0.75	28	23	19	18	12	8	108
0.85	15	11	9	16	10	6	67
0.95	10	6	8	5	6	10	45
1.05	4	9	5	8	4	8	38
1.15	4	4	9	5	5	1	28
1.25	4	4	1	5	3	3	20
1.35	2	0	2	1	1	1	7
1.45	0	1	1	1	0	1	4
1.55	0	..	0	1	1	0	2
1.65	1	..	1	..	1	1	4
1.75	0	..	1	..	..	1	2
1.85	1	..	..	..	..	..	1
1.95	0	..	..	..	..	..	0
2.05	0	..	..	..	..	..	0
2.15	1	..	..	..	..	..	1
	..	..	..	..	..	..	940
(e) Thirty-Minute Duration							
0.30	..	..	..	..	..	1	1
0.40	3	4	2	..	..	0	9
0.50	13	9	13	7	8	4	54
0.60	28	16	15	13	8	12	92
0.70	14	23	20	13	15	7	92
0.80	15	22	18	12	9	6	82
0.90	10	14	19	15	11	6	75
1.00	12	9	7	12	4	2	46
1.10	11	9	4	11	6	6	47
1.20	6	7	3	2	5	8	31
1.30	4	3	7	6	3	2	25

TABLE 12.—*Continued*

Inches of rainfall	Number of Occurrences						
	Kansas City	Topeka	Iola	Wichita	Dodge City	Concordia	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*e*) Thirty-Minute Duration (*Continued*)							
1.40	5	1	5	2	4	2	19
1.50	4	3	1	1	4	1	14
1.60	3	0	2	2	2	1	10
1.70	0	1	0	1	0	1	3
1.80	0	1	2	4	0	1	8
1.90	1	2	0	1	0	0	4
2.00	0	..	0	..	1	0	1
2.10	0	..	0	..	..	1	1
2.20	0	..	0	..	..	..	0
2.30	1	..	1	..	..	..	2
2.40	0	..	..	..	..	..	0
2.50	0	..	..	..	..	..	0
2.60	0	..	..	..	..	..	0
2.70	0	..	..	..	..	..	0
2.80	0	..	..	..	..	..	0
2.90	0	..	..	..	..	..	0
3.00	0	..	..	..	..	..	0
3.10	1	..	..	..	..	..	1
	..	..	..	..	..	..	617
(*f*) Forty-Five-Minute Duration							
0.625	9	2	3	4	1	3	22
0.775	16	9	9	5	4	3	46
0.925	14	16	13	3	8	3	57
1.075	7	7	11	6	6	5	42
1.225	5	10	5	6	9	1	36
1.375	1	4	2	4	3	2	16
1.525	9	5	2	4	1	3	24
1.675	8	0	3	2	6	3	22
1.825	1	2	2	1	1	1	8
1.975	0	0	1	0	0	0	1
2.125	4	1	0	4	0	0	9
2.275	0	0	0	3	1	1	5
2.425	0	2	1	1	0	1	5
2.575	0	1	1	..	0	0	2
2.725	1	..	..	..	0	0	1
2.875	0	..	..	..	0	0	0
3.025	0	..	..	..	1	0	1
3.175	0	..	..	..	..	0	0
3.325	0	..	..	..	..	1	1
3.475	0	..	..	..	..	..	0
3.625	0	..	..	..	..	..	0
3.775	0	..	..	..	..	..	0
3.925	0	..	..	..	..	..	0
4.075	1	..	..	..	..	..	1
	..	..	..	..	..	..	99
(*g*) Sixty-Minute Duration							
0.825	3	4	3	1	1	2	14
0.975	12	2	4	5	3	2	28
1.125	8	11	9	0	5	4	37
1.275	2	7	6	1	2	1	19
1.425	1	3	3	3	2	0	12
1.575	4	4	1	3	1	0	13
1.725	6	5	1	4	2	1	19
1.875	5	0	0	1	1	1	8
2.025	3	0	2	1	1	0	7
2.175	1	1	1	1	1	0	5

Inches of rainfall	Number of Occurrences						
	Kansas City	Topeka	Iola	Wichita	Dodge City	Concordia	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*g*) Sixty-Minute Duration (*Continued*)							
2.325	3	0	0	1	1	0	5
2.475	0	0	0	3	1	0	4
2.625	1	1	0	2	0	1	5
2.775	0	1	1	..	0	0	2
2.925	1	1	1	..	0	0	3
3.075	0	..	..	..	0	0	0
3.225	0	..	..	..	0	0	0
3.375	0	..	..	..	0	0	0
3.525	0	..	..	..	1	1	2
3.675	0	..	..	..	..	..	0
3.825	0	..	..	..	..	..	0
3.975	0	..	..	..	..	..	0
4.125	0	..	..	..	..	..	0
4.275	0	..	..	..	..	..	0
4.425	0	..	..	..	..	..	0
4.575	0	..	..	..	..	..	0
4.725	1	..	..	..	..	..	1
	..	..	..	..	..	..	184
(*h*) Eighty-Minute Duration							
0.8	7	..	2	2	2	2	15
1.0	8	1	7	0	4	2	22
1.2	2	3	5	5	1	1	17
1.4	4	2	1	1	0	0	8
1.6	3	3	2	0	1	1	10
1.8	3	1	2	2	2	..	10
2.0	3	1	0	0	0	..	4
2.2	1	1	1	2	1	..	6
2.4	0	1	1	..	0	..	2
2.6	1	..	..	..	0	..	1
2.8	0	..	..	..	0	..	0
3.0	0	..	..	..	0	..	0
3.2	0	..	..	..	1	..	1
3.4	0	..	..	..	..	..	0
3.6	0	..	..	..	..	..	0
3.8	0	..	..	..	..	..	0
4.0	1	..	..	..	..	..	1
	..	..	..	..	..	..	97
(*i*) One-Hundred-Minute Duration							
0.6	1	..	..	..	1	..	2
0.8	2	..	2	..	1	1	6
1.0	4	3	2	..	1	..	10
1.2	1	2	0	1	0	..	4
1.4	1	3	0	2	1	..	7
1.6	3	1	2	1	1	..	8
1.8	2	0	1	1	0	..	4
2.0	0	1	..	..	0	..	1
2.2	2	..	..	..	0	..	2
2.4	0	..	..	..	0	..	0
2.6	0	..	..	..	0	..	0
2.8	0	..	..	..	0	..	0
3.0	0	..	..	..	1	..	1
3.2	0	..	..	..	..	..	0
3.4	1	..	..	..	..	..	1
	..	..	..	..	..	..	46

It does seem reasonable to combine the records from a small area, such as the Sanitary District of Chicago, or even an entire State, where the rainfall characteristics are similar over the State. This the writer has done for the State of Kansas. There is some difference in the rainfall characteristics of Eastern Kansas when compared with the western part of the State. The average annual rainfall varies from about $17\frac{1}{2}$ in. in Longitude 101° to 40 in. in the southeast corner of the State; but the time of occurrence is similar, being greatest in the warm months. As regards annual rainfall, the coefficient of variation and the coefficient of skew are not greatly different for the eastern as compared to the western part of the State. For forty-two stations in the Eastern Division, thirteen in the Middle Division, and eleven in the Western Division, the average coefficient of variation, the average ratio of the minimum to the mean, and the average ratio of the maximum to the mean is as shown in Table 11. Thus, it appears that the annual rainfall is less and more variable in the western part of Kansas. The differences in the statistical parameters, however, are not great. The monthly distribution is similar over the State, although the total depth, month by month, is greatest near the eastern border.

In view of the foregoing, it seems permissible to combine the records of excessive precipitation for the five U. S. Weather Bureau stations within the State, together with Kansas City, Mo., which is on the eastern border. The lengths of the records included in the study are: Concordia, 1908 to 1929; Dodge City, 1892 to 1929; Iola, 1906 to 1929; Kansas City, 1899 to 1929; Topeka, 1899 to 1929; and Wichita, 1903 to 1929—a total of 167 station-yr. These data include all storms for which the depth of precipitation is equal to or exceeds the quantities given by the authors in Table 3. From the records the frequencies in Table 12 were compiled. The values given in the left-hand column, "Inches of Rainfall," are the central values of groups; for example, for rainfall of 5-min duration, the depth, 0.275 in. includes all values from 0.25 in. to 0.29 in., 0.275 in. being the central value of the group.

From the data in Table 12 the usual statistical parameters were calculated for each station separately, with the result shown in Table 13. The remarkable agreement among the several stations, for the several durations, as regards the mean and the coefficient of variation, indicates that the data for the several stations are comparable. The somewhat greater variation in the coefficient of skew is not particularly significant considering its relatively large probable error when the coefficient is large and the sample is small.

The coefficient of skew for the Kansas City data is greater than the average of all the stations for all durations except those of 5 min and 100 min. This is the effect of a single storm, August 3, 1906, which was an unusual one.

TABLE 13.—EXCESSIVE PRECIPITATION IN KANSAS, U. S. WEATHER BUREAU STATIONS

(i = Mean intensity, in inches per hour; c_v = coefficient of variation; c_s = coefficient of skew.)

City	i	c_v	c_s	i	c_v	c_s	i	c_v	c_s
	5-min.			10-min.			15-min.		
Kansas City	3.06	0.37	1.15	2.54	0.39	1.68	2.24	0.39	1.93
Iola	3.09	0.43	2.31	2.57	0.39	1.86	2.19	0.39	1.55
Topeka	3.04	0.33	0.89	2.51	0.33	1.04	2.16	0.33	1.16
Dodge City	3.07	0.41	1.45	2.53	0.42	1.53	2.29	0.39	1.28
Concordia	3.19	0.37	1.42	2.65	0.39	1.31	2.29	0.42	1.51
Wichita	2.96	0.37	1.23	2.52	0.38	1.18	2.22	0.39	1.24
Average	3.07	0.38	1.41	2.55	0.38	1.43	2.23	0.38	1.44
	20-min.			30-min.			45-min.		
Kansas City	2.00	0.41	1.89	1.78	0.44	1.58	1.57	0.48	2.12
Iola	2.00	0.38	1.47	1.76	0.38	1.40	1.52	0.37	1.54
Topeka	1.94	0.33	1.25	1.72	0.35	1.24	1.55	0.37	1.52
Dodge City	2.09	0.38	1.20	1.84	0.35	0.88	1.67	0.36	1.64
Concordia	2.12	0.41	1.19	1.89	0.38	0.96	1.77	0.46	1.42
Wichita	2.01	0.37	1.12	1.88	0.35	1.02	1.78	0.39	0.55
Average	2.03	0.38	1.35	1.81	0.38	1.18	1.64	0.40	1.46
	60-min.			80-min.			100-min.		
Kansas City	1.58	0.47	1.86	1.39	0.50	2.06	1.45	0.47	1.02
Iola	1.36	0.38	1.65	1.30	0.32	1.06	1.02	0.31	1.32
Topeka	1.42	0.36	1.50	1.32	0.31	0.89	....	....	....
Dodge City	1.55	0.40	1.42	....	....	....	....	....	....
Concordia	1.47	0.52	1.54	....	....	....	....	....	....
Wichita	1.69	0.34	0.19	....	....	....	....	....	....
Average	1.51	0.41	1.36	1.33	0.38	1.34	....	....	....

When the records of the stations are combined the result given in Table 14 is obtained. A typical histogram and frequency curve is shown for the 5-min duration in Fig. 9. The frequency curve is fitted to the data by Pearson's Type III curve.[15]

TABLE 14.—EXCESSIVE PRECIPITATION; SUMMARY OF STATISTICAL PARAMETERS, KANSAS DATA

Total frequency, F (1)	Duration, in minutes, t (2)	Average intensity, in inches per hour, i (3)	Coefficient of variation c_v (4)	Coefficient of skew, c_s (5)
1 243	5	3.05	0.382	1.53
1 228	10	2.56	0.366	1.69
1 136	15	2.21	0.385	1.50
940	20	2.00	0.377	1.42
617	30	1.80	0.379	1.43
299	45	1.66	0.392	1.68
184	60	1.50	0.407	1.67
97	80	1.40	0.402	1.58
46	100	1.38	0.410	1.43
21	120	1.24	0.338	0.24
Average*	...	....	0.386	1.55

* The average does not include the 120-min. duration, because of the small total frequency.

15 *Transactions*, Am Soc. C. E., Vol. LXXXVII (1924), p. 148.

Using the average intensities, i (Column (3), Table 14), and the average values, $c_v = 0.386$ and $c_s = 1.55$, the ogive curves, Fig. 10, were constructed by the use of Foster's skew factors.[16] The "raw" data for the 5-min duration

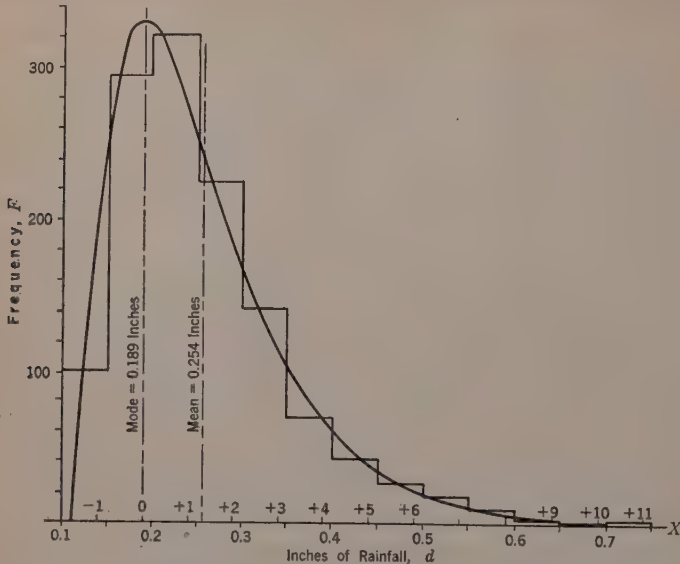

FIG. 9.—EXCESSIVE PRECIPITATION IN KANSAS; INCHES OF RAINFALL IN 5 MINUTES; U. S. WEATHER BUREAU STATIONS.

is compared with the ogive curve obtained by the use of Foster's skew factors, and with the curve obtained by the use of Pearson's Type III frequency curve, in Fig. 11. No comparison was made with the "raw" data

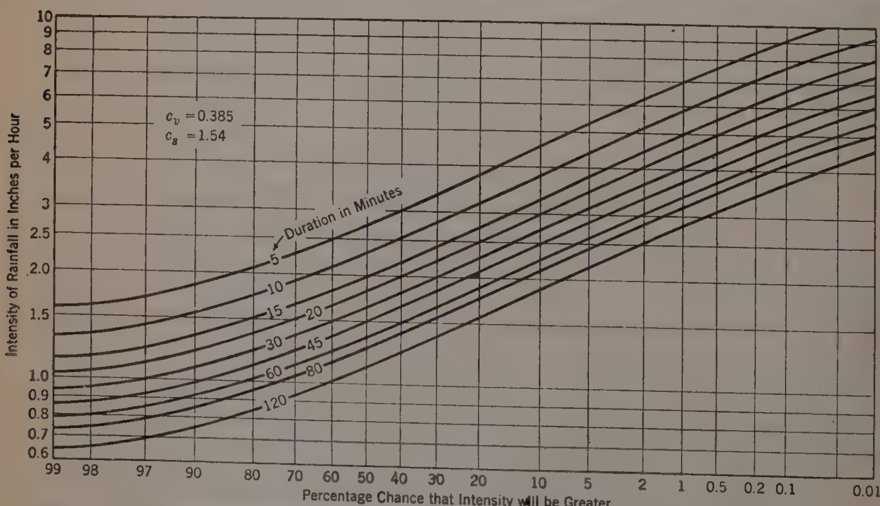

FIG. 10.—EXCESSIVE PRECIPITATION IN KANSAS; U. S. WEATHER BUREAU STATIONS.

[16] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 162.

for the other durations. The labor of computing the ogive curve by means of Pearson's equation is prodigious. An examination of the method used by Foster[17], in deriving his skew factors for the Type III curve, leaves one with the conviction that the ogive curve, obtained by the application of the skew factors, will agree closely with that obtained from Pearson's equation, as it does for the 5-min duration.

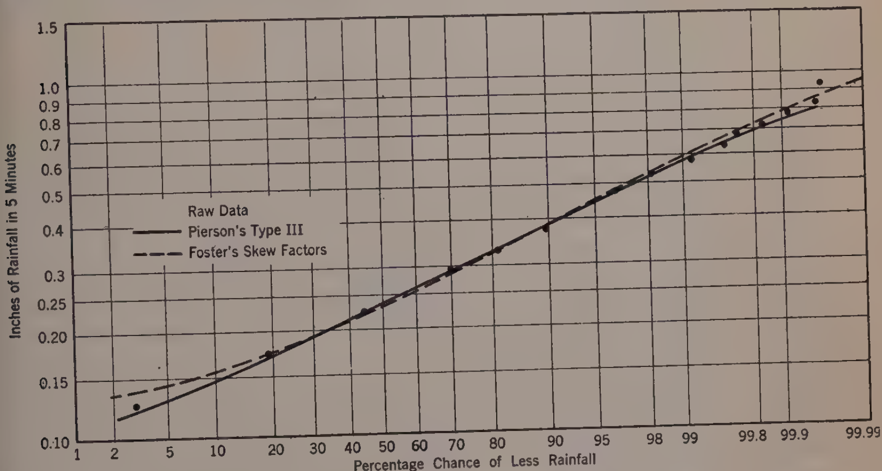

FIG. 11.—COMPARISON OF RAW DATA WITH FOSTER'S SKEW FACTORS AND PEARSON'S TYPE III FREQUENCY CURVE.

Equations 18(*a*) to 18(*g*) yield results which agree closely with Meyer's[18] equation for his Group 3; but a comparison with the results from the ogive curves is difficult. It depends upon what is meant by the term, "frequency". To state that a rainfall has a frequency of "once in 100 yr" does not necessarily mean that it has a 1% chance of occurring. There is some danger in assuming that because a certain maximum intensity has only been recorded once in 100 station-yr it, therefore, has a frequency of once in 100 yr. At Topeka, for the 5-min duration, an intensity of 6.9 per hr was experienced only once prior to 1929. In the interval, 1929 to 1935, inclusive, an intensity of 6.9 in. per hr was experienced once, and an intensity of 8.0 in. per hr was experienced once. Thus, prior to 1929 an intensity of 6.9 in. per hr had a frequency of once in 30 yr, but by the end of 1935 that intensity had a frequency of once in about 12 yr. Of course, by taking a large number of station-years such absurdities are "ironed out".

It would seem to be more logical to apply the methods of statistical analysis to chance events, and some statement involving the term, "percentage chance", would be preferable to the term, "frequency", as commonly used. To illustrate, there are on the average about four excessives per year, of 15-min duration, at Dodge City, whereas, at Topeka, there are on the average about eight. For any one of these excessives there is a 50%

[17] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 159 *et seq.*
[18] Hydrology," by Adolph Meyer, M. Am. Soc. C. E.

chance that it will exceed 2.0 in. per hr; a 10% chance that it will exceed 3.3 in. per hr; a 1% chance of 5.1 in. per hr; and 1 chance in 1 000 of an intensity of 6.7.

The Talbot formula has long been a favorite, because of its simplicity and its successful use in design. Considering the paucity of data at the time it was first proposed, it has given remarkable results in the design of storm sewers. It is not a good 'fit", however, to the record of a single storm. It can be stated in the form,

$$i\,t = A - i\,B \qquad \text{....................(28)}$$

in which $i\,t$ (intensity times duration) is the y-variable and i (intensity) is the x-variable. In this form, it is the equation of a straight line, A being the intercept on the y-axis and B, the slope of the line. In Fig. 12 is shown the intensity duration of the four greatest storms that have been recorded at Kansas City. Only one of these storms, that of September 6 and 7, 1914, is a good approximation to a straight line. The storms of June 8, 1910, and September 15, 1914, approximate a straight line if the 5-min and 10-min durations are omitted.

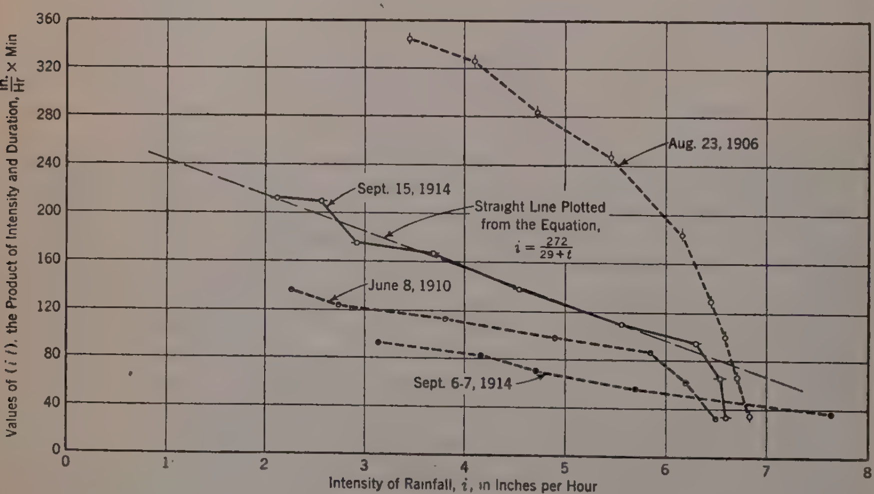

FIG. 12.—EXCESSIVE PRECIPITATION, KANSAS CITY, MO.

The result is little, if any, better when the combined data of several stations are considered. In Table 15 is listed the twenty greatest intensities from the record of 167 station-yr for the State of Kansas, arranged in order of magnitude without regard to the location of the station. These data, when plotted as in Fig. 12, are irregularly curved lines until the high orders of magnitude are reached.

The writer is forced to the conclusion that attempts to generalize, in the form of an algebraic equation, are futile as far as conditions in Kansas are concerned. Formulas are convenient, and, in the hands of an

TABLE 15.—MAXIMUM INTENSITIES OF RAINFALL IN KANSAS, ARRANGED IN THE ORDER OF MAGNITUDE

	DURATION IN MINUTES									
	5	10	15	20	30	45	60	80	100	120
1	11.64	8.22	6.60	6.45	6.14	5.47	4.74	4.09	3.44	2.26
2	9.60	7.08	6.40	5.55	4.58	4.37	3.52	3.20	2.92	1.75
3	9.12	6.72	6.28	5.22	4.54	3.93	3.47	2.56	2.12	1.60
4	8.40	6.54	6.20	5.10	4.26	3.68	2.96	2.42	2.11	1.58
5	7.80	6.30	5.84	5.07	3.98	3.51	2.93	2.37	1.92	1.56
6	7.68	6.18	5.48	5.04	3.84	3.51	2.92	2.34	1.85	1.50
7	7.20	6.12	5.32	4.89	3.80	3.36	2.81	2.25	1.85	1.46
8	6.96	6.12	5.24	4.80	3.78	3.31	2.73	2.19	1.79	1.44
9	6.84	5.94	5.20	4.65	3.72	3.24	2.65	2.13	1.72	1.42
10	6.84	5.70	5.16	4.62	3.68	3.16	2.65	2.12	1.67	1.26
11	6.84	5.70	5.08	4.41	3.66	3.15	2.64	2.11	1.64	1.23
12	6.72	5.70	5.00	4.41	3.64	3.05	2.56	2.06	1.61	1.22
13	6.72	5.70	5.00	4.29	3.64	3.04	2.55	2.03	1.58	1.15
14	6.60	5.70	4.92	4.29	3.64	2.95	2.50	1.94	1.58	1.02
15	6.60	5.70	4.92	4.14	3.56	2.93	2.44	1.90	1.55	0.97
16	6.48	5.70	4.72	4.08	3.56	2.93	2.43	1.88	1.52	0.97
17	6.48	5.64	4.72	4.08	3.54	2.92	2.40	1.88	1.47	0.92
18	6.48	5.58	4.64	4.05	3.50	2.91	2.38	1.87	1.41	0.79
19	6.48	5.58	4.52	3.96	3.44	2.89	2.37	1.85	1.41	0.68
20	6.48	5.46	4.52	3.96	3.40	2.84	2.30	1.84	1.39	0.66

expert, may serve a useful purpose, but they appear to be so easy to apply that they are likely to lead to absurd results in the hands of the inexperienced. It would seem that the histogram or the ogive curve is preferable.

CHARLES W. SHERMAN,[19] M. AM. SOC. C. E. (by letter).[19a]—The writer is particularly impressed by the almost unqualified faith of the authors in the assumption that the total number of station-year records may be used in studies of frequencies without regard to the matter of geographical locations, topographical conditions, or meteorological cycles.

Perhaps the first extensive use of combined records from various stations appeared in Part V of the Technical Reports of the Miami Conservancy District, entitled "Storm Rainfall of Eastern United States," which appeared originally in 1917. Quoting from the 1936 edition of this report:

"In order to utilize all existing records that possessed any value, even though they differed materially in length, a method was adopted which may be explained as follows:

"Assume a number of rainfall stations, say, five, located within an area possessing uniform rainfall characteristics. At station A complete records have been kept for a period of 70 years; at station B, for 40 years; at station C, for 60 years; at station D, for 80 years; and at station E, for 50 years; the aggregate of the period of records or sum being 300 years. Treating this aggregate as a single record for the area under consideration, it may be said by the above definitions of frequency, that the highest rainfall intensity recorded in the entire period has occurred with a probable frequency of once in 300 years."

[19] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[19a] Received by the Secretary April 5, 1937.

The most important words in this description are "located within an area possessing uniform rainfall characteristics." Too many studies have been made in which the writers have totally lost sight of this qualification, and the authors appear to have given little consideration to this requirement when they combined in their studies the records of stations as far apart geographically as Boston, Mass., Dodge City, Kans., Knoxville, Tenn., and Yankton, S. Dak. The topography and climatological conditions affecting quantities and intensities of rainfalls within this wide area vary so much that there is grave doubt whether a combination of the records is of any significance whatever.

Indeed, it may be noted that stations located only a few miles apart may be subject to such different conditions that it would be distinctly improper to combine the records. For instance, data relating to rainfall intensities at the summit of Mt. Washington, in New Hampshire, and at a point perhaps not more than 20 or 30 miles distant at a much lower altitude, certainly should not be combined. It is questionable also how far the records of several stations located within a district measuring not more than 10 to 15 miles in any direction should be combined in any analysis of frequency. Records from such gages are of great importance in studying the extent of the area covered by intense rains[20]; but that the records of ten such gages which have been maintained for 10 yr could be of equal value to a similar record of a single gage for 100 yr in any study of frequency, obviously is impossible.

Another item which is fully brought out in the foregoing quotation, but perhaps not emphasized as much as it should be, is that the record at each of the stations from which data are proposed to be combined, should be of considerable length. The typical example quoted relates to five stations, with a least period of record of 4 yr. Granted that the topographical conditions are such that rainfalls of like frequency and intensity may occur at each of these several stations, there can be no question that the combined record of the five stations is of much more value than that of a single station. If, however, the combined record totaling 300 yr in the example given, were based on 50 stations having an average length of record of 6 yr per station, and perhaps ranging from 1 yr to 20 yr at the different stations, the combined record would be of little greater value than that of the single station having the longest record.

A paper[21] presented by the late David L. Yarnell, M. Am. Soc. C. E., which was issued in 1935, contains a much more extensive analysis of Weather Bureau records of intense precipitation than that shown by the authors, and appears to be of distinct importance. Mr. Yarnell stated: "The methods followed and the results obtained differ considerably from those of the Miami Conservancy District."

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[20] "The Distribution of Intense Rainfall and Some Other Factors in the Design of Storm-Water Drains," by Frank A. Marston, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 535.

[21] "Rainfall Intensity-Frequency Data." *Miscellaneous Publication No. 204*, U. S. Dept. of Agriculture.

The importance of meteorological cycles must not be forgotten in any study of this kind. In this discussion the term, "cycles," is used without particular reference to its precise applicability. Whether or not changes of meteorological conditions occur with such regularity that they may properly be called cycles, there can be no question that wide differences of climatological and meteorological conditions have occurred within historic times. For the purposes of this discussion it may be considered that what is herein called a "cycle" probably has a duration of not less than 300 yr and may be as long as 1 000 yr. Any combination of the records from a number of stations, which have been obtained in the years during the part of a cycle when the rainfall is either high or low, cannot be accurate when consideration of frequencies of 300, 500, or 1 000 yr is discussed.

With regard to the type of formula which may best represent the relation between intensity of rainfall and its duration, for any given frequency, the authors state that the form, $i = \frac{A}{t + b}$, was found to be best. It might well have been noted that the form,

$$i = \frac{A}{(t + b)^d} \qquad (29)$$

would be a more satisfactory form and one applicable to all attempts thus far made, of which the writer has knowledge, to express this relation by a formula. If the exponent, d, is unity, Equation (29) is identical with Equation (3). If, at the same time, the value of b is 0, Equation (29) conforms to Professor Nipher's formula of 1885 for St. Louis, Mo. If b has a value of 0 and d a value differing from unity, the formula corresponds to that used by the writer[22] in 1905; and if b has a value other than 0 and d a value other than unity, the formula would correspond to that used by the writer[23] in 1931.

Conclusions (6) and (7) of the paper might perhaps have been considerably modified after consideration of the discussion in Mr. Marston's paper.[22] It appears probable that the data studied by the authors might have been analyzed by the methods suggested by Mr. Marston so as to provide information of great value regarding the variations in rainfall intensity over areas of moderate size.

Glen N. Cox,[24] Assoc. M. Am. Soc. C. E. (by letter).[24a]—Many data have been presented very ably in this paper, which should make possible closer predictions as to expected rainfall rates for any mid-western section. Nevertheless, it raises certain questions as follows: The U. S. Weather Bureau standard recording rain-gage is a tilting bucket type. Gages of this type are able to operate at a certain maximum rate due to the inertia

[22] *Transactions*, Am. Soc. C. E., Vol. LIV (June, 1905), p. 173.

[23] *Loc. cit.*, Vol. 95 (1931), p. 951.

[24] Prof. of Mechanics and Hydraulics, Louisiana State Univ., Baton Rouge, La.

[24a] Received by the Secretary April 5, 1937.

of the parts and will not record correctly excessive precipitation. The readings given by the record chart are low. This under-recording will begin when the intensity exceeds about 3.5 in. or 4 in. per hr. Then it becomes necessary to distribute any difference between the stick measurement and the chart value by empirical methods. The last sentence of the first paragraph under the heading "Stations Studied" reads: "No correction factor was used for the precipitation corresponding to any duration." In Table 4 (*a*), three occurrences of precipitation between 0.70 and 0.79 in. are listed. These values correspond to an intensity of about 9 in. per hr which is truly of cloudburst proportions and one which a gage of the tilting type can not record accurately. For this reason, one wonders as to the exact meaning of the foregoing quotation.

Some consideration should have been given to the time of year when certain rates could be expected. After all, the engineer is only interested in precipitation intensities inasmuch as certain run-off rates follow. Many factors other than rainfall intensity affect the run-off rate. For any one water-shed, the time of year, the temperature, extent and condition of vegetation, the condition of the soil, and previous precipitation are very important; and, in addition, the properties of the water-sheds vary greatly. It is entirely possible for a 1-in. slowly falling rain in early spring to cause a greater run-off than a 3-in. dashing rain of midsummer. After all, 1 000 cu ft per sec per sq mile only corresponds to a rate of 1.55 in. per hr. Run-off rates of this magnitude may follow after previous precipitation has saturated the soil and filled the surface storage.

Since April, 1933, the Louisiana State University, in co-operation with the U. S. Geological Survey, under the writer's direction, has maintained a number of U. S. Weather Bureau standard rain cans and one weighing type of recording gage. During this time, data have been obtained which permit the comparisons of Table 16. The number of times that the

TABLE 16.—Number of Times Values in Table 6 Were Exceeded

Frequency, in years	Duration, in Minutes						
	5	10	15	30	60	100	120
1	6	8	12	17	17	16	16
10	0	0	0	0	1	1	1
100	0	0	0	0	0	0	0

Weather Bureau designations were exceeded for excessive rates, for the 5, 10, 15, 30, 60, 100, and 120-min intervals, was 3, 11, 17, 17, 8, 1, and 0, respectively. During this period, in spite of the fact that Louisiana is subject to precipitation of cloudburst intensity, the maximum depth for the 5-min period was 0.42 in., which corresponds to an intensity of 5.04 in. per hr. These data show the relative improbability of obtaining the quantities designated by the author, or by the Weather Bureau, for the short intervals, and also the improbability of obtaining the Weather Bureau's designation for the longer periods.

GARRETT B. DRUMMOND[25], ESQ. (by letter)[25a].—A careful analysis of the method followed by the authors in determining the constants in their various equations raises the question of the actual dependability of empirical

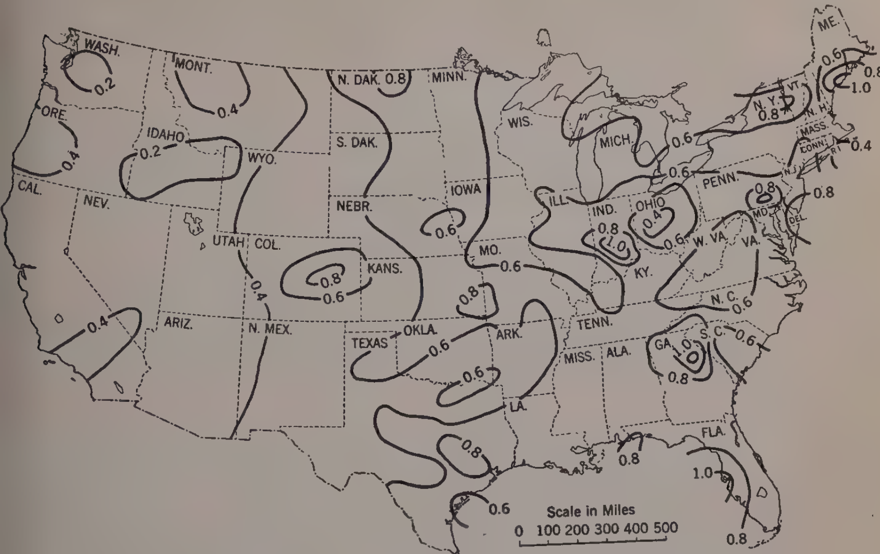

FIG. 13.—MAXIMUM RATE OF PRECIPITATION FOR FIVE MINUTES.

formulas. In their application it should be remembered that such formulas are reliable only within the range of data from which they are derived.

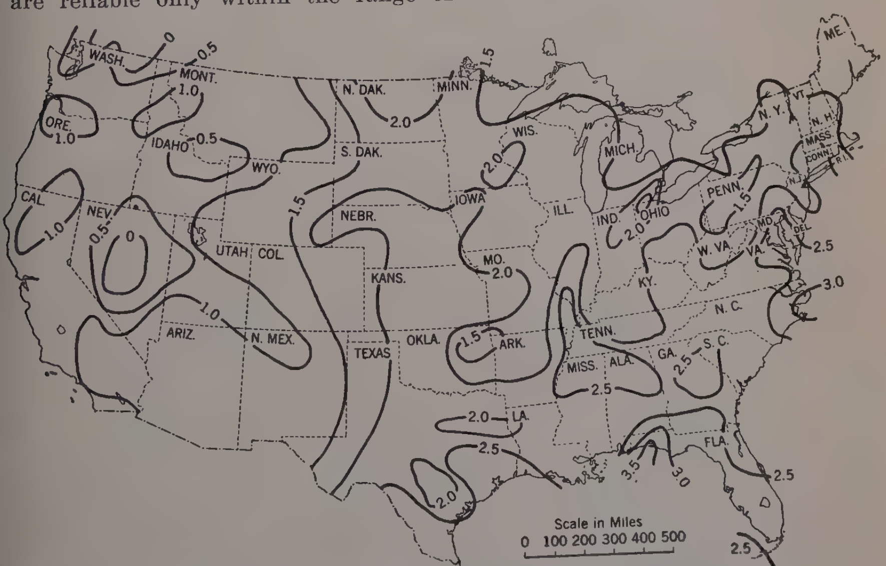

FIG. 14.—MAXIMUM RATE OF PRECIPITATION FOR THIRTY MINUTES.

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[25] Memphis, Tenn.

[25a] Received by the Secretary, April 8, 1937

Especially is this true when the mass of data is segregated arbitrarily into periods of durations, as is done in the case of rainfall intensities. Actually, the "picture" presented by such data is the complete regimen of rainfall. Therefore, by considering separately that falling within certain time limits, there is introduced an artificiality which must be taken into consideration when utilizing the results of such a study.

However, this should not detract from the importance of the study which has been submitted by Messrs. Schafmayer and Grant. Their efforts, although restricted to Chicago, point the way to other investigations which undoubtedly will add to the information now possessed concerning precipitation.

Regarding the use of empirical formulas for sewer design, especially those involving frequency, it is interesting to compare the results with a study of the maximum precipitation for various time-lengths. This, the writer has done for 5-min intervals to include 30 min. Figs. 13 and 14, for example, show maximum rates of precipitation for 5 min and 30 min, respectively.

EUGENE L. GRANT[26] ASSOC. M. AM. SOC. C. E. (by letter)[26a].—An implied conclusion of this study is that the intensities and frequencies of extreme storms of short duration do not vary greatly over a large area of Central United States. Although the area bounded by a line passing through St. Paul, Minn., Detroit, Mich., Knoxville, Tenn., Memphis, Tenn., Dodge City, Kans., and Yankton, S. Dak., contains wide variations in annual rainfall and other climatic characteristics, it is apparently homogeneous with respect to the frequencies of the intense storms which were studied. This implied conclusion should interest all students of meteorology and hydrology; it is of immediate practical importance in that the curves in the paper—particularly the very useful Fig. 5—may be applied to the economic design of storm drains throughout this entire area.

Because of the importance of this conclusion, and because the combining of a 33-yr record for ten stations into what amounts to a 330-yr record seems to be justified only on the assumption that the storms at each station are independent events which are subject to the same system of chance causes, it is desirable to check this conclusion in all possible ways. One such check was the observance by the authors of the close correspondence between the (unpublished) graphs for the individual stations. Other checks may be devised which make use of the methods of mathematical statistics, and which give results expressed in terms of probability. Two types of tests of this character which have been made by the writer seem to confirm the conclusion that the observed differences between the ten stations studied in detail are such as might be expected as chance variations in samples of the size observed, if there were no variations in the cause systems affecting the ten stations.

[26] Associate Prof. of Civ. Eng., School of Eng., Stanford Univ., Stanford University, Calif.

[26a] Received by the Secretary April 9, 1937.

The first of these tests is the so-called χ^2 test of goodness of fit devised by Karl Pearson and elaborated by Mr. R. A. Fisher[27] which has been applied to each of the seven sections of Table 4. A sample calculation relative to the 30-min storms of Table 4(*d*), is shown in Table 17, in which f_o = number of storms of a given magnitude observed at a given station in the 33-yr period; and f_e = number of storms expected, assuming uniform conditions.

TABLE 17.—TEST FOR CONSTANCY OF CAUSE SYSTEM AT TEN RAINFALL STATIONS

City	Number of Storms with the Following Precipitation (in Inches) During 30 Minutes							
	From 0.90 to 0.99		From 1.00 to 1.09		From 1.10 to 1.19		1.20 and more	
(1)	Number observed, f_o (2)	$\frac{(f_o-f_e)^2}{f_o}$ (3)	Number observed, f_o (4)	$\frac{(f_o-f_e)^2}{f_e}$ (5)	Number observed, f_o (6)	$\frac{(f_o-f_e)^2}{f_e}$ (7)	Number observed, f_o (8)	$\frac{(f_o-f_e)^2}{f_e}$ (9)
Knoxville, Tenn.	6	0.74	5	0.23	2	1.97	12	0.60
Memphis, Tenn.	13	2.38	10	2.33	9	2.78	4	3.27
Indianapolis, Ind.	10	0.26	8	0.52	7	0.62	9	0.04
Cincinnati, Ohio	11	0.74	5	0.23	1	3.39	7	0.70
Detroit, Mich.	7	0.26	5	0.23	4	0.28	9	0.04
Chicago, Ill.	4	2.38	8	0.52	6	0.12	11	0.20
St. Paul, Minn.	4	2.38	4	0.78	4	0.28	10	0.02
Cairo, Ill.	9	0.03	8	0.52	9	2.78	10	0.02
Yankton, S. Dak.	13	2.38	6	0.01	5	0.01	11	0.20
Dodge City, Kans.	8	0.03	3	1.65	5	0.01	13	1.20
Number expected, f_e*	8.5	....	6.2	....	5.2	....	9.6	....
Value of χ^2	....	....	....	....	....	....	....	37.13†

* Assuming uniform conditions. † Sum of Columns (3), (5), (7), and (9).

The χ^2 test involves the classification of the observed data into groups or "cells"; the number expected in each cell according to the hypothesis to be tested is then computed. In this case the hypothesis is that all the ten stations are alike with respect to the occurrence of intense storms of each magnitude, so that the number expected at each station is the average number of storms observed at the ten stations for each precipitation. The departure from expectancy in each cell is then squared and divided by the number expected in the cell; the sum of these computed values is called χ^2. Probability theory then gives the relation between χ^2, the number of "degrees of freedom" (the total number of cells compared minus a correction for the number of ways in which the numbers expected are made to agree with numbers observed), and the probability, *P*, that if the expected system of causes were really operating, a departure from the expected number as great as, or greater than, that observed would occur as a matter of chance in random sampling. The purpose of the test, of course, is the determination of this value of *P*.

In Table 17 it has been necessary to group together all storms in excess of 1.20 in. at each station; experience with the test indicates that it

[27] A general explanation of this test may be found in "Probability and Its Engineering Uses", by T. C. Fry, Articles 102–104. The specific application to a problem of the type presented in testing Table 4 for homogeneity is based on unpublished notes on "Statistical Inference", by Prof. Harold Hotelling, of Columbia Univ., New York, N. Y.

cannot be expected to give satisfactory results if the expected frequency in any cell is fewer than five. A similar condensation of cells has been made in testing each section of Table 4.

Table 17 gives a χ^2 value of 37.1. There are 36 degrees of freedom, 40 cells minus 4 restrictions imposed by the condition that the total number expected in each precipitation group agree with the total number observed in that group. Entering a table of χ^2-values, such as may be found in many books on statistical methods,[28] these two values give a probability, P, of 0.4. This is interpreted to mean that if there were no real difference in the cause systems affecting 30-min excessive storms at the ten stations, a variation from uniform results as great as or greater than, that observed might be expected four times out of ten. In other words, the observed data are not inconsistent with the hypothesis of uniformity. The results obtained from χ^2-tests on the other sections of Table 4, are shown in Table 18; they seem to confirm this conclusion. The best fit to the assumption of uniform causes is found in the storms from 60 to 120 min in duration.

TABLE 18.—RESULTS OF χ^2-TESTS APPLIED TO EXCESSIVE RAINFALL RECORDS OF TEN STATIONS

Duration of excessive storms, in minutes	Test of Data in Table 4 for Homogeneity				Value of P obtained from comparison with the Poisson law
	Cells compared	Degrees of freedom	Calculated χ^2	Value of P	
5	30	27	23.6	0.7	0.08
10	40	36	43.0	0.2	0.97
15	40	36	41.0	0.3	0.7
30	40	36	37.1	0.4	0.3
60	30	27	14.9	0.97	0.5
100	30	27	20.8	0.8	0.3
120	30	27	13.8	0.98	*

* Not determined.

The second statistical test made by the writer involved the checking of an hypothesis concerning the relation between the average number of excessive rainfalls per year of each duration, and the probability of any given number of such excessive rainfalls occurring in any single year. This hypothesis is that this probability may be determined by the Poisson law[29], which expresses the probability of any given number of occurrences per period as a function of the average number of occurrences, m. The Poisson law gives the probability of no occurrences as e^{-m}, of exactly one occurrence as me^{-m}, of exactly two as $\frac{m^2}{2} e^{-m}$, of exactly three as $\frac{m^3}{3 \times 2} e^{-m}$, of exactly four as $\frac{m^4}{4 \times 3 \times 2} e^{-m}$, etc.

[28] "Statistical Methods for Research Workers", by R. A. Fisher, Table III, pp. 96–97, or Fry's "Probability and Its Engineering Uses", Table VIII.

[29] For mathematical derivations and discussions of the Poisson law, see Fry's "Probability and Its Engineering Uses", Articles 83–88. For examples of a number of other series which are fitted by the Poisson law, see "Applications of Poisson's Probability Summation", by Frances Thorndike, *Bell System Technical Journal,* Vol. 5 (1926), p. 604.

The characteristics of extreme rainstorms would seem on theoretical grounds to agree with the assumptions underlying the Poisson law, at least in so far as the rainstorms at any one station are concerned. In brief, these assumptions are that the quantity measured is the number of occurrences of a particular event, each sample consisting of a very large number of independent trials of the event in which the probability of the event occurring on any individual trial is very small. Extreme storms are presumably independent events; there are a great many short intervals of time during a year which might have such storms; the probability of an extreme storm in any one of them is very small indeed.

In testing the Poisson law hypothesis it was necessary for the writer to examine the original rainfall data assembled by Messrs. Schafmayer and Grant in order to construct frequency distributions which showed in how many station-years there were no excessive storms of a given duration, and in how many there were one, two, three, four, etc. The expected number of years which would have each number of storms was then calculated by the Poisson law, using the observed average. Table 19 gives the data for the storms of 10-min duration, and illustrates the method followed. Dividing the total of Column (3) by the total of Column (2), the average number of storms per year, m, is found to be $\frac{396}{330} = 1.20$; $e^{-m} = 0.3012$; $me^{-m} = 1.20\ (0.3012) = 0.3614$; $\frac{m^2}{2} e^{-m} = \frac{1.20}{2}$ $(0.3614) = 0.2168$; $\frac{m^3}{3 \times 2} e^{-m} = \frac{1.20}{3} (0.2168) = 0.0867$; etc.

TABLE 19.—POISSON LAW FITTED TO NUMBER OF 10-MINUTE EXCESSIVE STORMS PER STATION-YEAR

Number of excessive storms N, in year (1)	Observed number of station-years with exactly N storms, f (2)	Total storms in each group fN (3)	Probability of exactly N storms by the Poisson law (4)	Expected frequency in 330 station-years by the Poisson law (5)
0	102	0	0.3012	99
1	114	114	0.3614	119
2	74	148	0.2168	72
3	28	84	0.0867	29
4	10	40	0.0260	9
5	2	10	0.0062	2
Totals	330	396	......	330

It will be noted that the number of 10-min storms shown in Table 19 for the 330 station-yr is necessarily the total of 396 shown in Table 4(*b*) for the same station-years. The correspondence between the observed frequencies shown in Column (2), and the theoretical frequencies from the Poisson law shown in Column (5), is extraordinarily good. Similar checks between observed and expected values—although not quite as close as this one—were found for the frequency tables for the storms of other durations. The χ^2-test was used to compare these observed and expected frequencies; the values of P obtained from these tests are included in Table 18.

It would seem reasonable that the relationship herein discovered between the average number of storms per year of a given magnitude and duration, and the probability of any given number of such storms occurring in any one year, justifies the use of the Poisson law for similar calculations relative to extreme storms of any frequency. For instance, any storm shown on Fig. 5 with a frequency, F, of 5 would correspond to an average of 0.2 storm per yr. The probability of no such storms in any one year may be computed by the Poisson law to be 0.819, and of exactly 1 yr to be 0.164; thus, in only 17 yr out of 1 000 would two or more such storms occur in the same year. Such computations would seem to be more of academic than of practical importance, although they might afford a kind of doubtful comfort to the engineer who has just seen his storm sewers overflowed by one such storm.

The important practical conclusion from the Poisson calculations is that they furnish another check on the hypothesis that the excessive storm characteristics are the same at all of the ten stations studied. It is a mathematical property of the Poisson law that two or more Poisson distributions with different averages do not add up to make another Poisson distribution; thus, the check between the total results for the ten stations and the Poisson law is good evidence of similarity between the stations.

The points of similarity and the fundamental differences between this study of rainfall intensity, and the many interesting studies which hydraulic engineers have made of flood frequencies, are worthy of comment. Although the authors have made use of no "probability curves" such as have been used in many flood studies, the same general philosophy underlies the approach to both problems. The evident fact that the authors' studies of excessive rainfall frequency seem much more conclusive than any published studies of flood frequency, results from certain characteristics of the rainfall problem which are not found in the flood problem. These advantages may be recognized if the implications of a frequency or probability approach to a problem of this type are noted.

The statistical or frequency definition of probability may be stated as follows: Assume that if a large number of trials of an event are made under the same essential conditions, the ratio of the total number of trials in which a certain event happens, to the total number of trials made, will approach a limit as the latter number is increased indefinitely; then the limit thus described is defined as the probability that the event shall happen under these conditions.

In the flood studies such as those made by the late Allen Hazen,[30] M. Am. Soc. C. E., the trial may be thought of as a year; the event which either happens or fails to happen is a flood of a given magnitude. Several assumptions are made, either consciously or by implication, in flood studies leading to probabilities for floods of any given magnitude. Among them are: (1) There really is a natural law which relates to the probability of a flood of a given magnitude at a given point on a given stream at a particular time; if it were possible to secure records for a great many years during which time there was

[30] "Flood Flows", by Allen Hazen, John Wiley & Sons, 1930.

no fundamental change in climate or in the characteristics of the stream's drainage area, the ratio of occurrences of a given flood to total years would actually approach a statistical limit; (2) successive trials are made under the same essential conditions or nearly enough so for practical purposes; this implies among other things that they are independent of one another; that is, the flood magnitude in one trial is not related to a flood magnitude in preceding or succeeding trials; and (3) there is a definite relation between the probability or frequency of a flood of given magnitude and the magnitude of the flood, which relationship would be apparent if one had a sufficiently long record. On the basis of a limited record an attempt is made to fit some kind of probability curve to the available data, extrapolating if necessary in order to obtain desired probabilities. Thus, it is common for investigators using these methods to calculate the probability of the 1-yr-in-100 flood from a 30-yr record—or sometimes even the 1-yr-in-10 000 flood.

The logical weakness of such extrapolation beyond the limits of a poorly defined frequency curve is obvious. Different forms of curves may seem to fit the limited data fairly well, and yet have widely different implications outside its range when they are used as a basis of extrapolation. Thus, the methods developed by different investigators of flood frequencies give very different results when applied to the same problem. One investigator may designate as a 1-yr-in-57 flood what another calls a 1-yr-in-500 flood.[31]

Two attempts have been made by investigators to avoid extrapolation by increasing the data used from a limited number of years of record. One method has been to combine the records for a number of streams; the other has been to take all the flood peaks each year in order to obtain a great many floods from a record of short duration. Neither of these devices seems reasonable with respect to flood probability studies. The water-shed characteristics of the various streams are sufficiently different so that floods on separate streams are not under "the same essential conditions;" the successive floods on a given stream in a given year are certainly not independent events.

In studies of intensity and frequency of excessive rainfall, however, it does seem reasonable to use both these devices. As was evidenced by the application of the χ^2-test to the data in Table 4, the excessive rainfalls of short duration at widely separated stations seem to come from the same system of chance causes. Excessive rainstorms are local in character; the storms at different stations at a considerable distance from one another are obviously independent events. Thus, it is legitimate to combine the 33-yr records of ten stations to secure what in effect is a 330-yr record.

It also seems reasonable in these rainfall studies to make use of all the excessive storms that have occurred in a given year. Although two or more flood peaks in a year on the same stream would seem to be definitely related, two or more excessive rainstorms of short duration in the same

[31] See the discussion by C. S. Jarvis, M. Am. Soc. C. E., in *Proceedings* Am. Soc. C. E., Papers and Discussions, October, 1927, p. 2028, for this particular example. The wide differences in conclusions obtained by different methods of extrapolation are familiar to any student of this subject.

year at one station might be expected to be completely independent. This expectation is confirmed by the close fit of the number of storms per year to the Poisson law.

Finally, the investigator of the frequencies of excessive rainfalls who wishes to use his conclusions in the economic design of storm sewers has a great advantage over the investigator of flood frequencies who wishes to use his conclusions in the economic design of structures subject to flood. In theory at least, design for long-run economy of both storm sewers and flood spillways would seem to require consideration of the cost of providing each possible capacity, the expected frequencies with which such capacity would be exceeded, and an estimate of the probable resulting damage. However, the conclusions of economy studies in the two fields, as to the frequencies against which it is economical to design, are likely to be quite different. A typical conclusion might be that a spillway should be designed against at least the 1-yr-in-100 flood, whereas a storm sewer should be designed, at most, against the 1-yr-in-5 excessive rainfall. In the first instance extrapolation to estimate the 1-yr-in-100 flood from a record of 30 yr or less involves much guesswork as to the form of the frequency function. In the second instance, as the authors have demonstrated, a 330-yr record defines its own frequency function so that it is possible to estimate the 1-yr-in-5 rainfall with a high degree of confidence, and there is no necessity for extrapolation.

Adolph F. Meyer,[32] M. Am. Soc. C. E. (by letter).[32a]—This paper gives encouragement to those who have labored long in favor of the use of station-year records from a relatively large area in preference to the present-day inadequate records of intense precipitation at a single station, for the purpose of predicting rainfall frequency. The fundamental soundness of the method of determining frequencies from station-year records is demonstrated by the fact that, notwithstanding the recent severe droughts, the last eighteen years of rainfall records (1929-37), when added to the earlier nineteen years of record for stations analyzed by the writer elsewhere,[33] do not result in any material change in the conclusions respecting the frequencies of given rates of intense precipitation to be expected in the several time intervals from 5 min to 2 hr.

A comparison of the frequency of intense precipitation, as determined from the authors' graphs in Fig. 3, for 659 station-yr, with the frequency of the same rates of intense precipitation for the City of Chicago, Ill., taken from charts presented by the late David L. Yarnell, M. Am. Soc. C. E.,[34] is most striking. The values are almost identical, with the exception of the 100-yr, 2-hr rate shown in the Fig. 3 of the paper. It is the writer's judgment, however, that curves, and not straight lines, should be used to represent the actual facts shown by the observational data in Fig. 3.

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[32] Cons. Hydr. Engr.; Engr. (Meyer Governor Co.), Minneapolis, Minn.

[32a] Received by the Secretary April 19, 1937.

[33] "The Elements of Hydrology", by Adolph F. Meyer, 1917, Group 3, p. 198.

[34] *Miscellaneous Publication No. 204*, U. S. Dept. of Agriculture.

If curves are used, the 2-hr precipitation of 100-yr frequency is substantially reduced and comes into much better agreement both with the Yarnell charts and with the writer's formulas, first published twenty years ago.[35]

The writer has found further satisfaction in the authors' use of definite rates of precipitation for the various time intervals as measures of what constitutes "excessive rainfall" rather than the use of percentages of annual rainfall.[34a] It has always been the writer's contention that there is no material relationship between intense precipitation during short-time intervals and annual precipitation. Stations with low annual precipitation are quite likely to have very intense precipitation for short-time intervals, although the frequency of such excessive rainfalls is usually greater in regions of moderate and heavy rainfall.

Another point cited by the authors which is worthy of emphasis is the fact that the maximum rainfall determined from the original rainfall recording gage charts for any given time interval is greater than the maximum determined from the published tabular data. The writer has called attention to this fact[35] in analyzing approximately 300 storms for Minneapolis and St. Paul, Minn., with a view to determining the correction which should be applied to the tabular data published by the U. S. Weather Bureau. The data published for Minneapolis and St. Paul give results 9% too small for the 5-min interval, 11% too small for the 2-hr interval, and from 2 to 4.5% too small for the intermediate intervals from 10 min to 60 min. These errors of about 10% for the 5-min and 120-min intervals are far in excess of the differences between conclusions drawn from the writer's 19-yr record,[33] as compared with those drawn from the present 37-yr record.

Another fact which bears emphasis, because of a deficiency in the earlier published data of the Weather Bureau and of errors frequently made by those who use such data, is that the accumulated precipitation for 60 or 80 min, for example, must also be considered as having fallen in 100 and 120 min, even if the rain did not continue that long, or was not excessive toward the end and, therefore, was not reported. Where the rain did continue, but the rate was not excessive, the later records of the Weather Bureau now give the necessary information.

Most of the questions to which attention has been directed herein result in increasing the quantity of rain which may be expected in a given time interval. If designs are to be conservative, these several factors must be taken into consideration.

When it is noted that the formula heretofore used in Chicago gave frequencies varying from 3 to 20 yr, which were supposed to represent approximately 5-yr frequencies, it must be apparent that the adoption of new values of frequencies for given rates of intense precipitation will result in better balanced and more economical sewer designs for the City of Chicago.

[34a] Corrections for *Transactions:* In Table 4, heading to Column (9) change "Egypt" to "Ill."; in caption to Fig. 4, change "65 Station-Years" to "659 Station-Years"; in Line 4 following Fig. 7, change "contents" to "constants"; and in Fig. 8, change "Equation (18)" to "Equation (19)".

[35] "The Elements of Hydrology", 1917, p. 165.

The writer believes that the Yarnell charts,[34] are by far the best measure of the frequency of given rates of intense precipitation over most of the United States, available to-day.

Clinton L. Bogert,[36] M. Am. Soc. C. E. (by letter).[36a]—A very complete study of rainfall intensities is reported in this paper. The authors have extended the analytical methods of comparing rainfall intensities and frequencies.

In pursuing statistical methods, engineers should not become so engrossed in the formulas and curves derivable from the data at hand, as to lose sight of the ultimate aim of the study, which is to obtain the run-off from a given area under the worst conditions.

This run-off is dependent on two factors of about equal weight, namely, the rainfall intensity with which this paper is concerned, and the run-off factor. Assuming that the studies are being made to establish the size of drains, the drains will be improperly proportioned if an error in judgment is made in selecting either of the factors. The run-off factor depends upon the slopes and the extent of the built-up and paved areas in the drainage district. Zoning will tend toward making this factor slightly more determinate, but there is no telling when, in the future, Court or legislative action will alter zoning restrictions and automatically put the engineer's assumptions of the future character of the district awry.

Another variable in the problem is the frozen ground in winter which is not considered in arriving at a run-off coefficient. A snow-covered lawn with an ice glazing is equivalent in run-off capacity to a paved area. In most cases of inadequate drains, the flooding will be found to occur in the early spring, due to high run-off from frozen ground and glazed snow.

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[36] Cons. Engr. (Sanborn & Bogert), New York, N. Y.

[36a] Received by the Secretary May 8, 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOW CHARACTERISTICS IN ELBOW DRAFT-TUBES [1]

Discussion

BY MESSRS. R. E. B. SHARP, AND L. F. HARZA

R. E. B. SHARP[24], ESQ. (by letter)[24a].—It is noted that the losses for Bend No. 5 were not included in the tabulation following Equation (22). It would be of interest to have this information. A comparison of Bends No. 1 and No. 4 as made and tested by the author does not show the true relation. The latter shows the smaller loss, but to give a true picture of the relative advantage of adopting the shape of Bend No. 4 as compared to the circular shape, the same radius of curvature of axis should have been used for both bends. For Bend No. 4, $\frac{R}{d} = \frac{15}{6} = 2.5$, whereas this value for Bend No. 1, equals 1.333. If the loss with Bend No. 1 is corrected from $\frac{R}{d} = 1.333$ to $\frac{R}{d} = 2.5$, in accordance with tests reported by Dr. Albert Hoffmann[25], the following approximate values are derived:

(*a*) Loss $= \frac{0.15\ V^2}{2\ g}$ (Mockmore) $\times \frac{0.125}{0.16}$ (Hoffmann tests with smooth walls) $= \frac{0.117\ V^2}{2\ g}$

(*b*) Loss $= \frac{0.15\ V^2}{2\ g}$ (Mockmore) $\times \frac{0.14}{0.17}$ (Weisbach) $= \frac{0.1237\ V^2}{2\ g}$

and,

(*c*) Loss $= \frac{0.15\ V^2}{2\ g}$ (Mockmore) $\times \frac{0.09}{0.11}$ (Alexander) $= \frac{0.1225\ V^2}{2\ g}$

The foregoing corrected values compare favorably, from the stand-

NOTE.—The paper by C. A. Mockmore, M. Am. Soc. C. E., was published in February, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: April, 1937, by F. T. Mavis, M. Am. Soc. C. E.; and May, 1937, by Jerome Fee, Assoc. M. Am. Soc. C. E.

[24] Hydr. Engr., I. P. Morris Div., Baldwin-Southwark Corporation, Philadelphia, Pa.

[24a] Received by the Secretary April 20, 1937.

[25] "Loss in 90° Pipe Bends of Constant Circular Cross-Section," by Albert Hoffmann, *Transactions*, Munich Hydr. Inst., *Bulletin* (pub. by A.S.M.E.), Fig. 1.

point of the circular shape, with those obtained by the author for the elliptical shape of Bend No. 4, in which he found the loss to be $\frac{0.13\ V^2}{2\ g}$.

The tests of the model draft-tubes, as presented by the author, are of interest as such. Tests of this nature may serve as guides for the construction of model draft-tubes for tests with model turbines, but should not be used as the bases for the construction of large tubes in the field. The reason for this, of course, is the impossibility of distributing, properly, the flow at the entrance to model tubes. It is noted that although Model Draft-Tube No. 2 gave the most favorable results, the author did not include data as to its dimensions or the area of its various sections. The lack of these data prevents a comparison and study of this design.

The writer does not think that the author is justified in submitting general recommendations for the designs of elbow draft-tubes on the basis of the tests submitted in his paper. His recommendation that the vertical leg be trumpet-shaped is not borne out by his comparative tests (Bend No. 2 as compared to Bend No. 4). Not only is the efficiency of Bend No. 2, with the conical vertical leg, higher than Bend No. 4, with the trumpet-shaped vertical leg, but, in addition, the outflow loss from Bend No. 2 is decidedly greater than that from Bend No. 4, so that if correction for this fact were made, the efficiency of Bend No. 2 would be relatively increased to a value still greater than Bend No. 4.

The Company with which the writer is connected abandoned long ago the Prasil principle of uniform deceleration, as it was demonstrated in practice that where the diverging angle of the tube to meet this requirement exceeded allowable values (7° to 8°) the water did not follow the tube walls, and harmful eddies were formed. Any impact plate at the end of the tube alters the conditions, resulting in flow in a radial direction. However, this condition does not apply to the vertical leg of a draft-tube.

Furthermore, the writer cannot agree that the trumpet-shape will reduce the cavitation tendency of the runner. It is true that the throat rings of some turbines of the Kaplan type tend to corrode. This, however, does not extend down into the tube, even with 7° to 8° flare, but appears to be a function of the movable guide-vanes above, as the corrosion is limited to parts in the line of flow from the guide-vanes.

Although a splitter is often beneficial, the author should draw no conclusions on that basis because Draft-Tube No. 2 showed better results with, than without, a splitter. This tube was apparently designed for a splitter, and the areas proportioned accordingly. Removing the splitter greatly increases the areas around the bend of the tube — at the very region where the rate of diffusion should be a minimum.

L. F. HARZA,[26] M. AM. SOC. C. E. (by letter).[26a]—It seems to be a general principle that water will flow with the least loss of energy if permitted to conform as nearly as possible with its natural tendencies.

[26] Cons Engr., and Pres., Harza Eng. Co., Chicago, Ill.

[26a] Received by the Secretary, May 6, 1937.

The logic of a wide, shallow design, such as Bend No. 4 (see Fig. 3) can be easily demonstrated by projecting a free stream of water tangentially against a curved sheet of metal. The stream will spread out into a thin sheet as deflected around the curve. The writer is surprised, however, at the small reduction in the loss experienced in Bend No. 4, as compared with the conventional elbow, Bend No. 1. It seems possible that considerably more improvement would result from a still shallower and wider section at the bend if not, indeed, a section that is actually flat on top and bottom. The flattening of Bend No. 4 would not seem to be enough to prevent the formation of the double spiral which could hardly exist in a thin wide section.

The saving of head in Bend No. 4 as compared with Bend No. 1 is only the equivalent of the loss occurring in about 8 in. of straight pipe, which would scarcely seem to justify the more expensive construction of the flattened bend in practical pipe manufacture.

In general, the writer is in agreement with the stated draft-tube principles, except perhaps that the vertical cone should be as long as possible, with short-radius elbow and, consequently, a sharp inside curvature. The writer would indeed agree also with this principle were it not for the better results apparently shown by later tests with the long-radius, sweeping elbow which was finally adopted for Bonneville Dam. It is to be regretted that a parallel test of this design, in a 6-in. inlet size, was not incorporated in the author's comparison. Apparently, it was not included because of the development of this design subsequent to the completion of the 6-in. model series.

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DISCUSSIONS

A NEW THEORY OF RAIL EXPANSION

Discussion

By Messrs. C. W. Baldridge, George W. Hunt, Frank B. Walker, Arthur N. Talbot, Randon Ferguson, A. N. Reece, G. M. Magee, and H. D. Hussey

C. W. Baldridge,[13] Esq. (by letter).[13a]—Under the title, "A New Theory of Rail Expansion", Mr. Africano gives a mathematical analysis of the forces resulting from expansion and contraction of steel rails in track, with a discussion of the probable performances of long rails or continuously welded rails. Tests of long rails have been attracting the attention of engineers since 1890 at least. The earliest tests, of which the writer has found record, consisted of an installation of 3 miles of rail on the Lynchburg and Durham Railroad, near Gladys Station, Va. A description of the test[14] shows the installation to have been made between March 19 and April 25, 1892. Three miles of 78-lb rail of the girder type, with the ends fitted tightly together, were jointed by having the bars riveted as tightly as possible. The track was then "buried in" with earth to the level of the top rail on the outside and to the level of the under side of the head on the inside of the track.

The report provoked considerable discussion, and established some values regarding the stresses resulting from such jointing of rails, which approach those calculated in later studies. Unfortunately, the promoter of the test claimed too much for the plan. He announced that continuous rail, buried in as it was in the test, would eliminate all need for track maintenance, and his attempt thus to handle the test resulted in the track getting into very bad condition. When the line was acquired by the Norfolk and Western Railway Company about three or four years after the installation of the test, the rails were removed and bolted-joint track was substituted for them.

Note.—The paper by Alfred Africano, Jun. Am. Soc. C. E., was published in February, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: April, 1937, by Messrs. Chester F. Gailor, and E. F. Kenney.

[13] Asst. Engr., A. T. & S. F. Ry., Chicago, Ill.

[13a] Received by the Secretary March 9, 1937.

[14] *Engineering News*, 1892, p. 388.

The history of the test seems to have ended with the publication of a reply by the Editor,[15] to a letter of inquiry regarding the test. In the reply, a list of the articles which had been published concerning the test was given.

Almost contemporary with the test on the Lynchburg and Durham Railroad was a test of long rails made on the Michigan Central Railroad, under the direction of A. Torrey, Chief Engineer of that line.[16] One section was composed of: One rail, 800 ft long; one rail, 500 ft long; two rails, 250 ft long, each; and two rails, 100 ft long, each. Originally, it was intended to construct these long rails by electrically welding 30-ft. lengths, but for a preliminary study Mr. Torrey constructed the rails by holding the individual 30-ft lengths together by means of the splices while the drilling for the turned bolts was done. The rail ends were not planes, but warped surfaces; consequently, the joints could not be held in intimate contact.

The rail was laid in the autumn of 1893 in a main-line freight track where the traffic was in one direction only, and consisted of all the east-bound freight business of the main line of the Michigan Central Railroad. The speed over this track at that time did not exceed 20 miles per hr, but brakes were often set and engines sometimes reversed, as it was in the vicinity of an interlocker.

The points to be determined by the tests were (quoting Mr. Torrey):

"(1) Whether a long rail would move freely enough on ties to be safe under sudden and extreme changes of temperature;

"(2) What provisions would have to be made to hold a long rail from creeping under traffic in one direction only;

"(3) Having been forced to use rails held in abuttal instead of welded, whether the joint would be stiffer with the rail ends in abuttal than the ordinary joint is;

"(4) Whether the life of the rail would be appreciably greater if the opening between the heads of the rails was eliminated and the splice relieved from the wear occasioned by the movements of the rail due to temperature changes."

Referring to Item (2), it was reported that under the speed of such traffic as this track accommodated, the rails did not drag the ties, since they were securely connected to other ties for a distance of 28 ft at the point of fastening. The answer to Item (3) was found in the result of tests made at the University of Michigan, Ann Arbor, Mich., which began with the assumption that the stiffness of the full rail was 1. The stiffness of the joint with abutted rail ends was $\frac{1}{2}$, and the stiffness of the joint with $\frac{1}{8}$-in. expansion and without the turned bolts was $\frac{1}{3}$, comparatively. The tests were made with the rails upright and inverted, and the proportionate stiffness was the same for each position.

From December 26, 1893, to January 20, 1894, measurements of the movement at the expansion joints were made in the morning. From January 20 to mid-summer, measurements were made at 7:00 A. M., on

[15] *Engineering News*, March 22, 1906.

[16] *Railroad Gazette*, July 27, 1894, p. 518.

Sundays, and at 7:00 A. M., and at noon, on week days. From December 26, 1893, until April 14, 1894, the theoretically proper range of movement was based on temperatures obtained from the United States Signal Office. From April 15 to mid-summer, the theoretically proper range of movement was based on temperatures obtained from a thermometer embedded in a piece of rail alongside the long rail. In the colder months the rail movement was considerably less than would be forecast from the changes of temperature during those months, if such forecast should be made in the light of the sensitiveness of the rail to changes of temperature, as shown between May 17 and May 20. Mr. Torrey concluded that the rail may or may not assume the air temperatures when the ground is frozen and stated finally that: "* * * defects of construction entered into it [the rail], which would naturally be discarded in its successor, if it has one, and the results as to smooth and noiseless passage of the wheels would probably be more favorable under better conditions."

Another study of longer rails was made public in a committee report on the subject, "Miter Cut Ends and Longer Rails," which was presented at the Convention of the Roadmasters and Maintenance of Way Association of America in the autumn of 1895, in which some comments of interest are to be found.

In recent years interest in the subject of the use of continuous, or very long sections of rail welded together, has resulted in actual tests being made on the Delaware and Hudson Railroad, in four different locations and under somewhat different conditions. Another installation of rails welded together for a full mile has been made on the Bessemer and Lake Erie Railway. These tests are making history and are attracting the attention which such studies deserve.

Although actual tests of continuous rails are necessary to prove conclusively whether or not such installations are dependable and practicable, some information regarding actions which result from contraction and expansion of long rails may be gleaned from the results of the shearing off of track-bolts in rail-joints during periods of severe cold weather.

In 1924, the writer had occasion to investigate a case in which, on a severely cold night following a mild day, nineteen joints in a distance of about 25 miles had sheared off both bolts in one end of the joint, thus allowing the rails to pull apart. The track was laid with 90-lb rail, all-steel joint bars and oil-quenched bolts. Inquiry as to maintenance revealed the fact that the joints had been bolted up as tightly as possible, using wrenches which, by means of pieces of pipe, had been lengthened from 46 to 48 in. The bolt-tightening had been done during an unusually warm spell late in the autumn. A few weeks later a sudden drop in temperature to − 18° F resulted in the shearing of the bolts at joints where for some reason the bolts were less tightly set, thus permitting the rails to slip through the joint bars. Notwithstanding the fact that the track was equipped with single-shouldered tie-plates and only four cut spikes per tie (two for each rail), none of the openings between rail ends widened to more than 5.5 in.

During the present winter (1936–37) a number of joints in track laid with 112-lb, all-steel joints, and high carbon, oil-quenched bolts, have sheared two bolts, allowing the rails to pull apart as far as the contraction would take them. In these cases, the track is also equipped with single-shouldered tie-plates and with four cut spikes per tie. The bolts had been installed by bolting machines when the rail was laid, followed by normal maintenance. Most of the adjacent joints showed that no movement had occurred through the bars, and in most cases the sheared bolts occurred in joints a mile of more apart; yet in these cases the opening between the rail ends was usually about 2 in. and none was reported to have opened more than 2.5 in., even when temperature readings below zero were experienced. These rails were laid using expansion shims based on rails that were tight at 100 degrees.

Taking the coefficient of expansion at 0.0000065 (which is the value generally used for railway rails), the width of opening found in these cases represents the contraction of only 320 ft of rail, and assuming that it was equally effective in both directions, it shows that the contraction of rail steel, due to zero temperature, is overcome by the resistance to movement of only 160 ft of 112-lb rail held by cut spikes in a normal manner.

In order to determine the results, of the forces created by the expansion and contraction of steel due to temperature changes, it is pertinent to begin with a single rail and then to build up a series of rails. It is easy to recognize that a single rail, 39 ft long, resting freely with no connections except the support under it, will have no movement, as a whole, due to contraction and expansion. Connect two rails of the same size, weight, and length, and each rail will exert the same pull upon the other. Add a third rail and the pull of Rail No. 1 upon Rail No. 2 will be equal and opposite to the pull of Rail No. 3, and since the resultant of equal and opposite forces is zero, there will be no movement. It becomes evident, therefore, that as long as there is no break in the connecting joints in a track the forces of expansion or of contraction nullify each other and there is no resultant force to produce movement. It is also evident that if the rails in a track are rigidly connected, the forces acting nullify each other, except for the short distance at each end where the resistance to movement produced by the weight of the rail and the grip of the spikes is less than the unbalanced force of contraction or expansion.

The widths of opening between rail ends, resulting from the shearing of bolts prove that such distances are not great and also that they are shorter for heavy rails than for lighter ones.

These actual results in track go far to confirm the accuracy of the mathematical determination of forces resulting from, and the resistances to movement due to, contraction or expansion of long rails, as given in Mr. Africano's paper. The results found in track as described herein, with the studies made by Mr. Africano, by the Kansas City Southern Railroad Company, by the Rail Committee,[17] and the Track Committee.[18]

[17] *Bulletin No. 391*, A. R. E. A., November, 1936, p. 233.
[18] *Bulletin No. 393*, A. R. E. A., January, 1937, p. 493.

of the American Railway Engineering Association, and by others, give favorable indication of the practicability of the use of continuously welded rails.

George W. Hunt,[19] Esq. (by letter).[19a]—First considering the condition of a rail perfectly free to expand, Mr. Africano proceeds to the condition in which the ends are restrained by means of ballast resistance to tie movement, and, finally, to the condition in which the ends are restrained partly by the ballast resistance and partly by the joints connecting the welded section with the adjacent track sections. The equations developed by Mr. Africano are applicable to all lengths of welded track, from a few rail lengths to a number of miles. Their purpose is to show the relation between: (1) Tie resistance; (2) joint resistance; (3) rail movement under various changes in temperature; and (4) to indicate the proper balance between these factors in order to avoid damaging strains and stresses after the rail is welded.

As Equation (18) contains all of the aforementioned factors ((1) to (4)), and represents the third condition, it may be accepted as the general equation. Equation (9) is simply a special case of Equation (18) when the joint restraint, P, is reduced to zero. For analytical purposes, P may be expressed in fractional parts of F, the total force required to restrain the rail, and incorporated in Fig. 4, thus making this graph to include all factors in the problem. For example, Equations (21) and (22) which determine the values of T for the case of no joint restraint, can be restated as follows to take joint restraint into account:

$$T = \frac{K' (\Delta t)^2}{\Delta l''_{max}} \qquad (25)$$

in which K' (see Equation (22)) equals $0.5\,K$ when $P = 0.3\,F$; $0.25\,K$ when $P = 0.5\,F$; and $0.0625\,K$ when $P = 0.25\,F$; and $F = 2\,816\,(\Delta t)$. In Fig. 5, the value of T can be read for various degrees of tie restraint. For example, suppose that $P = 0.3\,F$; then, for 130-lb rail, with a temperature change of 100°, $P = 0.3 \times 281\,600$ lb $= 84\,400$ lb. For a 60° temperature change, $P = 0.3 \times 2\,816 \times 60 = 50\,688$ lb. For these two temperature changes Fig. 5 shows that $(\Delta l'')_{max}$ (the rail end movement) has the values shown in Table 3.

TABLE 3.—Values of $(\Delta l'')_{max}$

Values of tie restraint, T, in pounds	Critical Length, $(\Delta l'')_{max}$, in Inches, for Temperature Changes of:	
	100° F	60° F
750	$1\frac{3}{8}$	$\frac{1}{2}$
1 000	1	$\frac{3}{8}$
1 250	$\frac{13}{16}$	$\frac{5}{16}$

It is the opinion of the writer that the influence of joint restraint has been under-estimated in the test at Mechanicsville, and that if the

[19] Maintenance Inspec., M. of W. Dept., Baltimore & Ohio R. R., Baltimore, Md.

[19a] Received by the Secretary March 24, 1937.

facts were known the joints at the free ends (so-called) were furnishing part of the resistance, and that the ties were furnishing considerably less than 2 000 lb. The opinion is based on his understanding that the joint gaps at the "free" ends were as much as $\frac{7}{8}$ in., at the temperature drops mentioned in this test.

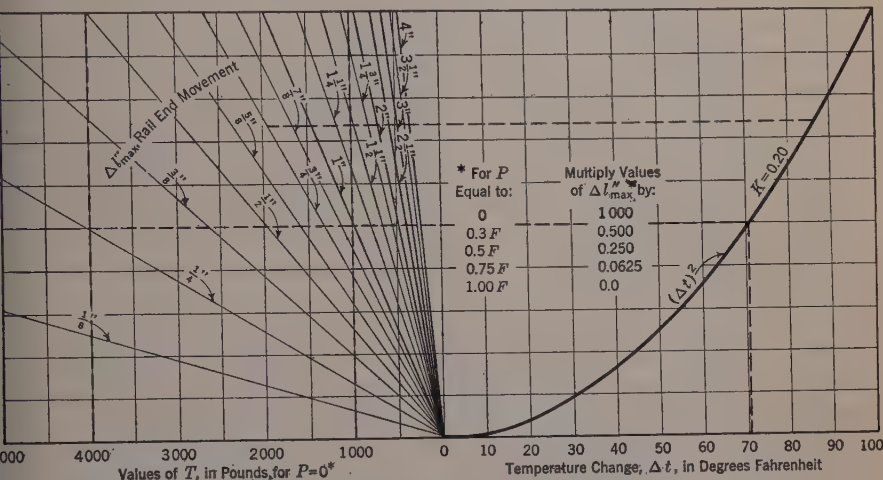

FIG. 5.—RELATION BETWEEN RESISTANCE, TEMPERATURE CHANGE, AND RAIL MOVEMENT UNDER VARIOUS DEGREES OF JOINT RESTRAINT.

Joint restraint in tension is composed of the frictional resistance between bars and rail, plus the bending resistance of the bolts. The frictional resistance in a 6-bolt joint may be taken as $(3 \times 20\,000 \times 2) \div 2 = 60\,000$ lb. The bending resistance of one bolt may be taken as 8 000 lb, or 24 000 lb for three bolts. On this basis the total joint resistance would amount to 84 000 lb, or about 30% of the force, F, required to restrain a 131-lb rail under a temperature drop of 100 degrees. It seems reasonable, therefore, to consider the ultimate value of P in tension as equal to $0.3\,F$. In this connection, it should be remembered that the rail-drilling, bar-punching, and bolt diameter for 131-lb and 112-lb rail sections are proportioned so as to allow only a $\frac{3}{8}$-in. rail gap before bringing the bolts into bending resistance. It follows, therefore, that when the joint gaps exceed $\frac{3}{8}$ in., the frictional resistance of the bars against the rail has been overcome and the bolts are exerting bending resistance. In compression, the ends of the rails come into contact before any bending strain is thrown on the bolts, so that the joint restraint in compression may be taken as 60 000 lb. The fact that a joint gap has greater latitude to enlarge under tension than to contract under compression is one reason why welded track should be laid or closed at temperatures that favor tensile strains, rather than compressive strains. The major reason is that the track is less likely to be thrown out of line by contraction in cold weather than by expansion in hot weather.

Mr. Africano conceives joint resistance as that force at which the joints in the adjacent unwelded section will begin to yield and continue to yield, sufficiently to accommodate $(\Delta l'')_{max}$, the rail-end movement. The actual value of the joint resistance is of little practical importance, serving only to explain the variation in the value of $(\Delta l'')_{max}$ as between theory and practice. In no case could it exceed the value of F; to do so the unwelded track would have to be as stiff as the welded track, a condition scarcely attainable. A high value for it would reduce the number of ties necessary to restrain the rail fully, but this would be to no purpose as long as a special track construction was used throughout. With increasing values it would reduce the value of $(\Delta l'')_{max}$ and to that extent reduce the movement at the rail gaps in the adjacent unwelded section. This objective is desirable as a precaution against tensile strains becoming concentrated in one joint and pulling the rails apart. The question of using expansion joints will depend upon what weight is placed on this hazard.

The question as to the temperature at which welded track should be closed is very important from the viewpoints of both rail stress and rail movement. A range of temperature from 20° below zero to 130° above zero will give a mean temperature of 55°; but as tensile strains are less objectionable than compressive strains, welded rail should be closed and anchored at a temperature between 70° F to 80° F. If this is done, the temperature change above the closing temperature need not exceed 60° and the rail movement in compression will be limited to that temperature change. Similarly, the temperature change below the closing temperature will be limited to 100° and the rail movement in tension to that temperature change.

The value for tie resistance, T, as applied to fairly well ballasted track, has probably been over-estimated in much of the discussion on this subject. There is a temptation to place too high a value on this factor, for rail laid at high temperature, on the theory that with a severe drop in temperature below the closing temperature, the ballast will freeze and thus increase the tie resistance, restrict the contraction movement, and build up the value for F in a shorter distance. This reasoning appears fallacious when it is considered that the rail responds very quickly to temperature change; freezing is a slower process and could scarcely occur in time to offer any resistance to rail movement. In fact, in the absence of sufficient moisture, the ballast may not freeze at all; it may have no higher resistance in cold weather than in warm weather. If the ballast did finally freeze, subsequent to the rail movement, and a thaw should set in, the temperature would have to rise to the closing temperature before any reversal of tie restraint could occur, and considerably above this temperature before the reversal of tie restraint reached any great magnitude. By this time the thaw, in all probability, would have begun to loosen up the ballast. It is felt, therefore, that the process of freezing and thawing can be disregarded as far as influencing the ballast

resistance, and that the factor, T, can be accepted as a fairly constant value for any given ballast section, throughout the year.

An approximation of this value can be made for loose ballast by taking one-half the weight of the rail, ties, and ballast, above the bottom of the ties for a distance of 22 in., and multiplying it by the coefficient of sliding friction, taken as unity. For loose ballast, $T = 0.5\ (192 + 175 + 755) = 561$ lb; and, for packed ballast, with the usual degree of cementation, it would seem that 1 000 lb per tie per rail, is all that could be depended on. Table 4 is illustrative of the manner in which $(\Delta l'')_{max}$, the rail end movement, varies with variation in temperature change, when $T = 1\,000$ lb and $P = 0.3\ F$. Items Nos. (3) and (4), Table 4, represent the conditions that are said to have existed at Mechanicsville, so far as temperature change and rail end movement are concerned. The division of restraint as between ties and joints are the results of the writer's construction and about this there may be a difference of opinion.

TABLE 4.—Variations in $(\Delta l'')_{max}$

Item No.	Tie restraint, T, in pounds (1)	Changes in temperature, Δt, in degrees Fahrenheit (2)	Restraining force, F, in pounds (3)	Joint restraint ($P = 0.3\ F$), in pounds (4)	Number of ties, N, required to develop force, F (5)	Critical length, l'', in feet (6)	Critical end movement, $(\Delta l'')_{max}$ in inches (7)
(1)......	1 000	100	281 600	Free end	432	792	2
(2)......	1 000	100	281 600	84 480	197	361	1
(3)......	1 000	82	230 910	69 270	162	297	$\frac{11}{16}$
(4)......	1 000	58	163 330	49 000	114	209	$\frac{5}{16}$

Where the welded track is isolated (that is connected at both ends by unwelded rail with the usual joints), these rail end movements, even in the case of Item No. (2) are not alarming. They would not result in any greatly enlarged rail gap, as in all three cases, the value of P is sufficient to distribute this movement through from two to four joints. Insulated joints, however, would present a problem, particularly in Item No. (2). Required to withstand a tensile force of 281 600 lb they would be able to withstand only 84 480 lb, and a rail gap of 2 in. would result if the joint failed. The only recourse, as a matter of precaution, would be to install several ordinary rails on either side of the insulated joint so that this movement could be distributed through several joints, or to adopt some extraordinary means of increasing the joint resistance. A similar problem will be encountered at all switch-point joints, on the frog side as well as at the heel joints of all frogs. Here Condition No. (1), Table 4, would obtain, due to the break in the continuity of the main-track rail by the switch-point, and the 2-in. rail end movement would have to be provided for either by a single expansion joint, or by the more simple expedient of installing about four or five rails with ordinary joints, in advance of the switch-point and heel, to distribute this movement. For obvious reasons, both economical and practical, welding through yards involving main-track turnouts should be avoided, and the welding should be terminated at the entering and leaving turnouts to yards.

Item No. (1), Table 4, shows that $(\Delta l'')_{max}$ amounts to 2 in. when the joint resistant is zero. It follows, therefore, that if a rail broke at a temperature drop of 100°, there would be a rail gap of 4 in. If no special track construction was used to utilize the tie resistance, the rail gap would be 8 in. This is one reason why it is advisable to bind the rail to the ties throughout, rather than just for a distance, l'', sufficient to develop the force, F.

It is apparent that if a rail broke, or if for any reason a section of rail were removed, it should be replaced by one of identical length, otherwise abnormal rail stress or rail end movement may result when the temperature changes radically in the other direction. The repair of a broken rail at a radical temperature change without restoring the rail gap, has the effect of creating a free-end condition at the break at that temperature as regards rail strain, without providing any means for rail movement when that temperature alters radically. What would happen under a radical change in temperature in the opposite direction is best illustrated by studying some specific case, such as that of a welded track laid at 80° F. A rail breaks at zero temperature, leaving a gap at the break of 2$\frac{3}{4}$ in., and is repaired by cutting out a distance sufficient to accommodate a new 39-ft rail and then re-welding. The temperature subsequently rises to 130° F. How will the rail stress and rail end movements at 130° compare with these same factors if the rail had gone through temperature changes and not broken? What happens can be

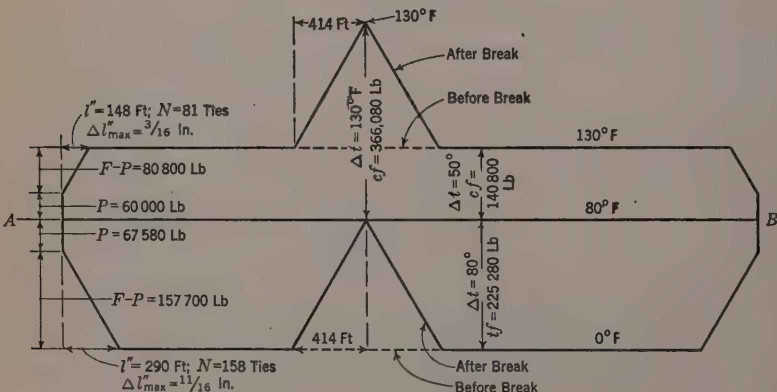

FIG. 6.

best illustrated by reference to Fig. 6, which shows that, had the rail not broken, the rise in temperature to 130° would have resulted in a unit compressive stress of only (140 800) ÷ 12.86 = 11 000 lb and the rail end movement would have been $\frac{3}{16}$ in.

The breaking of the rail at 0° F and its repair by adding 4 in. to the length would result in a concentrated compressive stress at the break of 366 080 ÷ 12.86 = 28 500 lb when the temperature rises to 130° F. This stress would hold as long as the break was far enough away from

End A for the ties and joint restraint to develop a force of 366 080 lb. In Fig. 6 this distance is 148 + 414 = 562 ft. Fig. 7 shows the conditions that would exist had the rail broken 300 ft from the end of the welded section. The theoretical gap at the break would be $1\frac{7}{8}$ in. at zero temperature which it may be assumed, was not restored when repaired. When the temperature rises, a movement of the rail end will occur equal to $1\frac{7}{8} + \Delta l''$.

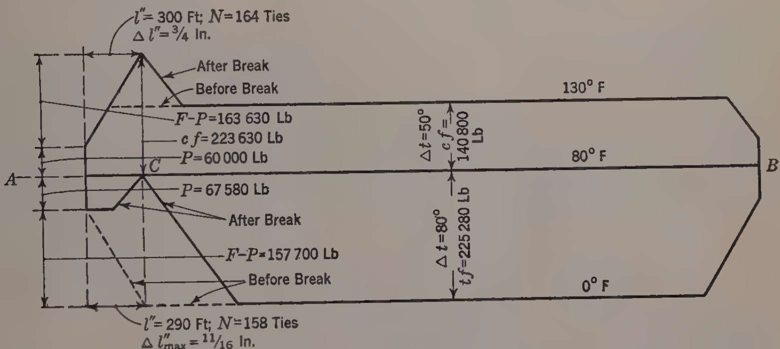

FIG. 7.

When a temperature of 130° is reached, a peak of maximum value for F will arise at Point C, Fig. 7, determined by the value of the joint resistance plus the tie resistance developed in 300 in. This value will be $60\,000 + (300 \times 12 \times 1\,000) \div 22 = 223\,630$ lb, or equivalent to a maximum unit stress of 18 400 lb per sq in. The rail movement at 130° will be equal to $1\frac{7}{8} + \Delta l''$. From Equation (18) the value for $\Delta l''$ is $\frac{3}{4}$ in., and the total rail end movement will be $2\frac{5}{8}$ in.

Should the break occur at or very close to End A, where the only force is the joint restraint, the rails would separate very little if they broke. Even if they separated considerably, the neglect of correction would have no effect on the stress in the welded section. The sole effect of this neglect would be to limit the room for expansion at higher temperatures, in the joints of the adjacent unwelded track.

This discussion shows that the repair of broken rails without restoring the original strained condition, may result in one of two conditions: Either the stress may be radically increased locally, adding to the hazard of buckling track, or the end movement may be increased radically, creating a necessity for expansion joints. Repairs in cold weather would create excessive compressive strains in hot weather, and repairs in hot weather would be likely to create excessive tensile strains in cold weather. It follows, therefore, that welded rail should be repaired wherever possible at or close to the temperature it was laid, and that any work of this kind will require the most rigid rules and careful supervision.

Although rail welded into continuous stretches should reduce greatly the necessity for lining and surfacing track, all such work would have

to be done with the utmost care. "Out of face work," such as raising track, cleaning cribs, and re-ballasting, should not be undertaken except at temperatures bordering on that at which the rail was laid, say, between 55° F and 95° F. ("Out of face" is a term used by track forces; it denotes a fairly long piece of track as distinguished from the one or two rail lengths. "Out-of-face surfacing," means the raising and tamping of all joints continuously; "spot surfacing" means just raising a joint here and there.)

With due consideration to the features mentioned, the writer can see no reason why welding of rail into continuous stretches cannot be practiced with perfect safety, with resulting economy, and greatly improved riding conditions.

FRANK B. WALKER,[20] M. AM. SOC. C. E. (by letter).[20a]—The mathematical formulas developed by the author are of interest to maintenance-of-way engineers, and the conclusions derived are apparently confirmed, at least in part, by the experience of the Eastern Massachusetts Street Railway Company in maintaining 210 miles of unpaved, welded track. At one time, there were 150 000 welded joints on that street railway.

The major part of the welding on this track was done from 1919 to 1923, but the change from trolley cars to buses during recent years has caused the abandonment and removal of most of it. The rail sections welded were 48-lb, 60-lb, and 75-lb tee-rail (Am. Soc. C. E. standard). The standard track consisted of 6-in. by 8-in. by 8-ft wooden ties, spaced 2 ft on centers; $5\frac{1}{2}$-in. by $\frac{9}{16}$-in. track spikes; and 6 in. to 12 in. of gravel ballast. There were no tie-plates. The rail welded in 1919, 1920, and 1921, had few expansion joints although some of the sections were more than a mile in length, as indicated in the following list:

Location	Feet of track welded
Brockton and Easton	15 600
Boston-Randolph	10 000
Brockton, Crescent Street	6 300
Salem-Danvers Line	8 300
Danvers-Danvers Center Line	6 400
Peabody-Danvers Line	11 600

No serious difficulties due to expansion or contraction were encountered in maintaining such long sections of welded track. Occasionally, during a very cold winter night, especially when the temperature was falling rapidly to near zero, a rail would break. Rail defects, together with high tension due to temperature changes, were believed to have been the cause, as they occurred only in old rail. New rail did not break under these conditions. It may be assumed that the difference in temperature from

[20] Chf. Engr., Eastern Massachusetts Street Ry., Boston, Mass.

[20a] Received by the Secretary March 26, 1937.

the time of welding to the time of failure was from 70° F to 100° F. Only one or two of the so-called "sun kinks" occurred during hot weather (see Fig. 8).

FIG. 8.—VIEW OF A "SUN-KINK" ON A WELDED TRACK.

When the rail broke, or when a sun kink appeared, expansion joints were installed (not necessarily at the break), about 20 rail lengths or 600 ft apart on tangents and at the ends of long curves. After the installation of expansion joints no sun kinks occurred, but rail breaks on old rail did occur occasionally. Sometimes these breaks were at long distances from the expansion joints, and at other times only a few feet, or a rail length, away. Where expansion joints were 660 ft apart, it was thought at first that the expansion would be 6 in.; but it was soon learned that the actual expansion did not exceed one-half the anticipated expansion. This phenomenon is mentioned by the author in connection with the experience on the Victorian Railways in Australia.

Hundreds of expansion joints were examined during hot and cold weather over a period extending from 1922 to 1934, and their movement, due to temperature changes only, did not exceed the 2 to 3 in. When a rail broke, it gave a very loud report and the ends separated by a distance never in excess of 2 in. Table 5 contains information concerning thirteen broken rails, the distance of the break from the nearest expansion joint, and the gap at the break.

The rail in Fig. 8 was a 60-lb tee-rail (Am. Soc. C. E. standard), laid on chestnut ties, 2 ft apart, without tie-plates, with $5\frac{1}{2}$-in. by $\frac{9}{16}$-in. track spikes, and about 12 in. of gravel ballast. There was a stone curb about 2 ft from the ends of the ties, the double track being in a reservation in the middle of a wide street on Broadway, in Lynn, Mass. The photo-

TABLE 5.—RECORD OF RAIL FAILURE IN WELDED TRACK*

Item No.	Date welded; summer of: (1)	Date of break; winter of: (2)	Distance of break from nearest expansion joint, in feet (3)	Gap at the break, in inches (4)	Item No.	Date welded; summer of: (1)	Date of break; winter of: (2)	Distance of break from nearest expansion joint, in feet (3)	Gap at the break, in inches (4)
1.....	1921	1925	0†	1.5	8.....	1921	1925	300	1.75
2.....	1921	1925	300	1.0	9.....	1920	1926	500	2.0
3.....	1921	1925	100	2.0	10.....	1920	1926	700	1.25
4.....	1921	1925	400	1.0	11.....	1920	1926	600	1.0
5.....	1921	1925	250	0.5	12.....	1920	1926	400	1.5
6.....	1921	1925	500	1.5	13.....	1920	1926	200	1.5
7.....	1921	1925	0†	1.5	....	......	......	....	....

* In each case the atmospheric temperature was reported as "falling"; and, each expansion joint was oiled.

† Break adjacent to joint.

graph (Fig. 8) was taken a few minutes after the sun kink appeared. The rail was welded in the summer of 1920. It was in service through the remainder of that summer, an unusually cold winter, and a late, cold spring

Some time in early June, 1921, after a cold night, the temperature suddenly increased to about 100° F, and this abrupt change produced a sun kink, in one track only. The kinking of the rails also moved the ties laterally. There was no noticeable bending or displacement of the track spikes. It is believed by the Company Engineers that this sun kink would have occurred even if a more substantial type of rail-fastening, such as larger or more track spikes, screw spikes, or even shoulder tie-plates, had been used. As far as had been observed previously the rail had been in perfect line and surface. There were no expansion joints at the time the sun kink occurred, but expansion joints were installed immediately and no further trouble occurred. The track was discontinued in 1935.

Another kind of difficulty occurred on a hot summer day when a man's leg was almost cut off when he pulled out too many spikes in renewing ties. The rail popped out of the remaining spikes, catching his leg against the adjacent rail.

It is obvious from the author's mathematical discussion and from the testimony of those experienced with maintaining welded track, that ballast must be of sufficient width, depth, and character to withstand the thrusts put upon it during hot weather; and no doubt the construction with tie-plates, and with better means of fastening rail than by ordinary

track spikes, is desirable. The rail will withstand, safely, the additional tension induced by a decrease in temperature. It is suggested that no welding be done when the temperature is extremely hot or cold.

ARTHUR N. TALBOT,[21] PAST-PRESIDENT AND HON. M. AM. SOC. C. E. (by letter).[21a]—To many engineers, this paper has brought the news that long stretches of continuous welded rail have been used on railroads without troublesome expansion and contraction difficulties. To those familiar with modern developments in track construction and the nature of the forces applied to the track it was not unexpected that, on a long stretch of continuous welded rail connected firmly to a suitable substructure of ties and ballast, the cumulative anchorage of the ties in the ballast at the two ends will reach a total force sufficient to hold an intermediate section stationary and fixed through the usual range of atmospheric temperature without changes in length, except for local effects. In the substructure used in present installations the rail is secured firmly to each tie in a manner such as to prevent slippage between rail and tie either longitudinally or transversely of track, and the ties are well bedded in ballast. This anchorage must be such as to provide adequately for the great force required to hold the intermediate section in a fixed position during temperature changes equal to those obtained in both summer and winter exposure and also to provide adequate transverse support against lateral buckling during the highest temperatures. These requirements imply a construction differing from the ordinary run of common track, in which the rail can be lifted from the tie a short distance and is held by anchors along its length only roughly and not fixedly with respect to the ties; that is, the rail must be fastened to the tie so that there will be neither longitudinal nor transverse movement between them, and the relation between tie and ballast bed must be one of firmness and security.

It is obvious that an analysis of the action of welded rail under such conditions must be based on known or assumed premises and, as in all reasoning, the validity of the analysis will be contingent on the validity of the premises. A start must be made to learn what underlying bases to use.

In beginning an analysis in this matter, one naturally first makes inquiry as to the possible ways in which the end portions of the track may act. Many questions arise. In developing the anchorage that will fix the unmoving intermediate part of the length of the welded stretch, does the anchorage act elastically? Does the anchoring force push the ties and ballast longitudinally along the track with the changes in temperature, even in small amounts? Is the nature and magnitude of the resistance the same in backward and forward directions—that is, with rising and falling temperatures? Do the ties near the end of the welded stretch, where the movement of the rail under temperature changes is

[21] Prof. Emeritus, Coll. of Eng. Univ. of Illinois; and Chairman, Committee on Stresses in Railroad Track, Urbana, Ill.

[21a] Received by the Secretary April 16, 1937.

greatest, give a greater resistance (greater rate of anchorage) than the ties at points farther from the end but within the moving length; or does the anchorage resistance have the same value (rate) all along this end portion; or does some other law govern?

Whether or not the tie-and-ballast rate of anchorage is uniform along the length of the anchorage stretch, does the magnitude of the length of the anchoring portion vary with the magnitude of the change in temperature of the rail, or is it constant throughout any range of temperature? In other words if the rise in temperature from the laying and fastening temperature is at first 40° and then 40° more, does the 80° increase in temperature result in twice as long an anchorage length as the 40° increase?

Then, there are questions as to the source of the anchorage. Granting that the rail does not slip over the tie (considered as essential in this form of construction) is the anchorage due to friction of the ties on the ballast by reason of pressure of the tie on, and in, the ballast so that the anchorage rate will have a constant value throughout any given anchoring stretch; or does the anchorage utilize some compressive resistance in the ballast, ties, and fittings? If the latter, does the movement of the tie within the ballast, bed vary with the longitudinal anchorage force developed? How can the rail at the end of the stretch move without a movement of the ballast? For sensible movements of the rail end, will the tie move sufficiently to leave a crack behind it which would be closed on reversing the temperature change? Furthermore, if the ultimate resistance is reached, what will be the nature of the failure (all materials have their ultimate resistance)? Will it shear along a plane under the ties, or what?

These queries and others, although important, must mostly await further experience and investigation. Without definite and explicit information concerning some of them it is natural that in starting an analysis the easiest and simplest assumptions would be chosen first. The simplest and most probable assumptions to use would seem to be that for a given track: (1) The anchorage rate is uniform in magnitude over the length of the end portion that acts as anchorage restraint; and (2) the value of the rate of anchorage force is constant regardless of the magnitude of the change in temperature from the laying temperature of the welded rail.

These two assumptions are made by Mr. Africano, at least implicitly. They were chosen tentatively by the writer when, in 1935, he began considering the phenomena of long stretches of welded track preliminary to planning tests and observations on the track of the Delaware and Hudson Corporation for the Track Stresses Investigation of the Joint Committee on Stresses in Railroad Track, which is a co-operative investigation of the Society and the American Railway Engineering Association. The assumptions formulated at the time with other statements of the accompanying conditions in part were as follows:

"To study the general action of the track in a preliminary manner, use may be made of assumed anchorage conditions. The simplest one and the one most likely to come to mind is the assumption that each tie toward the ends contributes the same increment of force and these many increments

together accumulate a force of sufficient magnitude to restrain and anchor the intermediate portion of the rail fully so that it will not contract or expand under the given change in temperature. A second simple assumption may be made, which at first thought may be considered a corollary of the first one although it is not, that the magnitude of the restraining force developed at a tie along the two anchoring stretches of the long rail has a constant value regardless of the magnitude of the temperature change. The validity of these two assumptions can be tested from observations on the action of the track. At any rate they will be useful in discussing the possible magnitude and distribution of the anchorage forces and the movement of the end portions under temperature change."

In the study, the calculus method was first used similar to the method given in the paper, but the premises assumed were so simple that it seemed sufficient and preferable to use the analogy of constant increments of force and of constant acceleration, following the laws of mechanics, and at once write the equations for length of anchorage and for rail movement without the more formal phraseology of the calculus. The force transmitted through the joint at the end of the stretch was included readily in the general consideration. It should be stated that this discussion refers to stretches with a length more than sufficient to give full anchorage. Some of the further statements formulated at that time (omitting the derivations and using the notation of the paper under discussion) were:

"(1) The anchorage force required to restrain the intermediate portions of the length of rail so that it will not change its length with a change in temperature is:

$$F = \Delta t\, n\, E\, A \qquad (26)$$

this force being independent of the length of the fully restrained part and the unit stress there being $s = \Delta t\, n\, E$. (See Equation (2).)

"(2) Calling l'' the length of the end part of the rail (that giving the anchorage to the intermediate and fully restrained portion), under the assumption of a uniformly increasing stress in the rail from zero at the end of the rail to the full stress, s, at the other end of the anchorage and, likewise, to the full anchorage force, $\Delta t\, n\, E A$, the movement of the end of the rail will be one-half the full temperature expansion or contraction for the length, l''; that is, $0.5\, \Delta t\, n\, l''$. Likewise, the distribution of the anchorage force along l'' will be per unit of length,

$$\frac{F}{l''} = \frac{\Delta t\, n\, E\, A}{l''} \qquad (27)$$

Likewise, the length of the anchorage under the assumption made, is:

$$l'' = \frac{\Delta t\, n \cdot E\, A}{q} \qquad (28)$$

Even with the assumptions used further information will be needed.

"(3) From Assumptions (1) and (2) and Equations (26), (27), and (28), certain deductions may be made: (a) Assumption (2) implies that for any given change in temperature the full available anchorage force per unit of length (and likewise the anchorage force per tie) will be developed and applied on the anchorage portion, whether the temperature change is small or large; and by Assumption (1) this same maximum available anchorage

force per tie is applied uniformly along the anchorage portion; (*b*) the length of the anchorage portion with a given available anchorage unit per length or per tie will vary directly as the temperature change—twice as great a temperature change will require or use twice as long an anchorage length; and (*c*) the movement of the rail end for the same conditions will vary as the square of the temperature change—twice as great a temperature change will result in four times as great a movement at the rail end."

Naturally, the observational tests on welded track of the Stresses in Track Investigation were planned to cover the validity of the two assumptions. As the results of the tests are giving needed cumulative information as to the action of welded track, it seems germane to the discussion of this paper to present some of the results which bear on the assumptions made in the analyses, with a very brief statement of the methods used in the observations.

Reference points and readings to determine longitudinal and lateral movements of the rail along the track had been included in the program undertaken by the engineers of the railroad companies; it was thought not necessary to duplicate these observations. The method of test chosen was based on strain-gage measurements longitudinally along the web of rail (8-in. or 10-in. gage lengths) on gage lines distributed throughout the length of the stretches, with simultaneous measurement of the rail temperatures along the stretch and frequent comparison of the strain-gage with reference bars or comparators. The strain-gage measurements were made at several rail temperatures in summer and winter. The rail temperature was found, accurately and quickly, by an open-ended thermo-couple and portable potentiometer. An invar reference bar of known expansion coefficient, a short piece of rail similar to that in the track, and, later, a compensating comparator giving almost no variation in length with changes in temperature were used in the tests; temperature and temperature correction for both rail and instrumental equipment, of course, were of extreme importance in securing accuracy of data.

As an example of the distribution of the gage lines, their spacing on the mile stretch of the Bessemer and Lake Erie Railroad may be cited. Beginning at the ends of the welded stretch, gage lines were placed on both rails, at distances from ends, of 3, 10, 30, and 58½ ft; then at intervals of one rail length to the tenth rail; then at distances of two lengths to the twenty-fourth rail length; and finally at distances of four rail lengths to about the mid-point of the welded stretch. Additional gage lines were placed at points of special interest by reason of curves and other local conditions.

It was not convenient to make observations at the time of the rail laying; besides, later adjustments by track men and other changes, would have rendered data taken then of doubtful value for the purposes of the investigation. Observations were made in September, 1935, on one stretch of track of the Delaware and Hudson Railroad Corporation at Albany, N. Y., and on two stretches at Schenectady, N. Y.; the length of these welded stretches varied from 0.5 mile to 1.3 miles. Further tests on these stretches were made in January, 1936, August, 1936, and January, 1937.

Observational tests have likewise been made on the mile stretch of welded G.E.O. track of the Bessemer and Lake Erie Railroad Company near Pittsburgh, Pa., in July, 1936, and February, 1937. The rail on the Albany stretch was 130-lb and on the others, 131-lb. The alignment of these stretches contained various curves from 1° 30′ to 7 degrees. In the summer tests, rail temperatures were made as high as 120° F and, in the winter tests, observations were made at rail temperatures as low as 13° F. Tests were also made at medium temperatures. Observations in the early morning, or on cloudy days, were useful to form a basis of comparison for the changes in length in the rail. Generally, check sets of readings were made along the stretches.

A few of the results of the tests on the Bessemer and Lake Erie Railroad will give an idea of the nature of the information obtained in all the tests. In Fig. 9 are plotted the changes in length observed along the

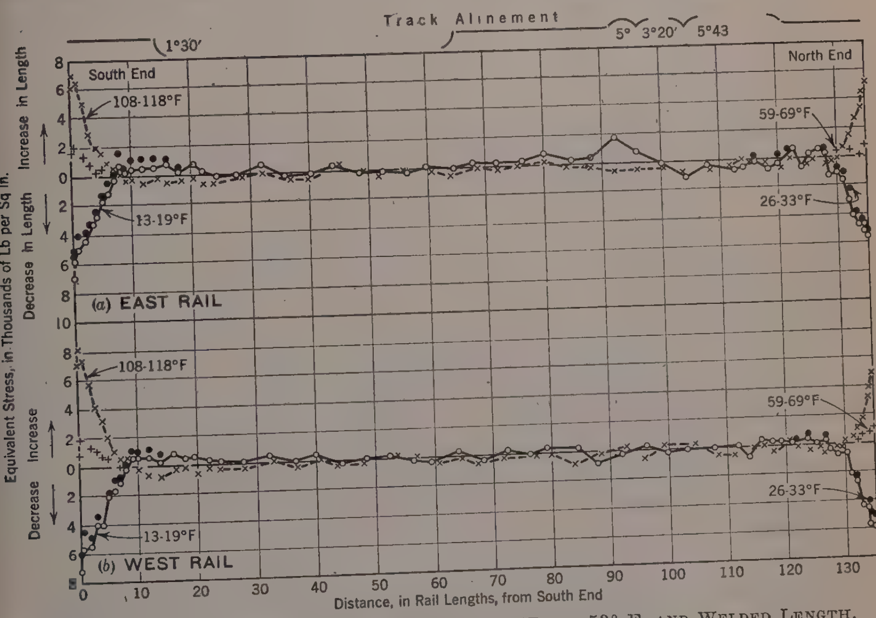

FIG. 9.—CHANGE IN LENGTH IN TERMS OF STRESS (BASE, 53° F, AND WELDED LENGTH, 1.0 MILE).

mile stretch for two representative series in summer and two in winter. The position along the stretch is shown in terms of rail lengths of 39 ft each. Naturally, the temperatures of the rail at any time varied somewhat as the conditions of shade and exposure varied, and these variations affected the stresses and the movements. The rail temperatures along the track for the series shown were 108° to 118°, 59° to 60°, 26° to 33°, and 13° to 19°, F. As the changes in length from one temperature to another are quite minute, for ease in picturing values the unit strains (inches per inch) have been translated into equivalent stress per square inch in steel for representation on Fig. 9—that is, that stress which would be developed

in the rail if an external force had produced the given change in length without any change in temperature. The observations before sunrise on a day in July with a rail temperature of 53° F, were taken as the basis or zero line for the diagram.

It will be noted that throughout the intermediate part of the welded stretch (about 0.9 mile) very little change occurred—only small local changes—the rail having been held closely to one length through all these variations in temperature. As the differences are generally so small, not all the points for the four series are plotted on the diagram. For the end portions of both rails, and for both the summer and the winter tests, the rails changed length through an average distance of about seven rail lengths. This change in length increases rather regularly from the points at the end of the intermediate portion (which may be thought of as the point of fixation) to the end of the welded stretch, showing a lengthening in summer and a shortening in winter, with some rounding off in the line

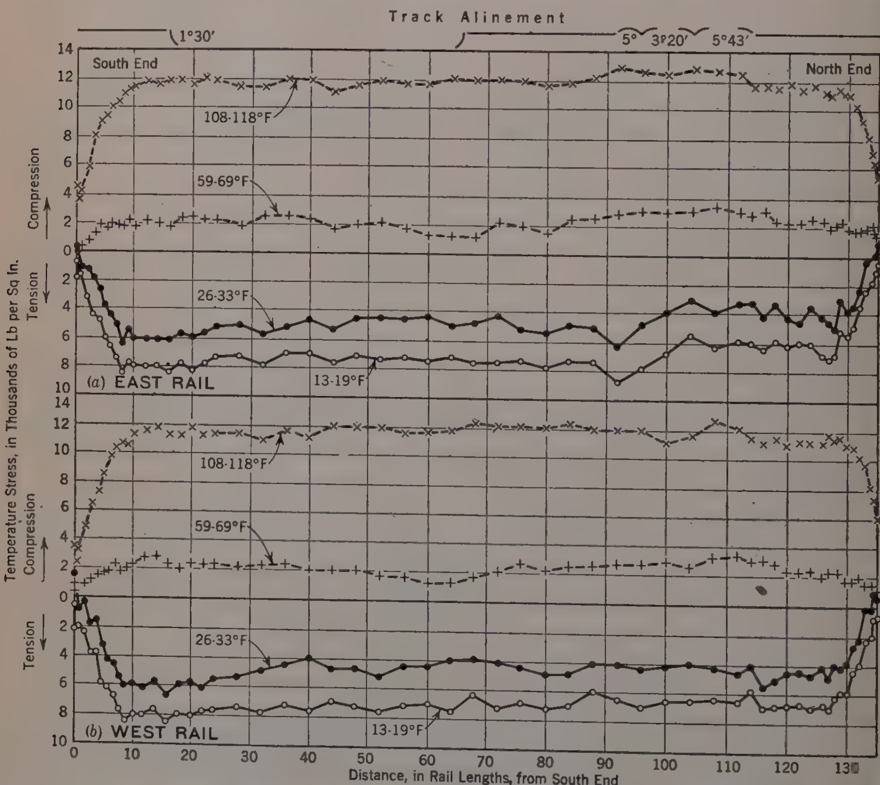

FIG. 10.—TEMPERATURE STRESSES IN RAIL (BASE, 53° F, AND WELDED LENGTH, 1.0 MILE).

of the end portion as it approaches the intermediate portion. The data indicate that the magnitude of the winter and summer temperatures has not particularly affected the length of track over which expansion and con-

traction occurs in each end portion, and that the long intermediate portion remains practically fixed in position with the various changes in temperature at the time the observations were made. It is evident that under these conditions there is no pull or drag on the ties of this intermediate portion, except as may be caused by differences in exposure and other local conditions.

In Fig 10 are recorded the stresses developed in the rail throughout the length of the stretch based on the rail temperatures and strain-gage readings at the time of observation as compared with readings at a base temperature of 53°F, which were taken before sunrise in the summer of 1936 with temperature conditions very uniform over the entire stretch. For the summer observations, at an average temperature of 113° F, the relative compressive stresses over the intermediate 0.9 mile are about 12 000 lb per sq in., and for the winter observation at a temperature of about 16° F, the relative tensile stresses average somewhat less than 8 000 lb per sq in., both values being based upon the aforementioned readings at 53° F; this means that the zero reference line represents the stress at 53° F. If the reference line were placed at 63° F, the temperature at which the rails were originally fastened in the track (the difference of 10° in temperature representing approximately 2 000 lb per sq in.), the corresponding compressive stress on the summer test would be approximately 10 000 lb per sq in. and, for the winter test, a tensile stress of aproximately 10 000 lb per sq in. For summer temperatures higher than those observed and for lower winter temperatures, in accordance with other data obtained, the stress may be expected to increase in proportion to the increase in change in temperature from the 63° base. For the winter tests, the end portions show a tapering off of stress over the distance of, say, seven rail lengths to nothing at the end. For the summer tests, there was a similar tapering off in seven rail lengths to a stress of 2 000 to 6 000 lb per sq in. at the end. This end compressive stress was doubtless transmitted through the joint connecting the welded stretch to the regular track beyond. Incidentally, it may be remarked that joints have been found to transmit tensile and compressive forces to the adjacent rail equivalent to as much as 4 000 and 6 000 lb per sq in. of rail section, respectively.

The data given in Figs 9 and 10, corroborated by data of the track tests at Albany and Schenectady, indicate that within the range of temperature observed the magnitudes of the change in temperature for different series having different temperature changes do not particularly affect the length of track over which expansion and contraction takes place at each of the end portions. It is apparent, too, that the central part of the stretch has remained practically fixed in position through the summer-to-winter interval. The change in length and in stress over the end portions of about seven rail lengths, is fairly regular with some rounding off near the point of fixity, although, of course, there are changes in position from time to time at the extreme ends of the welded rail and the movement here is not always regular. The length of the end portions which give anchorage to the intermediate portion, and the stress set up in the intermediate portion correspond

to an average anchorage or restraint of about 600 lb per tie per rail in each direction for the temperatures of 113° F and 16° F as counted above and below a laying temperature of 63° F. If the anchorage length remains fairly constant for temperatures above or below the aforementionad range as is indicated to be the case by the data of the various tests, the corresponding anchorage force would be proportionally greater than 600 lb.

It may be added that no indication was observable of any visible movement between the ballast and ties one way or the other at the times of the tests or of any particular change in length of the anchorage developed. It was found, however, that breakage in imperfect welds and readjustment of track in cool weather have, in places, changed the conditions of stress and anchorage considerably. It is not known, of course, what the ultimate shearing strength of the ballast bed may be. The track seems to have kept close to an elastic condition of strength.

As the data of the tests on this stretch of welded rail, and also on the other stretches tested, show that the rate of anchorage is fairly uniform over the end portions, Assumption (1) in the analyses is borne out by the tests. The tests, however, indicate that the length of the average anchorage portion remains about the same for different values of the change in temperature. This conclusion is not in accord with Assumption (2) of this discussion referred to previously as having been made by the writer in his earlier study of the problem, nor with an implied assumption of the paper that in a given track the anchorage force is a constant regardless of the value of the difference in temperature. In other words, the tests indicate that T is not the same for different values of Δt, but varies nearly as Δt.

As a consequence, it follows that Equations (8) and (9) and others in the latter part of the paper are true only when the value of T used corresponds to the given Δt. It also follows that Item (2) in the second paragraph following Theorem 1 (that the expansion varies as the square of the temperature difference for a given track) is not warranted. The tests on the tracks of the Delaware and Hudson Railroad and on those of the Bessemer and Lake Erie Railroad indicate that the expansion at the end of the welded stretch varies as the first power of the change in temperature rather than as the square, and also that the length required to develop the force, F, is largely independent of the magnitude of the change in temperature, referring, of course, to more than slight changes in temperature. Care should be taken in the use of the formulas given in the remainder of the paper where the reader may be led to think that T is implicitly used as constant for a given track. Caution in this respect is suggested in the use of Equations (18), (20), and (21). It may be implied from the wording in Example 4 that the value of T is the same for a temperature change of 100 ° F as for one of 58° F. This assumption is not in accord with the results obtained in the Stresses in Track Investigation. These several inaccuracies in the analysis and its applications affect the usefulness of the "theory" in a way that seems to warrant calling the attention of the reader to them.

These comments are made in the light of what has been learned so far in the observational tests of four stretches of long welded track, all of which are in a sense experimental. It is planned to continue the seasonal tests to learn the changes that occur at later dates when time and traffic may have had their effects. As stated by Mr. Africano further fundamental knowledge of the physical action of welded rail in track is needed. It will take time to secure this information. Much more must be learned of welds before welded track, with its higher investment in welds and sub-structure, will be generally accepted as a standard for high-grade track. That interest has been aroused in this type of construction is shown by the starting of an investigation on continuous welded rail at the Engineering Experiment Station, University of Illinois, through a co-operative agreement with the American Railway Engineering Association and the Association of American Railroads. The Investigation will study the strength, ductility, and durability of continuous welded rails made by different processes. It will doubtless find the quality and uniformity of the metal at the welds and at each side of them as well as the uniformity of surface and line and the wear of the joint under repeated rolling loads for both sample joints and joints cut from track.

It may not be out of place to suggest that Mr. Africano's recognition of the desirability of giving the track anchorage throughout the entire length of the welded stretch might well be made much stronger. Without continuous rigid support, breakage of rail in the intermediate portion of the welded stretch and other accidental events might lead to serious trouble. To make a further comment: The coefficient of linear expansion of steel varies over quite a range; the International Critical Tables give values in the range of rail steel from 0.0000055 to 0.0000072. For 131-lb rail tested at the University of Illinois, Urbana, Ill., the coefficient was found to be 0.0000063. This differs appreciably from the value, 0.0000073, given in the paper.

The author is to be commended for putting forth an analysis in a new field, even without having adequate experimental evidence of the action of continuous rail in railroad track.

RANDON FERGUSON,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—As a member of the test party of the Special Committee on Stresses in Railroad Track of the Society and the American Railway Engineering Association, the writer has been engaged on tests on several sections of long welded track. A short discussion on some of these tests has been published elsewhere.[23] Further discussion and description of various installations are also available in the reports of the Committee on Rail of the American Railway Engineering Association.[24]

Although the tests are not complete at the present time certain comments of a tentative nature may be made as to the results. It is doubt-

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[22a] Received by the Secretary, April 16, 1937.

[23] *Proceedings*, A. R. E. A., Vol. 37, 1936, p. 954.

[24] *Loc. cit.*, p. 465; and, also, *Bulletin 393*, A. R. E. A., p. 493.

ful whether the nature of the tie resistance is frictional, as assumed by the author. A frictional resistance would imply that the value of the tie resistance, T, is a constant for any given track for all temperature changes if end movement occurs, as it apparently does, and that the length of track giving anchorage would depend on the temperature change. Instead, the tests on the track of the Delaware and Hudson Railroad Company and the Bessemer and Lake Erie Railroad Company indicate that the degree of tie anchorage on a given track varies with the magnitude of the temperature change, and that the length of track giving anchorage remains constant for all changes of temperature both winter and summer. The tie resistance has also been found to vary somewhat near the ends for a given temperature; or, in other words, the force or stress diagram is not necessarily a straight line, as the author indicates in Equation (3). The tracks tested have been found to be quite completely anchored except for the end portions where the periodic expansion or contraction due to temperature occurs over a distance of about five to twenty 39-ft rail lengths.

It should be stated for the benefit of readers who are not intimately familiar with the existing installations of continuously welded rail, that as far as known all installations of welded track for standard railroad service have special fittings designed to anchor the rail firmly to each tie. A clip and bolt fastens the rail to the tie-plate on both sides of the rail and the tie-plate is fastened to the tie with screw spikes. This greatly increases the available longitudinal restraint as compared with the usual practice of applying three to six rail anchors per rail. Ordinarily, the rail anchors are effective in one direction only. Of course, the fastenings must be provided over the entire stretch whether or not they are doing any work. Any rail breakage, in a weld or elsewhere, at once requires the track adjacent to the break to be capable of withstanding forces of the same magnitude as those occurring at the normal end portions.

The tests have indicated the development of tie resistances of about 600 lb per tie per rail in each direction, with a rail temperature range of 50° above or below the laying temperature. The limiting value of the tie restraint is not known from the tests. Any appreciable slippage observed has been through the clips, and this occurred in a few cases when a rail broke or the end joints were loosened for adjustment. The ballast in all cases was snugly bedded against the ties with no evidence of the tie working back and forth along the track.

The range of temperature of the rail may be as much as 150° over a period of a year. In the absence of sunlight or protective covering, such as snow, the rail follows the air temperature quite closely. On a bright day, either winter or summer, the rail may be as much as 40° warmer than the air temperature if there is no breeze. With snow around the rail, the rail temperature has been found to be 10° to 15° warmer than the air temperature, thus minimizing the effect of low air temperatures.

A. N. Reece,[25] M. Am. Soc. C. E. (by letter).[25a]—In directing attention to factors influencing the amount of rail expansion, that have quite generally been neglected in the past by engineers in the United States, this paper serves a useful purpose. Not only does the new theory that Mr. Africano describes apply to long welded rail installations on the rigid type of track, but it shows equally well that expansion requirements for rails of ordinary length, secured only with cut-spike rail fastenings, are appreciably less than the theoretical requirements based upon the assumption that the rail is free to expand without restraint.

For example, in a 131-lb rail having a cross-sectional area of 12.86 sq in., a force of $12.86 \times 0.0000065 \times 30\,000\,000 = 2\,500$ lb, is required to restrain the rail fully through a temperature change of 1° F. Assuming that six-hole joint bars, with bolts tightened, will develop sufficient frictional resistance so that a force of 75 000 lb is required to slip the rails within the bars, then, for a temperature increase or decrease of $\frac{75\,000}{2\,500}$, or 30° F, there would be no rail end movement. This joint restraint would then be sufficient to prevent rail expansion or contraction through 60° of the seasonal variation in rail temperature, or almost throughout one-half the range. Thus, only one-half the expansion allowance, calculated for the rail without restraint, need be allowed. To secure full advantage of this condition, however, it would be necessary to lay the rail at temperatures ranging from less than 30° of the maximum to more than 30° of the minimum.

The use of longer and continuously welded rail offers a prospect of material improvement in the track structures, and studies, such as that of the author, are invaluable in acquainting engineers with a new field into which they are destined to "travel."

G. M. Magee, Esq.[26] (by letter).[26a]—In recent years, the expansive action of rails subject to joint and tie restraint has been given intensive study in Germany by several investigators. In addition to theoretical analyses of the temperature change and restraining forces, considerable research work was conducted to determine the actual values of joint and tie restraints. The adoption of a new track construction as standard on the German National Railways in 1925, provided a tie-plate fastened to each tie with four screw spikes, a compressed wooden shim between rail and plate, and a firm fastening of rail to each plate by two clamp bolts, clamps, and double-coil spring washers. This construction has also been introduced in the United States.

It was found that with this type of fastening, the rail did not slip through the plate, but that where longitudinal movement of the rail did occur, the rail, ties, and ballast within the cribs moved as a unit, presumably slipping along a horizontal plane at the bottom of the ties. In 1929, Ammann

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[25a] Received by the Secretary May 3, 1937.

[26] Asst. Engr., Kansas City Southern Ry., Kansas City, Mo.

[26a] Received by the Secretary May 3, 1937.

described[27] an interesting experiment in which a panel of track was especially constructed on crushed rock ballast, a concrete buttress placed at one end, and known longitudinal forces applied to the rail ends by hydraulic jacks, in order to determine the tie resistance to longitudinal movement of the rail. Later investigations were made on track in traffic use and under different weather conditions. A tie resistance of 881 lb per running meter of rail was observed for 24 wooden ties per 15-m rail length.

As a result of the analysis of expansion movement with proper consideration of tie and joint restraint the rail lengths in use on this type of track construction were generally increased from 15 to 30 m (approximately, 49 to 98 ft, respectively) by welding two rails when the old rail was serviceable, or by purchasing new rail of 30-m length. Trial installations were also placed of 60 and 120-m lengths.

Long welded installations of rail have been placed in open track in the United States on two railroads. The first installations were placed on the Delaware and Hudson Railroad, using a type of construction similar in principle to the German fastening, but differing principally in that two spring clips and bolts secured the rail to the tie-plate and no shims were used. An installation of long welded rail has also been placed on the Bessemer and Lake Erie Railroad, using the German type of construction.

The new theory of rail expansion described by Mr. Africano explains the apparent phenomena of placing rail with no provision for expansion and shows that, in this welded construction, expansion movements may be analyzed by known engineering principles.

Although, as Mr. Africano points out, the rigid rail fastenings are required only at the ends of long welded rail to restrain the expansion, it should not be thought that these fastenings are not needed at the intervening ties. Their function is a highly valuable one at the intervening ties to prevent excessive opening of the rail in the event of rail breakage in cold weather and to resist track buckling in hot weather. Should the track begin to buckle at a point of weakness, the rail is held from running into this point of weakness and a "sun-kink" is thus prevented in its beginning stages.

The rail stresses due to restraint of the expansion must not be disregarded. Should rail be laid, welded, and secured in the hottest weather, a maximum tension stress due to temperature effects in the coldest weather of approximately 30 000 lb per sq in. should be anticipated. This would be additive to the rail bending stresses and would place too large an added burden on the rail. The laying of this type of track at mean temperatures would equalize the disadvantages of stress due to rail tension in winter and track-buckling forces in summer. The maximum rail-tension stress of 15 000 lb per sq in., due to temperature effect, which would be developed in this method of laying, is probably not too severe for the heavier rail sections that weigh 131 lb per yd, or more.

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[27] "Latest Investigations in Regard to the Effect Which the Longitudinal Forces Exert on the Rails", by Prof. Dr.-Ing. Ammann, Karlsruhe, Germany, *Technical Weekly Traffic Review*, Issue No. 47, Berlin, November 20, 1929.

The effect that wave action of traffic may have on tie resistances for long welded track are not yet well determined. Preliminary measurements indicate that with traffic in one direction, the rail is likely to show somewhat more movement at one end of the long welded rail than at the other. It does not seem, however, that this will appreciably alter the analytical determinations.

There is also some question as to whether the tie resistance to horizontal movement may be considered as an elastic resistance similar to the modulus of track resistance, u, to vertical depression. It is to be expected that the tie resistance is more in the nature of a frictional, rather than of an elastic, resistance.

The author is to be commended for his invaluable analysis of this new and important subject.

H. D. Hussey,[28] M. Am. Soc. C. E. (by letter).[28a]—The assumption is made in this paper that long welded rails are laid in tangent, level track, and the author has developed a method of computing the stress in the rails and the end expansion, for a change in temperature and a known tie resistance. When the rails are laid on a curved alignment, the radial component of the stress in the rails produces a force which tends to displace the ties laterally. This radial force on each tie is:

$$F_r = \frac{2\, s\, A\, c}{r} \qquad (29)$$

in which, in addition to the notation of the paper, F_r = radial force on one tie, in pounds; and r = radius of curvature. Mr. Africano has demonstrated that s = 219 lb per sq in. for 1° change in temperature. It is seen that F_r decreases as the radius of curvature increases, and that, in a straight track, it becomes equal to zero. For example, assume that 131-lb rails are laid on a 1° curve and that the tie-spacing, c, is 22 in. Then, by Equation (29), F_r = 1.80 lb for 1° change in temperature. For 100° change in temperature, F_r = 180 lb. This force must be added to (or subtracted from) the radial force on the tie that is due to a train passing around the curve. When the temperature is higher than that at which the rails were laid, these forces will be additive, but when it is lower, they will counteract each other.

Equation (29) can be used in the computation of vertical forces exerted by rails that are curved in a vertical direction, as in the case of vertical curves at the top of a hill or the bottom of a valley. A horizontal and a vertical curve occurring at the same point is an unfavorable combination.

Research on long welded rails should include the determination of the lateral resistance of ties in track on curved alignment and the action of curved rails.

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[28a] Received by the Secretary May 8, 1937.

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DISCUSSIONS

ECONOMICS OF HIGHWAY-BRIDGE FLOORINGS OF VARIOUS UNIT WEIGHTS

Discussion

BY MESSRS. HENRY C. TAMMEN, MILLER MCCLINTOCK, JOSEPH G. SHRYOCK, AND C. CALOR MOTA

HENRY C. TAMMEN,[4] M. AM. SOC. C. E. (by letter)[4a].—In view of the many new developments in light-weight highway-bridge floorings during recent years and the interest of engineers in such floorings as evidenced by their increasing use, this paper is very timely. It should be of material help in the analysis of the economics of such floorings.

The author develops the relative economy of two floorings by the comparison of sums composed of two items: (1) The construction cost of the flooring and of certain special floor parts affected by the flooring design; and (2) the construction cost of the remainder of the superstructure and of the substructure.

For any flooring the amount of the first item (the cost of the flooring and certain special parts, shown in Table 2) is to be determined by each engineer for himself. It is to be expected that, for the same flooring and for the same structure, different engineers will reach somewhat different results. This might follow from variations in the design procedure (which is not yet standardized for many of the floorings); from variations in fastenings of the flooring to its supports (regarding which there are wide differences of opinion); from variations in minimum metal thicknesses; or from other causes. Some engineers will find it difficult to accept thicknesses of $\frac{1}{4}$ to $\frac{3}{16}$ in., or even less, as proper for a primary element carrying heavy loads such as a flooring, after having thought and worked for years with such members in terms of $\frac{5}{16}$ in. and $\frac{3}{8}$ in., or even greater thicknesses. The question will arise as to whether such thick-

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NOTE.—The paper by J. A. L. Waddell, Hon. M. Am. Soc. C. E., was published in February, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: April, 1937, by Jonathan Jones, M. Am. Soc. C. E.

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[4a] Received by the Secretary April 20, 1937.

nesses should be considered satisfactory at all, and also, whether a non-corrosive coating should be applied to the metal or whether the safe life of the flooring making use of such thicknesses should not be considered very much less than that of, say, the standard flooring.

To give a true economic comparison between several floorings, the writer would consider it proper to add to the flooring costs suitable amounts to cover an allowance for amortization of the flooring cost in cases where the safe life cannot be taken the same for the several floorings being compared. Similar allowances should properly be made also for differences in anticipated maintenance costs, including painting, snow removal (where applicable), and other maintenance items. For open-grate floorings the average annual maintenance painting cost may be in excess of 1 ct per sq ft, requiring the annual interest on an investment of 20 or 25 ct per sq ft, or more. For light-weight floorings having only a flat-bottom steel surface to be painted the cost will be only a fraction of the foregoing values, whereas for concrete floorings this cost disappears. Clearly the capitalized maintenance costs may vary substantially with the flooring type and may require consideration in any determination of comparative economy of several floorings.

With reference to the costs making up the second item (shown in Figs. 1 to 8) there should be no question regarding the portion of this cost which represents the superstructure. The superstructure cost is quite definite for each span length and, without doubt, the author's determination of it is as accurate as can reasonably be made. However, there may be material variations for different structures in the portion of this cost which represents the substructure.

Inspection of the cost curves for simple spans shows that minimum costs are found for spans between 200 ft and 300 ft for the various types of floor, for various roadway widths. Clearly, these are the economic span lengths for the grade line and foundation conditions which have been assumed by the author. Granting that, in actual structures, the span lengths in general conform closely to the economic, it would follow that for such economic structures the substructure costs and the saving effected by the use of lighter floorings which would result from the use of the cost curves, are too high for spans less than the foregoing and too small for greater spans. This applies also to the cost curves for cantilever spans where the several curves show minimum costs for spans between 700 ft and 800 ft.

In arriving at his substructure quantities, the author has used increasingly smaller piers with the lighter flooring. Many times the conditions are such that the pier shafts and, more frequently, the base sizes which would be adopted for standard concrete flooring are minimum sizes to give stability for the dead, live, wind, water, ice, and other load conditions for which provision must be made, and would not be decreased even if a light-weight flooring were used.

To make the cost data more generally useful in the analysis for structures with foundation conditions varying from those assumed in the paper, it would be helpful if the superstructure and substructure costs could be separated, and it is hoped that the author will find it possible to add this information in his closing discussion.

The writer has been particularly interested in the statements made with reference to economy of light-weight floors on movable spans. He has had occasion to make use of light-weight floors many times on such structures. The earlier applications on vertical lift spans involved merely the use of a light-weight aggregate in the concrete floor-slabs, these slabs being supported on closely spaced cross-beams to reduce slab thicknesses to a minimum. Even with the limited weight reduction practicable by this means, it was found possible with economy to ship such light-weight aggregate from its point of production in Kansas City, Mo., as far as San Francisco, Calif., and Canada, and in the latter case also to pay a duty on the aggregate.

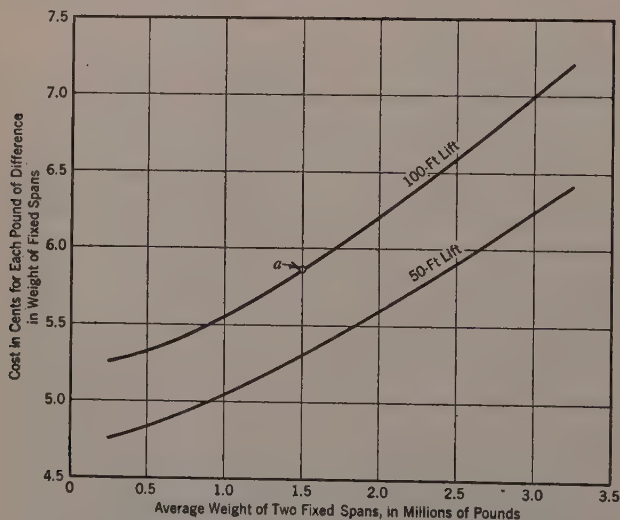

FIG. 11.—DIFFERENCES IN COST OF CONVERTING TWO FIXED SPANS OF DIFFERENT WEIGHTS INTO VERTICAL LIFT SPANS.

Some years ago, the writer prepared estimates of quantities and costs for a group of vertical lift spans; from these he determined the cost of converting an ordinary fixed span into a vertical lift span and, finally, the increase or decrease in this conversion cost resulting from an increase or decrease of 1 lb in the weight of the fixed span. The results, revised to conform to present design practice, are shown in Fig. 11 in convenient form for use. They are based on towers with inclined rear legs supported on truss approach spans and on the conventional method of operation, with the machinery at the middle of the lift span, but should apply very

generally also to spans with "tower drives". The make-up of the total cost, in cents, for Point a, Fig. 11, is as follows:

Tower and span metal	0.327 lb @ 5.5 ct $\times$ 0.75 =	1.35
Counter-weights	1.153 lb @ 1.0 ct $\times$ 0.75 =	0.87
Tower sheaves, shafts, and bearings	0.111 lb @ 15.0 ct $\times$ 0.75 =	1.25
Ropes	0.044 lb @ 20.0 ct $\times$ 0.75 =	0.66
Balance chains	0.052 lb @ 8.0 ct $\times$ 0.75 =	0.31
Operating machinery	0.028 lb @ 20.0 ct $\times$ 0.75 =	0.42
Motors, engine, house, etc.		1.00
Total		5.86

The same unit prices were used for all other points. The factor, 0.75, used with the assumed unit prices conforms to Equations (2), (3), and (4), given by the author which means that any reduction or increase in cost resulting from a change in quantity of an item is calculated by assuming that the quantity change takes a unit price equal to three-fourths that for the original quantity. In making a comparison of lift-span costs for two floorings, enter the curves with the average weight of the fixed spans for the two floorings. The difference is weight of the two fixed spans multiplied by the unit cost taken from the curves, will then represent the total cost saving in the movable span superstructure by use of the lighter of the two floorings under consideration.

Referring to Fig. 11, the average weight of two fixed spans may be expressed as $\frac{A + B}{2}$, in which A is the weight of the heavier fixed span; and B is the weight of the lighter of the two. Similarly, the total difference in cost may be expressed as $(A - B)\ C_x$, in which C_x is the cost per pound, taken from the curves of Fig. 11. The costs given by these curves include only the superstructure, and the savings shown by them for light-weight floorings are in addition to any savings for fixed spans that may be shown in the analysis presented by the author.

Fig. 11 indicates clearly the value of light weight in the flooring of a vertical lift span. The economy in a bascule span by use of light-weight flooring, in general, will be somewhat less than on vertical lift spans, due to the absence of ropes and to the fact that the dead weight of the span and counterweights has only a minor effect on the operating machinery and power requirements. The latter are determined largely by the wind loads. The saving in counterweight cost on bascules by the use of light flooring, on the contrary, may be several times that for lift spans, due to an increased ratio of counterweight weight to span weight and, frequently, also to a reduction in the unit weight of counterweights. The conditions for bascule spans are so variable that it is not practicable to show, by curves, the effects of weight changes in the span.

Miller McClintock,[5] Esq. (by letter)[5a].—Shortly after the first installment of an open steel-mesh flooring was made in the University Bridge, in Seattle, Wash., the writer, attracted by the possibilities of such light-weight construction for the decking of elevated, limited-way, structures, visited Seattle and made observations of the characteristics of the installation. On the basis of these observations and other tests and considerations, he recommended the use of this type of open steel-mesh flooring for a system of elevated limited highways then being designed for the City of Chicago, Ill. The incorporation of this type of light-weight flooring made it possible to reduce the size of various structural units and resulted in substantially lowering the estimated unit costs. The cost element, however, although important, was not the main factor which justified the recommendation. The writer was seeking a flooring completely flat in character and without pitch or crown for the purpose of improving the riding characteristics of the structure. To obtain such a flooring, it was necessary to design a decking which would be self-draining. This function of the open steel-mesh decking was very satisfactory and, in turn, eliminated certain structural costs in the design of drainage and run-off systems. The added economy of self-snow-cleansing was not without importance in the consideration.

A further very important economy was found in the capacity of this open steel-mesh flooring to pass a substantial percentage of normal light and air. In view of the fact that many of the elevated limited way structures, as designed, were contemplated for construction over existing streets, this was an element of very considerable significance. It eliminated in all or in part the necessity for day-time illumination for street surfaces under elevated structures and materially reduced contemplated damages to abutting property owners.

In the writer's opinion, this paper indicates a trend in thinking which should have substantial influence upon future designs of bridges and similar types of structures. There appears to be a particular need to obtain a simpler and less expensive type of construction for the thousands of grade-crossing separations which will be required in the near future.

Joseph G. Shryock,[6] M. Am. Soc. C. E. (by letter).[6a]—This timely and useful paper brings out many points of interest to the bridge and structural engineer, and emphasizes the fact that the lighter the floor, the greater will be the difference of cost per square foot over that of the conventional standard designs, and that there is doubtless a point on long-span bridges where the saving in weight of the floor-slab is offset by a sacrifice of lateral stiffness and rigidity of the structure.

All forms of solid floor construction, whether of concrete or steel, or a combination of both, form a horizontal girder capable of taking and absorbing a large percentage of the lateral stresses; this attribute is lacking in the open type floor.

[5] Director, Bureau for Street Traffic Research, Harvard Univ., Cambridge, Mass.

[5a] Received by the Secretary April 21, 1937.

[6] Vice-Pres., Director, and Chf. Engr., Belmont Iron Works, Philadelphia, Pa.

[6a] Received by the Secretary, April 29, 1937.

All standard bridge specifications call for a specified $\frac{3}{8}$-in. minimum thickness of material, even for lacing-bars and redundant members, and only permit less thickness for lining and filling vacant spaces.

In 1900, the author presented[7] specifications for steel highway bridges which read as follows: "No metal less than $\frac{5}{16}$ of an inch in thickness shall be used, except for filling plates; and in important bridges this limit shall be increased to $\frac{3}{8}$ of an inch." Even with modern bridge steel, corrosion is still an important problem, and leaves only a relatively small factor for permanent strength on any $\frac{3}{16}$-in. member.

It would be advisable, therefore, to design an open-grate floor to meet standard specification requirements, as to minimum thickness of material and minimum size of structural rivets. To add a supplementary jack system to support the conventional open-grate floor properly, doubles the number of pieces in the floor system and their maintenance as well. Owing to the importance of future maintenance on all major structures, the modern engineering tendency is to use as few members as possible of rolled sections without lacing, thereby not only reducing secondary stresses to a minimum, but the cleaning and painting of the structure as well.

Whether it is of masonry or steel, solid or open, the floor-slab is a structural member of major importance; it receives not only the maximum concentrated wheel loads, but a greater percentage of impact than any other member in the bridge, and, in addition, is subject to abrasion or wear from moving loads.

On long-span bridges, each pound per square foot saved in the dead load of the floor becomes an important factor. Savings, therefore, of as much as 50% of the weight of the standard type of floor would effect economies without undue sacrifice of stiffness and rigidity. To reduce the weight of the floor to only 25% of the standard, however, could scarcely be secured without a sacrifice of these important requirements.

C. Calor Mota,[8] Esq. (by letter).[8a]—In a course of bridge design for civil engineering students this paper is indeed a very useful one, and it is especially adapted to the practical use of the bridge designer. The most striking characteristic of the paper is its accurateness and completeness. All possible factors that may cause a change of metal weights in the trusses and floors per linear foot of bridge, are considered carefully and also any variations from the basic assumptions adopted. There is no question but that, in order to obtain the partial unit-cost curves of the paper, an enormous mass of data was necessary which only a designer of long experience could make possible. Table 2 is indeed a valuable tool for computing the total cost of structure per linear foot.

This paper, similar to that entitled, "Weights of Metal in Steel Trusses"[9], by the same author, constitutes a creditable contribution to technical literature in the economics of bridge engineering.

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[7] "De Pontibus", 1900, p. 226.

[8] Head, Dept. of Civ. Eng., Univ. of Puerto Rico, Mayaguez, Puerto Rico.

[8a] Received by the Secretary May 14, 1937.

[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1.

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DISCUSSIONS

NATIONAL ASPECTS OF FLOOD CONTROL A SYMPOSIUM

Discussion

By Messrs. Fred C. Scobey, Howard T. Critchlow, T. T. Knappen, M. C. Tyler, Gordon R. Williams, Arthur T. Safford, W. G. Hoyt, J. D. Arthur, Jr., John H. Meursinge, H. K. Barrows, E. D. Hendricks, and Edward W. Bush

Fred C. Scobey,[35] M. Am. Soc. C. E. (by letter).[35a]—In the Symposium under discussion reference is made to the use of snow surveys, in forecasting floods. When sufficient snow-survey data have been accumulated for any major water-shed then, of course, excessive snow depth and water content will be indicative of a heavy run-off. This information can be directly applied to streams in which floods are always, or nearly always, dependent on snow melting, such as the Colorado River of the Far West. For streams in which floods are developed by torrential rains alone, or by hard rains on top of snow, such as occur in the Gila River water-shed, of course a snow survey does not apply or merely indicates a contributing factor.

In 1936, snow surveys were extended in the West, by many agencies acting through the U. S. Bureau of Agricultural Engineering as a clearing house, until now they cover nearly all the Western States. The purpose of this information is to forecast run-off by the Bureau of Agricultural Engineering through its Irrigation Division. Primarily, the interest of this agency lies in irrigation and supplemental power, although, of course, the information developed is equally valuable to hydro-electric interests, to those responsible for city water supplies, and to the operators of the flood-control reservoirs that lie in the Southwest and other parts of the country.

The writer believes that the snow-survey data, thus collected, will induce its extension until all the major streams, at least in the West,

Note.—This Symposium was presented at the Fall Meeting of the Society and at the meeting of the Waterways Division, Pittsburgh, Pa., October 13 and 14, 1936, and published in March, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the Symposium.

[35] Senior Irrig. Engr., U. S. Dept. of Agriculture, Berkeley, Calif.

[35a] Received by the Secretary May 8, 1937.

have sufficient courses established and under observation to enable their performance for the following season to be known. The purpose is not in terms of a snow survey to determine merely how much snow lies on the ground, but wholly as a point on a definite curve from which one can forecast the probable run-off from that stream during the following year. A long-time accumulation of such points gives a curve of reasonable accuracy.

HOWARD T. CRITCHLOW,[36] M. AM. SOC. C. E. (by letter).[36a]—It is scarcely necessary to emphasize the importance of the collection of fundamental data to which Mr. Wolman refers as the bookkeeping of the water-resources problem. There is an economic aspect to this problem, namely, that its solution requires money and that such money must be obtained, in most cases, from Government agencies. The writer has had a little experience with trying to get money from State legislatures for this purpose in order that the State may co-operate with the Federal Government in the collection of this necessary information. Engineers who appreciate the importance of these fundamental data, should make that fact known to the people who have control of the "purse strings" in the Federal Government and in the State Government. It does not require large sums, but it does require sufficient money to increase the facilities for collecting this information. This subject is discussed at some length,[37] in a report of the National Resources Committee on "Deficiencies in Hydrologic Data."

T. T. KNAPPEN,[38] M. AM. SOC. C. E. (by letter).[38a]—One gains the impression from reading the interesting group of papers in this Symposium that flood-control engineering is as yet a poorly developed branch of the profession. The economic aspect of the problem seems to be the least developed in the opinion of most of the authors, although the engineering side provokes a rather varied discussion. There is manifested a feeling that the problem must be dealt with on broader lines than in the past, that technically it must be considered as a major problem in stream planning and development, and that, economically, new values must be reflected in the determination of justification if the popular demand for adequate protection is to be met.

These demands point the need for a critical overhauling of the methods of handling these problems in the past and, to-day, there is every evidence of this development in that section of the profession connected with flood control. Essentially, the problem may be divided between: (1) Hydrological studies, concerned with snowfall, rainfall, run-off, and flood characteristics; (2) planning, which includes the investigation and design of flood-control structures, such as reservoirs, walls, dikes, floodways, and

[36] Engr. in Chg., State Water Policy Comm., Trenton, N. J.

[36a] Received by the Secretary April 14, 1937.

[37] Rept. of the Special Advisory Committee on Standards and Specifications for Hydraulic Data: Submitted by the Water Resources Committee on April 18, 1936, to the National Resources Committee.

[38] Prin. Engr., North Atlantic Div., U. S. Engrs., New York, N. Y.

[38a] Received by the Secretary April 23, 1937.

channel improvements; (3) flood routing, which is used to determine the effect of such works on flood stages; and (4) economic studies, which determine the justification for the construction of flood-protection works and relative merits of the various plans considered.

Hydrological Studies.—The recent experience with disastrous floods on many of the important rivers of the United States has modified the American engineer's ideas as to the probabilities of their occurrences. Many engineers now are dubious of the value of frequency studies developed on the short recorded experience available, particularly in view of the constantly changing condition of water-shed development. A natural development is an investigation in the meteorological field that will indicate the possibilities of maximum precipitation conditions and a possible upward revision of intense rainfall frequencies. Another major consideration is the change in water-shed development that has been occurring over many years, the determination of its effect on run-off, and a consideration of curative or alleviating measures. The necessity for more basic data along these lines is apparent to every engineer dealing with these problems, and it is to be hoped that the curtailments that have resulted in the partial cessation of these activities in many parts of the country will come to an early end.

Planning.—Rarely to-day does one hear of proponents of levees, reservoirs, or channel improvements declaring that their method is the only one to use. On the other hand most engineers realize that any major plan may combine all of these, as well as other methods, such as floodways, outlets, and water-shed control. Furthermore, it is becoming more apparent, as emphasized by several of the authors, that flood control cannot be separated from other developments in water conservation and must be considered in civic planning and regional development plans.

Mr. Uhl has indicated that, in the New England States, flood control can be secured as an incident to power development with a reasonable degree of safety. Others may question the dependability of such control, but regardless of individual opinion, at least in many instances economic planning requires that the two be considered together. Another important factor is the need for increased stream flow for sanitary reasons which may justify a greater measure of conservation than power alone. The reasonable combination of these needs with those for flood control, together with a careful economic valuation of available storage and its most beneficial use, is an important regional problem. The solution of such problems has an important bearing on the planning of flood-control structures.

Flood Routing.—Only through accurate flood routing can the effect of flood-control works be determined and their economic justification established. Messrs. Morse and Thomas take a rather pessimistic view of the developments along this line and suggest that model studies under way at the Carnegie Institute of Technology, in Pittsburgh, Pa., offer the best solution to this problem. They seem quite hopeful that these models will give accurate results in the determination of the effectiveness of proposed works. Although the writer feels that the use of these models is unques-

tionably a wise move and that they will give valuable results, he is also of the opinion that recent developments in the analytical handling of these problems have eliminated their objections partly and that excellent results are now being obtained with the methods developed on the Muskingum flood-control project, in Ohio.

It should be emphasized that the analytical flood-routing methods can be checked by applying them to known past floods. When properly made, these checks have resulted in accurate reconstruction of past floods. This procedure seems parallel to that used in the model studies, in which the model is adjusted for known past floods.

The analytical method is based upon the principle that constant ratio exists for any reach of the river, between the valley storage in the reach and a weighted flow determined from inflow and outflow. The equation may be written,

$$K = \frac{t\,[0.5\,(i_2 + i_1) - 0.5\,(o_2 + o_1)]}{X\,(i_2 - i_1) + (1.0 - X)\,(o_2 - o_1)} \quad \text{..............(3)}$$

in which K = ratio of storage increment in the reach, in day-second-feet, to corresponding weighted flow increment, in cubic feet per second; t = time unit of computation, in days, or in fractions of a day; X = fraction of weighted flow increment that is derived from the inflow increment; i_1, i_2, etc. = total instantaneous inflow, in cubic feet per second, to a reach at the beginning of successive time units (t); and, o_1, o_2, etc. = corresponding instantaneous outflow to a reach at the beginning of successive time units. The numerator in Equation (3) is the storage increment, in day-second-feet, and the denominator is the corresponding weighted flow increment, in cubic feet per second. If the denominator is rewritten in the form: $(o_2 - o_1) + X\,[(i_2 - i_1) - (o_2 - o_1)]$, it is apparent that the first term represents the "prismatic" storage below the normal surface slope, and the second term represents the "wedge" storage increment. It will be noted that this eliminates at least part of the objection expressed by the authors to the analytical method. This method was brought to its present development by Thomas S. Burns, M. Am. Soc. C. E., and Frank B. Harkness and G. T. McCarthy, Assoc. Members, Am. Soc. C. E., in the flood-routing studies on the Muskingum project.

The analytical flood-routing method as now applied is believed to give fair results on the main stems of important rivers but in most cases its application is not practicable on the tributaries particularly where many streams of about the same relative importance are coming together. For such cases, some method of developing modified hydrographs for reservoir systems is needed, which will account for the location and the collection characteristics of the controlled areas as well as their extent. Possibly, the technique of model analyses can be developed for the accurate solution of such cases. At the same time there is a need for an analytical method that can be used at least for preliminary studies.

Economic Studies.—Mr. Jacobs states that: "The construction of flood-protection projects on a monetary economic basis alone is a thing of the

past. Where the lives of human beings are endangered and where great suffering may occur as the result of floods, protective works should be constructed." This statement is qualified later in the paper by the following: "It is also essential that those who must pay for the protective system have full knowledge of the cost, of what burden they must bear, and of what security the final developed plan provides."

On the other hand Mr. Uhl states,

"Attempts have been made to rationalize the second problem, namely, that of flood prevention or reduction, on a purely economic basis. If definite information were available, the solution would be relatively simple; the maximum expenditure economically justified to prevent recurrence of certain flood damages would be the capitalized present worth of the damages over the period of flood frequency causing them."

Somewhere between these points of view the answer to the problem must lie. Taking the case of an individual community planning to protect itself, it can determine its policy knowing the costs and the degree of protection. The people of that community may see more to the problem than the capitalized value of average annual damages. They may do as the people of Dayton did, after the 1913 flood—envision a blighting of their community, an almost complete abandonment of the flooded areas, with the loss in private investments therein far in excess of the capitalized annual direct and indirect flood losses based on frequency calculations. Cities such as Pittsburgh, Pa., Hartford, Conn., Binghamton, N. Y., Springfield, Mass., Wheeling, W. Va., Cincinnati, Ohio, Louisville, Ky., and Paducah, Ky., can take little satisfaction from statistical computations that show the average annual losses to be small based upon frequency studies. They are faced with the plain facts of tremendous property depreciation and restricted development, creating a situation in which the assurance of protection against future floods has a value in excess of the capitalized average annual value of flood losses.

The writer would like to suggest that the answer to the problem lies neither in the abandonment of all economic rules nor in the strict application of the criterion of capitalized value of average annual flood losses in the accepted sense, but rather that it may be found in the determination of benefits which may be concerned with the restoration or maintenance as well as the appreciation of property values depreciated by flooding, with an added evaluation of the benefits to Government property and of the benefits due to elimination of indirect damages outside the flooded areas. This attitude is based on the assumption that the intangible values not ordinarily evaluated, such as fear of loss of life, mental distress, tendency to abandon flooded areas, etc., as well as direct flood losses, are reflected in such depreciation.

The evaluation of these benefits although difficult, may be worked out and the need for flood protection established in such places as the Ohio River, Connecticut River, and Susquehanna River Valleys where recent disastrous floods have demonstrated the vulnerability of the valley towns. On the other hand, may there not be many other places as vulnerable that

have not felt the disaster of a major flood to "drive home" the necessity for, and value of, flood protection? Despite the previous floods, the citizens of Pittsburgh were not sufficiently flood-minded before 1936, to demand protection and to be willing to pay at least part of the costs. Prior to the 1937 flood, it would have been most difficult to convince the people of Louisville or Paducah that their cities were in danger of serious flooding. Certainly, there is a necessity for the development of sound engineering and economic methods for determining need for protection, not only for places which floods have attacked, but also for those places which, although equally vulnerable, have not felt the scourge of a major flood in recent times.

Assuming that the need for flood protection can be evaluated on a broad enough basis to justify protection for the more populated sections, it should be possible to apply the developed data to the internal economics of various projects. There are many problems to be solved therein, such as the relative balance between reservoirs, channel work, and levees, the economic size for any reservoir, the height to which levees should be built, etc. The solution of these problems requires the careful development of the basic data by zones and elevations as well as extensive flood-routing studies to assign values to the units under consideration. Furthermore, frequency studies are necessary as a basis for the determination of average annual values.

The economic problem is not a simple one in any case and on large river systems becomes highly involved; but only by the application of thorough field investigations and office studies can adequate and sound economic, as well as engineering, studies be developed.

M. C. TYLER,[39] M. AM. SOC. C. E. (by letter).[39a]—An excellent account of the situation with respect to flood-control work by the Federal Government has been given by Colonel Covell. He has traced the legal and legislative procedure through which the United States has passed to reach the stage at which the Congress has authorized many worthy and beneficial projects for the protection of lives and property from the ever-recurring ravages of excessive flood flows caused by periodic extraordinary precipitation.

It was inevitable that, sooner or later, the people would demand that the National Government assume some obligation for the protection of citizens from the injuries and devastations caused by floods which frequently arouse so much human interest, first in one section of the country and then in another. Flood-control works, of necessity, affect the navigation of rivers in some manner, and it is logical that the Federal Government should accept its proper function with respect to flood control.

The Constitution of the United States confers upon Congress the power "to regulate commerce with foreign nations, and among the several States, and with the Indian tribes," and by this provision there was transferred

[39] Brig.-Gen., Corps of Engrs., U. S. Army; Asst. to Chf. of Engrs., War Dept., Washington, D. C.

[39a] Received by the Secretary April 22, 1937.

from the States to the Federal Government the control of all the navigable waters of the country for the purpose of navigation. At first, it was seriously doubted whether the power to regulate, comprehended the right to improve; that is, whether the improvement of rivers and harbors was a subject of national concern and of constitutional appropriation. In May, 1824, Congress passed the first Act to improve navigation entitled, "An Act to Improve the Navigation of the Ohio and Mississippi Rivers." About that time the luminous decision of Chief Justice Marshall, in the case of Gibbons *vs.* Ogden, established clearly and indubitably the power of Congress with respect to the waterways of the country which could carry interstate commerce, and removed completely the doubts of earlier years. That decision of the Supreme Court of the United States marked the birth of a permanent Federal policy of river and harbor improvement—a policy which has grown from year to year with the increase of population and needs of commercial transportation. Now, flood control also has been made a national activity by Federal law.

The general flood-control bill approved June 22, 1936, and the Overton Flood Control bill approved June 15, 1936, were studied for a period of almost two years by committees of Congress. They were based on the reports of the Chief of Engineers containing the results of studies extending over a long period of years. The Mississippi River Commission is continuously engaged in studying the flood-control problem on the lower river, and advance planning for additional works necessary to provide full protection has continued with the prosecution of the works authorized for construction by Congress. The voluminous and comprehensive data presented in the reports of the Army Engineers[40] provide the principal basis for national planning with respect to the development of water resources. They permit the preparation of a program for such development on a scale limited only by advisable expenditures. This program may go hand in hand with the development of land resources, reforestation, and measures for the prevention of erosion and retardation of run-off. It is very seldom that major flood-control structures conflict or interfere with other water uses.

The Flood-Control Act of June 22, 1936, embarked the Federal Government on a flood-control program which is destined to develop in an economic and orderly manner so as to afford great and lasting improvements throughout the land for the preservation of lives and property and the betterment of living conditions generally.

Since the enactment of the general Flood Control Act, the Army Engineer organization has been working vigorously in making detailed plans for beginning construction by contract of the flood-control works authorized. The War Department will be ready, when funds for actual construction are appropriated, to begin work without delay. In the design of dams and appurtenant works, the Chief of Engineers is fortunate in having the assistance of many of the ablest and most experienced consulting

[40] House Doc. No. 308, 69th Cong., 1st Session.

engineers in the United States, and hopes to secure the services of others as the need develops. Much help is also being afforded through co-operative arrangements with other Departments and with the hydraulic laboratories of universities.

Although the planning of flood-control works by the Federal Government is in a most favorable and constructive situation, this does not mean that everything desired is already accomplished. There is much hard work to be done, and innumerable difficult problems to be solved. However, the ground-work has been well laid. Questions of local co-operation as to financing appear difficult, but they can be solved with patience, judgment, and co-operative thinking.

GORDON R. WILLIAMS,[41] ASSOC. M. AM. SOC. C. E. (by letter).[41a]—In discussing the economics of flood control, Mr. Uhl states that, on a purely economic basis, the justified expenditure for the prevention of flood damage caused by a flood of a certain magnitude and frequency "would be the capitalized present worth of the damages over the period of flood frequency causing them." It seems desirable to clarify the procedure implied for the determination of future damages for which the present worth is to be obtained. There may be situations in which too much reliance on the figure for past damages would erroneously justify too large an expenditure for flood protection and then, again, it might lead to an entirely inadequate expenditure. As an example of the former situation consider the Winooski River Valley which is mentioned by Mr. Uhl.

The reconstruction that occurred in the Winooski River Valley after the flood of 1927 was undertaken with the intent of avoiding, as far as possible, a repetition of former damages. Consequently, highways were relocated on higher ground, concrete and steel truss bridges with large openings replaced light bridges with inadequate openings, spillways of dams were enlarged, and many people rebuilt their homes on higher ground. The result is that if the flows of 1927 occur again, the damage in the Winooski River Valley will not be as great as it was in 1927. In other words, some of the justified expenditure for the control of flood waters was spent at the time of reconstruction. Of course, if flood-control measures are adopted immediately after a flood there may be a different policy in regard to expenditures for reconstruction, but if reconstruction has already taken place weight should be given to the situation just considered. A similar situation may exist in other communities where there has been a permanent decrease in the value of property which lies in the path of flood waters.

In contrast to the foregoing example are the more numerous arguments for using, as a basis for computing justified expenditures, a value that exceeds past damages from the greatest flood. Again, taking the Winooski River as an example, one finds, that the U. S. Army Engineers

[41] Asst. Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C. Submitted with the approval of the Director, U. S. Geological Survey.

[41a] Received by the Secretary, April 14, 1937.

computed that an expenditure of $433 000 was justified for protection against the 100-yr flood. However, this sum should be increased because a structure which protects against the 100-yr flood also protects against damage caused by the smaller but more frequent floods. Actually this was done and the justified expenditure was increased to $1 055 000.

The problem of increasing the justified expenditure because of a possible increase in the value of property which will be benefited by flood control is analogous to the one of determining, theoretically, the justified expenditure for a trunk-line highway. Roughly, the procedure is to take a traffic count of vehicles that use existing facilities, and then compute the resulting revenue in taxes that will come from the sale of gasoline used by the vehicles. Obviously, this does not give a true picture of the situation which will result from the construction of the road because there is little doubt that the volume of traffic will exceed the original traffic count and, in addition, there will be an almost indeterminate saving to private individuals and to industries that benefit by the construction of the road. The original estimate of justified expenditure must be increased, but how much?

A comparable situation exists in flood-control planning. If reasonable protection is given to river valleys, the cities and towns affected may grow, industries may be attracted, property values will doubtless increase, and the wealth of the individual citizens will be augmented. In such cases the figure for the past flood damages becomes entirely inadequate as a basis for determining the price to pay for future protection.

This discussion has considered the purely economic justification without any allowance for the humanitarian aspect stressed by Mr. Jacobs. There is no question but that this consideration should be stressed, but not to the point where a community assumes a burden which may even threaten its financial ability and thereby interfere with the fulfillment of its normal commitments. Experience may prove that in some cases the soundest method to pay for future damages will be to establish an insurance fund maintained at least in part by small premiums paid by those who cannot, or will not, move out of danger.

ARTHUR T. SAFFORD,[42] M. AM. SOC. C. E. (by letter).[42a]—Engineers working on designs of river structures, officials and operating men in hydro-electric power stations, and owners and managers of industrial plants located on New England rivers will find the paper by Mr. Uhl extremely valuable, not only because of the important data, but because of its arrangement and the accurate conclusions contained therein. One must have had the experience of living and designing structures in and on New England rivers to interpret properly flood records. One needs only to read over the newspaper accounts and early reports of the 1927 and 1936 floods, to realize how little was known at the time about their causes, and the lessons to be drawn from them. Unless records of river heights and conditions given

[42] Engr., Proprietors of Locks and Canals on Merrimack River; Cons. Hydr. Engr., Lowell, Mass.

[42a] Received by the Secretary April 15, 1937.

out during a great flood are correctly reported by newspapers, or correctly broadcast, the result is to confuse and make it more difficult for the authorities to know when and how to act in protecting the public.

This is one of the many papers contributed by engineers far enough removed from the hysteria produced by the floods, to make them valuable as a permanent record. Even a year after the Great Flood (which name, for many years, will be given to that of March 20, 1936), opinions are expressed and plans made for eliminating future floods, which in many cases have no more basis than the hope that such a result is possible.

The writer's official responsibilities during the 1927 and 1936 floods were confined to the Merrimac River, at Lowell, Mass.; but he was in the employ of the Water Power Company there, during the floods of 1895 and 1896 mentioned by Mr. Uhl; and was familiar with the flood marks of 1785 and 1852. At Lowell, there are on record six or more floods during 150 yr which registered 10 to 13.5 ft on the Pawtucket Dam, and about 15 ft of back-water, but which did little damage beyond the quiet flooding of streets, cellars, and mill basements. Machinery and household goods, and goods in process left on the machines were inevitably wet; but the protective margin of a few feet above the peak of these floods was sufficient to leave intact all hydraulic structures of the Company, due to the foresight of Mr. Francis, by installing the Francis gate,[43] before the famous 1852 flood, as a result of his careful studies of former floods.

FIG. 18.—SAND BAY AND TIMBER DAM ACROSS THE BOSTON AND MAINE RAILROAD WEST OF LOWELL RAILROAD STATION.

The flood of March 20, 1936, on the contrary, which registered a final height of 102.02 ft, or 20.02 ft above the crest of the Pawtucket Dam, and 6.5 ft above any previous peak, produced an additional 8 ft of back-

[43] *Civil Engineering*, February, 1937, p. 143.

water. For the first time the Merrimac River was not held within its ordinary flood channel, but broke through at several points, went entirely out of bounds, and isolated large sections of the city. It was only prevented from wrecking parts of the business center of the city, by a skeleton dam, 3.5 ft above the Francis gate, and a timber and sand-bag dam across the railroad right of way into the city from the north (see Fig. 18).

After the water subsided, it was found that the city had had a very narrow escape from lasting injury. The great mass of fill accumulated over a period of 100 yr and more over low areas on the banks of the river or dumps was gone; many neighboring buildings were wrecked; certain streets in the path of the flood were left with half the buildings either moved or wrecked; the great Northern Canal wall of the Proprietors of Locks and Canals (which had been overtopped for the first time in its history) suffered serious injury in several places; and most of the conduits carrying electric power to the river mills were so thoroughly wet that for nearly a year thereafter, weak places kept appearing and had to be replaced.

FIG. 19.—ABOUT THREE-QUARTERS OF CANAL LINING REPLACED; REMAINING QUARTER OPEN SHOWING CHARACTER OF BREAK.

All these structures belonging to the Water Power Company and the industrial plants on the river, were repaired before the end of the year, although much of it had to be done by continuous week-end operations. Fig. 19 shows the repair work in progress, the unfinished part indicating the nature of the break.

From personal examination and published reports by utilities, insurance authorities, and others, apparently these same conditions, both during and after the Great Flood, obtained at Concord and Manchester, N. H., and at Lawrence and Haverhill, Mass.

The fact that, in the 1936 flood, the stage was 6.5 ft higher, and the quantity of water 65% greater than any previous flood, with existing margins of safety wiped out, would appear to make some improvement necessary if these communities are to survive without the anxiety of a repetition of this situation.

Any study of flood control for New England rivers would seem to require a knowledge of what quantity and height of water had occurred prior to this 1936 flood; what additional burdens this flood had placed on the industrial cities; and what reasonable improvements could be made in the nature of reservoirs, channel improvements, and dikes, to bring the local control down to the situation as it prevailed during 150 yr or so prior to 1936. For Lowell and the Merrimac River, this means a peak run-off of not more than 100 000 cu ft per sec instead of 173 cu ft per sec; a depth on the Pawtucket Dam of not more than 13.5 ft instead of 20 ft; and a corresponding reduction of 8 ft in the height of the backwater below the city.

Assuming flood reservoirs to be actually empty and 100 000 cu ft per sec to be the maximum at Lowell, and the 17 500 000 000 cu ft of natural flood pondage above Lowell, Manchester, Concord, and Bristol, N. H., to be not reduced, the equivalent of two additional Lake Winnepesaukees would be required.

Mr. Uhl wisely refers to several "partial" solutions, and they should be more generally considered as such. The promise of more than this simply deceives the public who must pay the bill, either by assessment or taxation. These structures may be storage reservoirs, improved channels and dikes, and free channel bridges instead of reinforced concrete arched structures. All these controls, built and maintained at great expense, may not be sufficient to reduce the maximum peak at the dams across the Merrimac River more than 2 or 3 ft, but even this may be worth while.

In recent years, New England has experienced storms which may be called torrential, and such storms appear to be inevitable both locally and over areas of several thousand square miles. They make the White Mountains and the Green Mountains look more and more like the roof of a barn with the run-off, ultimately, being approximately the same as the rainfall. Add to this condition the melting snow and ice cover, and the conditions existing prior to March 20, 1936, are re-created.

As a lesson learned from the 1936 flood, it would appear that it is the duty of any one in authority on these New England rivers: (*a*) To understand, fully, the conditions that under the proper combination of circumstances, may make for a heavy run-off; (*b*) to have complete records; (*c*) to know how much and how far permanent structures will stand the strain; and (*d*) to have ready at least a skeleton on which to build temporary barriers well above the permanent structures. Office records in

the form of diagrams and photographs should be available and studied from time to time, as freshets (even mild ones), occur; and lessons learned from each freshet should have their place in the records.

In addition, an important element in preparing for New England floods is time—time to transfer human beings, livestock, and necessary and valuable possessions to higher ground. In Vermont, in 1927, besides property damage approaching \$40 000 000, there was a large loss of human life (87 were drowned, 55 in the Winooski River Valley), for the most part due to the suddenness of the rise of the water and the difficulty in maintaining communications.

Very few lives have been lost in floods of the Merrimac River although the time of rescue often has been short. The peak of the 1936 flood at Plymouth, N. H., was on March 19, from 8:00 A.M. until noon, and at Lowell on March 20, from 5:00 P.M. to 11:00 P.M.; but the Lowell peak had been forecasted early on the morning of March 19, the Francis gate had been dropped, and work had begun on the protective dams above that point by 7:00 A.M.

The reports and predictions of flood heights from the up-stream stations, maintained by the public service companies and other authorities, were so frequent and reliable that not only could the highest points to be reached be estimated, but the length of the peak. There was ample time to warn officials of the city, the U. S. Works Progress Administration, Red Cross, State Police, and the National Guard; and as a result of the splendid co-operation which American people always have given when they know what and when there is an emergency, the City of Lowell did not have an appalling loss of life. It speaks for the value of the operating forces of the public utilities, beyond that of temporary activities. Most of the information broadcast by radio, in so far as it was insufficient or mistaken, probably increased the anxiety and may have done more harm than good. The radio can be made a splendid addition to the telephone and telegraph, if in the hands of proper authorities.

Some conclusions, which appear to the writer to be warranted by the experiences of the March, 1936, flood, and its record-breaking heights, flow, and currents, are:

(1) Engineers and others, concerned with flood work, should think in terms of maximum run-off, possible from heavy rains, warm weather, and melting snow, and not of what heights had been reached through a limited period of years. They should be "conditioned" to the idea of the 100-yr flood. The precipitation and water equivalent of the Pinkham Notch record for March, 1936 (29 in.) may occur the next time in almost any of the mountain valleys of New England.

(2) More records should be taken every half hour, or so, for the upper rivers and principal tributaries, as well as the main river stations, in order to be able to predict whether peaks will be distributed or be concentrated.

(3) Abutments should be higher; if the last few feet interfere with approaches to power or gate-houses, or if there are railroad cuts, such as

those at Lowell, Mass., and Manchester, N. H., concrete ribs, with holes drilled for plank flash-boards, may be substituted. At the Old Guard Locks, at Lowell, an additional 3.5 ft above the Francis gate was taken care of without excitement by that form of construction, built after a study of the 1927 flood appeared to warrant it.

(4) Every growing industrial city or town, situated on a New England river, should have a map showing flooded areas for the maximum flood. This should be used as a picture in the acceptance of streets, in building bridges, in laying out sewers and in granting building permits. The occurrence of ice jams, accumulating pressure, and wrecking bridges and other structures, is always a possibility before the winter is over. No structures should be built which will add to the flood menace of any district.

(5) The valley storage which obtained during the 1936 flood from Lawrence, Mass., to Franklin, N. H. (estimated at 17 500 000 000 cu ft) was about 50% greater than the combined storage in Lake Winnepesaukee and other reservoirs that were already full and discharging before the danger of a higher peak was over. There is no advantage in depending upon flood reservoirs unless they are nearly empty before the extreme test which they are expected to meet.

(6) The duty of setting permanent markers on New England rivers for the 1927 and 1936 floods, should not be allowed to be forgotten. The fact that the 1936 flood carried 65% more water than ever before, may make these records good for a century or more.

(7) Federal and State operating activities (such as the Weather Bureau, Geological Survey, U. S. Engineer Corps, State police, etc.) should have on file the records of the 1936 flood; and, at times of future floods, they should refer to the unusually good records of the utilities operating hydro-electric power stations as well as to their own. Snow records should be much increased in number.

This discussion does not do justice to Mr. Uhl's paper and is too personal. Having lived through the strain of the week and more of the floods that occurred from March 13 to 20; and afterward waiting for the full extent of the damage to be known and another two months before repair work could be begun, it is not easy to study any problem of flood control for the Merrimac River impersonally; but it leaves one with a profound respect for the forces of Nature, and the difficulty of competing safely with them.

The declivity of New England streams is in their favor. The Pemigewasset River, at Plymouth, could not have passed 65 000 cu ft per sec on March 19, 1936, or the Merrimac River, at Lowell, 173 000 cu ft per sec the next day, except for the slope maintained to produce the high velocities. Velocities as great as 20 ft per sec were not uncommon.

Local flood control by dikes, and river channel improvements on limited areas, should include the sewerage problem, because of many individual outlets into the river. When such outlets "back up" they can be even more dangerous to the public than quick flooding by clean water.

From the insurance standpoint: If the damage to the Merrimac Valley from the 1936 flood was roughly \$20 000 000, and a flood of such intensity occurred only once in 100 yr, the sum of \$400 000 at 4% compound interest would pay the equivalent at the end of 100 yr; if once in 200 yr, the sum of \$8 000. There would appear to be a field for flood damage insurance, as it might be written by companies operating on a national scale, to protect at least a part of the investment in industrial plants, stores, warehouses, and their contents against loss.

W. G. Hoyt,[44] M. Am. Soc. C. E. (by letter).[44a]—No city of the United States of equal size is situated so unfortunately with reference to rapid concentration of flood waters as is Pittsburgh, Pa.; yet, with respect to flood damage, Messrs. Morse and Thomas describe Pittsburgh as a "lucky city," a description that will be concurred in by all engineers who have studied cause and effect of major floods in the basins of the Allegheny and Monongahela Rivers However, there seems to be no valid reason why Pittsburgh's luck will always hold. Eventually, Nature's roulette wheel will record a double zero. When that occurs it is sincerely to be hoped that a catastrophe will be averted by means of a "superior type" of reservoir control combined with an efficient system of flood-forecasting.

Whether or not a super-storm can occur in the Monongahela and Allegheny River Basins simultaneously is a question which, perhaps, meteorologists will some day be able to answer. It is true that no such outstanding storm has thus far been recorded—the Miami (Ohio) storm of March 24 and 25, 1913, and the storm of January, 1937, having had their centers to the west, and the storm of March 17 and 18, 1936, to the southeast.

If any of these storms had been centered over the Upper Ohio River Basin greater floods would have resulted than the record-breaking flood of March, 1936. However, even if it is assumed that precipitation over the basin which has been recorded in the past is an index of future possibilities, Pittsburgh has thus far apparently been fortunate, and especially so in March, 1936. During February, 1936, there were three thaws in the Monongahela River Basin and one in the Allegheny River Basin which materially reduced the water content of the snow then on the ground. Conservative estimates indicate that the reduction in water content may have amounted to between 1 in. and 2 in. in depth over the Allegheny River Basin and to between 2 in. and 3 in. over the Monongahela River Basin. The minor flood peaks resulting from these thaws broke up and removed the unusually thick channel ice. To some extent at least flood run-off must have been reduced by increased infiltration as a result of the reduction of frost in the ground. All these conditions were favorable to Pittsburgh.

[44] Hydr. Engr., (Prin.), Conservation Branch, U. S. Geological Survey, Washington, D. C.

[44a] Received by the Secretary April 21, 1937.

Messrs. Morse and Thomas refer to the turning of rain to snow above Oil City, Pa., during the storm of March 17 and 18, 1936. An analysis of the snowfall, rainfall, and run-off during the flood period discloses how important this freak of temperature was in lowering the flood crest. Apparently, much of the rain turned to snow not only in the Allegheny River Basin above Oil City, but also in the Monongahela River Basin above Morgantown, W. Va. The effect of this occurrence was materially to reduce the resultant run-off from that storm. That this condition existed is evidenced by the fact that in the upper basins of both the Monongahela and Allegheny Rivers the run-off during the period, March 20 to 31, after the flood crest had passed Pittsburgh, was in excess of the precipitation, indicating a postponement of run-off because of the snow of March 17 and 18, equivalent to possibly 2 in. in depth of water over the basins. The peak run-off during the period, March 20 to 31, in these areas, was greater than the peak run-off during March 17 and 18, when Pittsburgh was experiencing its maximum stage—a stage very considerably lower than that which would have occurred if the rain had not turned to snow over nearly half the drainage basins.

Messrs. Morse and Thomas seem to have departed from an excellent presentation of facts into the realm of conjecture when they referred to "the man-made floods." It is true, that at Pittsburgh, Man is occupying the valley and encroaching on the channel that Nature has designed to carry the floods and, to this extent, the flood damage is man-made. Getting into the way of a flood and being damaged by it however, are far different from making a flood. No evidence is presented to indicate that the flood of March, 1936, was increased as a result of changes made in forests or other vegetative cover of the basin, or as a result of any of Man's activities.

Do engineers play fair with the public who look to them for facts and advice if they lead the public to believe that Man is accountable for the excessive run-off of a major flood, such as that of March, 1936, in the Upper Ohio River or that the run-off following major storms, involving the unusual timing of the meteorologic events which create floods, can be controlled materially, except by engineering structures?

Lt. Col. J. D. Arthur, Jr.[45] Corps of Engineers, U. S. Army (by letter).[45a]—The present status of Federal flood-control projects has been fully outlined by Colonel Covell. It is to be noted that the system of reservoirs for flood control on the Ohio River, either authorized or under construction, is distributed over a wide area, on many tributaries. Little has been published as to the methods of operating this system to provide the maximum benefits.

The operation of the fourteen reservoirs now (1937) under construction in the Muskingum Valley will be placed in the hands of the Muskingum Watershed Conservancy District, a political sub-division of the State of Ohio. It is

[45] Dist. Engr., U. S. Engr. Office, Zanesville, Ohio.

[45a] Received by the Secretary April 30, 1937.

only human to assume that it will be operated to provide the maximum protection to the residents of the Muskingum Valley. However, will such operation provide the maximum protection to the residents along the Ohio River below Marietta? Engineers know that the Miami Conservancy District System provides full protection to the residents of that Valley, but they likewise know that, with its uncontrolled outlets, benefits to localities along the Ohio River will not necessarily accrue. In fact, it is easy to imagine conditions under which it might prove to be decidedly harmful.

When the system is extended to other tributaries, each with a local flood problem in its valley, does it not follow that, unless there is a single office with full authority to control operations on all tributaries, the system will not be operated for the greatest good of the greatest number? The construction of the system must be the primary objective, but studies should be initiated at once of the best methods of operation.

The problem is a difficult one, and it is not simplified by the requirements of local participation in costs. If the residents of the Muskingum Valley, or of Pittsburgh, for example, are taxed for a portion of the costs, might they not well ask why they should be interested in what happens at Cincinnati? Perhaps this phase of the subject did not receive proper attention when the policies enunciated in the Flood Control Act of 1936 were adopted.

JOHN H. MEURSINGE,[46] ASSOC. M. AM. SOC. C. E. (by letter).[46a]—Except China, there is probably no country in the world which has suffered so much from floods as the low lands along the North Sea, which are better known as Holland, or The Netherlands.

The first levees in Holland were built by the Romans to protect their military highways. As the construction of levees has been continued ever since, a protective system has been developed, which probably has no equal in the world. It is small wonder that some Dutch engineers do not consider the levee system of the Mississippi as very effective; but in an interesting comparison between flood problems in Holland and in the United States,[47] C. E. W. van Panhuys, District Engineer for the Netherlands Government, states that engineers in the United States have done everything technically and financially possible for such a young country.

In connection with this Symposium, the question may be raised as to whether American engineers could benefit by Holland's long experience with floods. The answer is "yes," always bearing in mind the fact that the Dutch engineers are mainly concerned with delta problems, so that dams and reservoirs are of minor importance to them. The purpose of the collapsible dams constructed on the Meuse River about 1917, was to improve navigation in summer. They are made to disappear during the flood season.

[46] Structural Designer, General Petroleum Co., Vernon, Calif.

[46a] Received by the Secretary, May 6, 1937.

[47] "Overstroomingen langs Ohio en Mississippi", *Ingenieur*, Royal Inst. of Netherlands Engrs., March 5, 1937.

Furthermore, the area considered is rather small—about one-five-hundredths of the Mississippi River water-shed. However, it is not the size that counts, but the effectiveness of the protection system. Since about 1880 human lives have scarcely ever been lost in Holland on account of floods.

For each river, the Dutch engineer defines two channel sections: The so-called winter section and the summer section. Both are protected by levees. The small dikes along the summer section simply protect the valuable grazing grounds in the winter section against the summer flash floods. The winter section is from 1 500 to 3 000 ft wide, and the summer section from 450 to 675 ft wide. Within these sections no encroachments are allowed except brick kilns, which may excavate enough clay to equal the volume of the buildings they occupy.

Occasionally, a severe flood will overtax the winter section and spillways are provided to meet this emergency. To understand the function of these spillways the geography of the Meuse-Rhine delta must be considered. Most of the lands have elevations which vary between the low-water and high-water stages of the rivers. If a dike collapses much land would be threatened with submersion except for the fact that it has been divided into many different diking districts or "polders," each of which are protected by minor levees. Such a "polder" is an administrative unit, administered by the property owners themselves. They are under the supervision of the provincial authorities, however, who create and consolidate the districts if desirable. The district maintains all its levees, makes minor repairs, and operates the pump-houses which take care of the drainage of the seepage water.

In case of a dangerous flood, each "polder" sends its guards to the dikes. Their services come under the control of the engineers of the Netherlands Government who take charge of the entire river as long as the emergency lasts. If a dike threatens to collapse, or actually breaks, the engineers have the same authority as is generally granted to military commanders in time of war.

The advantage of many diking districts is obvious. When a spillway discharges its water over the dike, it submerges one "polder" only. This district is occasionally sacrificed for the benefit of the others.

In spite of these spillways, breaches in dikes still occur. Hence, each district has been provided with a so-called "terp." This is nothing but a properly located levee, which is high and large enough to take care of the population and live stock of the "polder" in case the district happens to be flooded. Frequently, the church and some public buildings are located on the "terp"; and it hurts no one's feelings when the church is converted into a cow barn during times of emergency.

The methods of making emergency repairs have changed very little in the past hundred years. The first sign of danger is an increased seepage flow in the dike. This water is likely to wash away so much of the material that all the fingers of all the Dutch boys would be unable to stop it. Mats made of willow branches, staked down on the inside talus of the dike, are more effective. They keep the material in its proper place, at

the same time allowing a free flow of seepage water. This method does not cause any increased water pressures inside the dike, which might produce a breach.

Although breaches seldom can be stopped until the floods have passed, occasionally the "hay-wagon" may solve the problem. One or several wagons tightly packed with hay are dumped, wagon and all, in the breach. Whereas a more solid material would be swept away immediately by the in-rushing waters, the hay permits water to flow through, and the wagons may form a foundation on which sand bags can be piled for temporary repair.

Most of the breaches in Holland have occurred in the dikes along the Meuse, the most troublesome river in The Netherlands. In its low stage the run-off may be as low as 7 000 cu ft per sec, and in its flood stage as high as 100 000 cu ft per sec.

In 1923, the construction of a series of collapsible dams was begun to improve navigation conditions in summer. While this construction was at its peak a run-off occurred which was 15% greater than the previous recorded maximum. It flooded all the works and broke levees everywhere.

The cause of this unusual occurrence has been explained in a report by the late Dr. C. W. Lelij,[48] submitted when he was District Engineer for The Netherlands Government. Dr. Lelij's investigation showed that deforestation and newly cultivated lands had had no important influence on this run-off, because the average run-off of the floods had been decreasing in the previous fifty years.

This remarkable flood was caused by the following conditions: From November 24 to December 20, 1925, much snow had fallen in the mountains of the Meuse water-shed. Immediately thereafter a heavy thaw had set in, and simultaneously a heavy rain soaked the entire shed. Remarkably enough, the peak of the storm moved with the peak of the flood from the south to the north.

In other words the determining factor in this flood was the weather; and as no one has yet learned how to control the weather, the Dutch engineers (who have accumulated an enormous mass of run-off data) are as much "in the dark" concerning the possible maximum run-off as their American colleagues. The authors of the Symposium suggest the gathering of more data, but the writer believes that the value of data should not be over-emphasized, because some of the floods of the future may be much worse than data will ever reveal.

This is not the only important consideration stressed in Dr. Lelij's report. It calls attention to the fact that the bottom of the winter section of the Meuse River was rising at a rate of 2 mm (0.08 in.) per yr. In the past 750 yr, this bottom has risen 5 ft.

At some other places conditions were worse. At the end of the Nineteenth Century many railroad bridges had been built. To compensate for the piers the area of the winter section had been increased by excavat-

[48] "Rapport betreffende de Verbetering van de Maas voor groote Afvoeren", door Dr. C. W. Lelij, Hoofd Ingenieur van den Rijkswaterstaat, s' Gravenhage, *Algemeene Landsdrukkerij*, 1926.

ing the channel. In 1926, when Dr. Lelij's report was published those places had been entirely filled in again. This condition, undoubtedly, had contributed its share to the higher floods.

American engineers probably never will know the possible maximum run-off of the Ohio or Mississippi Rivers. In the centuries to come the lands adjacent to the river, which are flooded regularly, will become higher also; and the floods, subsequently, will grow more severe. Undoubtedly, there will be flood disasters as long as people keep making homes along rivers. As a perfect protection system seems an impossibility, it may be advisable to consider first a more adequate protection of human lives.

It may be worth while to spend some money in creating an efficient emergency organization that could be mobilized on short notice. Places of refuge, where the flood victims could find temporary shelter, may fit in to this plan, and although from a technical and economic standpoint the results may not seem great, the most important part of the job would have been done at a relatively small cost.

H. K. Barrows,[49] M. Am. Soc. C. E. (by letter).[49a]—A clear picture of flood conditions in New England, a very full discussion of the interesting features of the flood of March, 1936, and much information about earlier great floods are contained in the comprehensive and excellent paper by Mr. Uhl.

TABLE 10.—New England Flood Data

Stream (1)	Drainage area, in square miles (2)	Maximum discharge, in cubic feet per square mile (3)	Approximate total flood discharge, in inches (4)
(a) Flood of November, 1927			
Ellis River, near Jackson, N. H.	28	528	17.0
Jail Branch, East Barre, Vt.	38	303	9.7
Baker River, near Wentworth, N. H.	52	288	9.2
White River, near West Hartford, Vt.	695	202	6.5
Winooski River, near Essex Junction, Vt.	1 000	111	3.6
(b) Flood of March, 1936			
North Branch of the Hoosic River, near North Adams, Mass.	39	99	17.6
Middle Branch of the Westfield River, near Goss Heights	53	160	....
East Branch of the Pemigewasset River, near Lincoln, N. H.	104	164	20.2
Deerfield River, near Charlemont, Mass.	180*	180	....
Connecticut River, near Montague, Mass.	7 840	28.9	8.8
Merrimac River, near Lowell, Mass.	4 424	39.1	9.9

* Omitting the controlled area of 182 sq miles up stream from Harriman Reservoir.

The flood of November, 1927, centering principally in Vermont, Northern New Hampshire, and Massachusetts, was due to excessive rainfall, reaching a maximum of probably 12 in. in the Green Mountains and resulting in especially high run-off from the smaller streams. Some note-

[49] Prof., Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.

[49a] Received by the Secretary May 19, 1937.

worthy data are given in Table 10(a). The flood hydrograph[50] in these cases was approximately triangular in form, beginning at about 2% of the peak flow, rising uniformly to 100% at the twelfth hour, and then decreasing uniformly to 20% of the peak flow at the thirty-sixth hour. This would correspond to total flood run-offs, as shown in Column (4) Table 10(a), ranging from about 17 in. for the smaller drainage area to 6.5 in. for White River (695 sq. miles).

As will be noted from Fig. 3, the unit flow of White River is materially in excess of the enveloping curve presented by Mr. Uhl, indicating the unusual magnitude of this peak flow for such an area. This is probably due, in part at least, to the shape of the drainage area of this stream, which has three approximately parallel tributaries tending to concentrate flood flows in the lower river.

The flood of March, 1936, was due in part to heavy precipitation and in part to melting snow, and it covered a much larger area than that of 1927, resulting in what may be called a "main river" flood of unusual magnitude, but with tributary streams at lesser peak flows than in 1927. Table 10(b) shows a few of the peak flows and total flood run-offs.

The Lower Connecticut River flows exceeded those of 1927 by about 33% at Montague City, Mass., and 42% at Thompsonville, Conn. On the Merrimac River, at Lowell, Mass., the peak flow of March, 1936, exceeded by 67% that of the previous highest (in April, 1852).

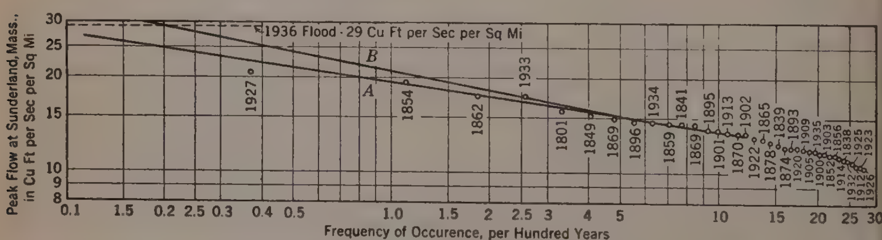

FIG. 20.—FLOOD FREQUENCY CURVE, CONNECTICUT RIVER, AT SUNDERLAND, MASS., 1801–1935.

The writer has made a study of flood frequency on the Lower Connecticut River, a graph of which is shown in Fig. 20. Records of yearly flood flows are available for the period, 1880 to 1936, at three Massachusetts stations, Holyoke, Sunderland, or Montague City. A record of flood stages at Hartford, Conn., extends back to about 1801 and by a plot of flood flow, in cubic feet per second per square mile, at one of the foregoing three stations to flood stage at Hartford, it was possible to relate the latter to the flood flow at these stations (about 8 000 sq miles of drainage area). In this manner all peak flood flows of importance were determined approximately for the period 1801-1936, or 136 yr, upon which the plot of Fig. 20 was based. Two interpretative extension curves are shown on Fig. 20. Curve A gives weight to the five or six upper points whereas Curve B is a more conservative interpretation, giving little weight to the 1927 point and to the other relatively high points.

[50] Rept. of the Cons. Engr. to the Advisory Committee of Engrs. on Flood Control, Vermont, 1928.

This shows the flood of 1936 (not plotted as a point) to have been the actual greatest based upon observations for about 150 yr of record, and by inference from the plot it would have a frequency of about once in 500 yr. The 1927 flood shows a frequency of about once in 90 yr.

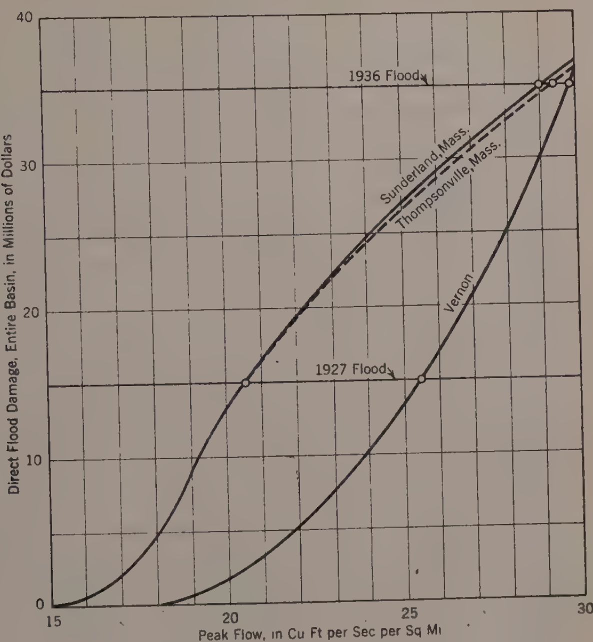

FIG. 21.

On Fig. 21 are plotted peak flood flows in the Lower Connecticut River, at Sunderland and Thompsonville, against direct flood damages; a curve is also shown using Vernon, Conn., as an index station. In each case these curves have three points for their definition; (1) Damages in the flood of 1936; (2) damages in the flood of 1927; and (3) the stage or point of discharge at which flood damages are negligible. This last point has been fixed by personal experience and judgment.

Combining the results of the upper extension curve in Fig. 20 and the Sunderland curve in Fig. 21, Table 11 was prepared. The various steps in the computations are obvious and require no further explanation. As will be noted, the lesser half of the flood stages contributes mostly to the average yearly flood damages because, although flood damages are not relatively high, their greater frequency of occurrence effects a relatively larger yearly increment of damage. A flood of the order of that of March, 1936, with about 29 cu ft per sec per sq mile, has a high damage ($35 000 000), but its frequency is so rare that the yearly increment of damage from such a flood is relatively small.

Roundly, the average yearly flood damages on the Connecticut River, as determined by this study, are $400 000. This is direct damage only

and allowance for indirect losses of all kinds would double this figure, or possibly raise it to $1 000 000 yearly.

Flood protection on the Connecticut River including a system of reservoirs as well as dike or levee systems in certain of the lower cities, such as Hartford, Springfield, etc., will probably cost about $50 000 000. This would mean a yearly carrying charge of $2 500 000 to $3 000 000 for such

TABLE 11.—DETERMINATION OF YEARLY DIRECT FLOOD DAMAGES, CONNECTICUT RIVER BASIN

Peak flood discharge, in cubic feet per second per square mile (8 000 sq. miles	Flood frequency years, (from Fig. 20)	Direct damage, in dollars	Increment of damage, in dollars	Average frequency, in years	Yearly increment of damage, in dollars	Average yearly flood damage, in dollars
15	19	0				0
			700 000	23	30 000	
16	27	700 000				30 000
			1 500 000	31.5	48 000	
17	36	2 200 000				78 000
			2 500 000	41.5	60 000	
18	47	4 700 000				138 000
			4 300 000	55	78 000	
19	63	9 000 000				216 000
			4 000 000	72	56 000	
20	81	13 000 000				272 000
			6 600 000	105	63 000	
22	130	19 600 000				335 000
			5 400 000	165	33 000	
24	200	25 000 000				368 000
			4 200 000	250	17 000	
26	300	29 200 000				385 000
			3 800 000	365	10 000	
28	430	33 000 000				395 000
			4 000 000	510	8 000	
30	590	37 000 000				403 000
			3 000 000	695	4 000	
32	800	40 000 000				407 000

works. It is obvious, therefore, as Mr. Uhl states, that a complete plan of flood relief on the Connecticut River is not economically justifiable based upon flood benefits alone. Other benefits from the use of reservoirs besides those of flood relief, principally from the use of water for power purposes, but also from the point of view of recreational and sanitation benefits, must bear a substantial portion of the cost of a reservoir system to justify its construction.

E. D. HENDRICKS,[51] M. AM. SOC. C. E. (by letter).[51a]—New York State has a varied topography. Its surface drains into the Mississippi, through Chesapeake Bay, Delaware Bay, and New York Bay, into the Atlantic Ocean, Lake Erie, the Niagara River, Lake Ontario, and the St. Lawrence River. It contains the lowest divide into the great central basin between Canada and the Gulf of Mexico. The writer has been familiar with the Oswego and Mohawk drainage areas for many years, both from personal observation and from the records that have been kept since 1898, the year

[51] Senior Claims Engr., State Dept. of Public Works, Albany, N. Y.

[51a] Received by the Secretary May 22, 1937.

when the U. S. Deep Waterways survey was started across New York State from the Hudson River to Lake Ontario. This survey was followed by the New York State Barge Canal surveys and the record has been continued with a greater or less continuity until the present time.

For the ten years (1927–1937) records in the Oswego River drainage area have been kept by Foster B. Crocker, M. Am. Soc. C. E. The control of the principal tributaries of the Oswego River, namely, the Seneca, Clyde, and Oneida Rivers, is vested in the State Department of Public Works as these streams are canalized and are part of the New York State Barge Canal System.

As a result of the intense rainfall of July, 1935, and of the much greater volume of run-off that occurred in March, 1936, this drainage area was subjected to the two major floods generally described by Messrs. Harrington and Johnson.

Flood control that has been accomplished in New York State prior to the great floods of 1935 and 1936, except in isolated cases, has been incidental to other purposes. The isolated instances with which the writer is familiar are the Hornell and Canisteo flood abatement projects on the Canisteo River, which were completed before 1927 and which, in the case of the 1935 flood, proved inadequate. The Sacandaga Reservoir was as much a water-storage project for power interests as a flood-control project and a large part of its cost is being paid for by power corporations. In the writer's opinion the deepening of the Hudson River down stream from Albany, making that city a seaport, had as much effect in reducing the flood level at Albany as the Sacandaga Reservoir.

The 1936 flood was the greatest of which there is a record on the Mohawk River in respect to the rate of flow. However, it did a relatively small amount of damage compared to other floods, such as those of October, 1903, March, 1904, March, 1910, March, 1913, and March, 1914.

The floods of 1904, 1910, and 1914 were accompanied by ice gorges. The flood of 1913 on the Mohawk had climatic conditions similar to that of 1936—a period of warm weather early in March followed by high water, followed in turn by a rain in the latter part of the month on saturated or frozen soil. The physical conditions, however, were greatly changed in 1936; that is, the canalization of the Mohawk River had been completed, and a number of additional storage reservoirs were in operation on the tributaries, the level of which reservoirs had been drawn down early in March. The result was that although a greater quantity of water was retained in reservoirs on the head-waters, a smaller quantity was retained on the flood-plains of the river, resulting in an increased discharge at the mouth with much lower elevations up stream; for example, the estimated discharges, in cubic feet per second, were: 1913, 110 000, and 1936, 130 000.

Neither the reservoirs on the Mohawk nor the canalization were for the purpose of flood control. However, nearly all flood damage has been eliminated between Schenectady and Utica, a distance of 80 miles, where prior to the canalization the ground floors of property located within the flood-plain were flooded every few years.

Although the primary purpose of the control of the Seneca and Oneida Rivers was to render them navigable with a minimum depth of 12 ft, an indirect benefit has been to reduce flood elevations and to reclaim large tracts of land, particularly in the Montezuma marshes through which the Seneca River flows. The property owners along these streams, however, have had a different opinion; and as a result have filed claims against the State alleging that as a result of the canalization of the aforementioned streams and their control by the State, flood conditions have been aggravated with increased damage to them. Test cases have been tried in the New York State Court of Claims for the years 1935 and 1936, and in the preparation of the State's defense a great mass of data concerning previous floods has been collected, together with an intensive study of climatic conditions, rate of inflow, progress of the flood, yield of the tributary streams, etc.

The drainage area of the Oswego River is shown on Fig. 11. It consists of 5 120 sq miles at Oswego. A daily discharge record is kept by the U. S. Geological Survey at that place. The State maintains a discharge station at Fulton, 11 miles up stream (area 5 017 sq miles). At Three Rivers, 23 miles up stream from Oswego, the Oneida (area, 1 493 sq miles) and Seneca (area, 3 445 sq miles) Rivers unite to form the Oswego.

The State maintains a gaging station at the Caudenoy Dam, 12 miles up the canalized Oneida River, where the drainage area is 1 377 sq miles. This dam maintains the water surface in Oneida Lake, 4 miles farther up stream, and is provided with a flood-gate which, with a guard-gate above Lock 23, permits the lowering of the water surface in the lake during the winter months and the control of the outflow from the lake.

Seneca River has its first dam at Baldwinsville, 12 miles up stream from Three Rivers. This dam is supplied with a 30-ft flood-gate. Nine miles up stream from Baldwinsville, the State has a gaging station consisting of two recording gages situated at each end of a 4 300-ft rock-cut channel, known as State Ditch. The discharge from the drainage area of 3 073 sq miles is computed by the slope-area method. Thirty miles up stream is the junction with the Cayuga-Seneca Canal. The main canal proceeds westerly up the Clyde River, and 6 miles farther up stream the State maintains a gaging station at Clyde, which has a drainage area of 880 sq miles. Thirty miles farther up the Clyde, at Lock 30 at Macedon and the westerly limit of the water-shed, the water brought into the drainage area from Lake Erie and Genesee River is measured through calibrated sluice-gates. This quantity averages about 135 cu ft per sec during the navigation season, April to November, inclusive. It varies between 100 cu ft per sec and 170 cu ft per sec, depending on the number of lockages at Pittsford, 17 miles west of Macedon.

Returning to the main channel of the Seneca River: The Cayuga-Seneca Canal proceeds up the Seneca River southerly a distance of 4 miles to the outlet of Cayuga Lake, the surface of which is controlled by six Tainter gates. The outflow through these gates has been calibrated and a daily record is kept. The drainage area above this station is 1 572 sq miles.

The canal turns westerly from Cayuga Lake, following the course of the Seneca River to Waterloo, a distance of 8 miles, where the control works regulating the surface of Seneca Lake are maintained and where a daily record of the outflow from the lake is kept. The drainage area above Waterloo is 749 sq. miles.

In addition to these gaging stations maintained by the State the U. S. Geological Survey, under the supervision of Mr. Harrington, as District Engineer, maintains three stations on the Oswego River water-shed, all of which are near the head-waters, in addition to the one at Oswego at the outlet of the drainage area (see Table 12).

TABLE 12.—GAGING STATIONS MAINTAINED BY THE UNITED STATES GEOLOGICAL SURVEY

Station		Drainage Area	
At (1)	On (2)	Name (3)	Area, in square miles (4)
Taberg, N. Y.	The East Branch of Fish Creek	Oneida River	189
Auburn, N. Y.	The outlet of Owasco Lake	Seneca River	208
Ithaca, N. Y.	Fall Creek	Cayuga Lake	124

Storm of July, 1935.—The writer is convinced, from personal observations and Court testimony, that the storm of July 7 and 8, 1935, was more extensive than is shown by Fig. 12[52], which is based on reports from rainfall stations of the U. S. Weather Bureau. In the writer's opinion, the precipitation was concentrated at or near the higher elevations where there are no Weather Bureau stations, and passed over the low divides or valleys with a heavy precipitation to concentrate with double the precipitation on the hills beyond; that is, extending from the New York-Vermont line east of Mt. McGregor, the area of high precipitation extended westward along the southern escarpment of the Adirondacks to Stewart's Landing, 20 miles northwest of Gloversville. South of the Mohawk the storm concentrated on the peaks of the Cherry Valley Hills, on the Mohawk-Susquehanna divide, and Oak Hill, which is the divide between the Mohawk and the Schoharie Rivers. Thus, streams entering the Mohawk from the south and west of the Schoharie, show peak flows of more than 400 cu ft per sec per sq mile which resulted from a rainfall of not more than 6 hr duration.

However, the water-shed of the Oswego River is well supplied with rainfall stations in addition to the U. S. Weather Bureau stations, most of which are maintained by the New York State Department of Public Works and by the Oswego River Watershed Corporation. In addition to precipitation records, snow survey records are kept.

[52] *Water Supply Paper 773-E*, U. S. Geological Survey.

The run-off that resulted from the rainfall of July 7 and 8, 1935, does not appear to have been equalled for summer flows on the Seneca River in the period in which records have been kept—that is, for 37 yr. Fortunately, no unusual run-off occurred on the water-sheds of the Oneida and the Clyde Rivers. The precipitation on the Oneida water-shed was less than one-half that on the Upper Seneca and the precipitation on the Clyde water-shed was no more than that from an ordinary summer storm. In fact, at least one-third of the area draining into Cayuga and Seneca Lakes did not experience a really destructive rain.

The remainder of the Cayuga and Seneca Lake areas had a rainfall that resulted in an inflow at the rate of 80 000 cu ft per sec for 8 hr on Seneca Lake, from a drainage area of 749 sq miles. Cayuga Lake had a rate of inflow of 100 000 cu ft per sec for 8 hr from a drainage area of 823 sq miles. Due to the storage utilized in both lakes, the peak outflow from Seneca Lake was 4 400 cu ft per sec and that from Cayuga Lake was 16 000 cu ft per sec. The elevation reached in Cayuga Lake was 386.0 and in Seneca Lake, 449.34. The peak discharge at Baldwinsville was 12 000 cu ft per sec. The greatest damage on the navigable waters resulted to truck gardeners cultivating thousands of acres of muck land in the former Montezuma marshes which had been rendered tillable by the canalization of the Seneca River between Cayuga Lake and Jack's Reef.

Greater damage occurred on the streams flowing into the lakes and has been described elsewhere by Mr. Johnson.[52] Over these streams the New York State Department of Public Works exercises no control.

The discharge at Jack's Reef or at Baldwinsville had been exceeded a number of times in the late fall or spring months, both before and since 1918, the date of completion of the Barge Canal. There is no record of a greater summer run-off. The summer elevation in Cayuga Lake had been exceeded several times prior to canalization, but with an outflow of less than 4 000 cu ft per sec. In August, 1922, the lake reached an elevation of 386.1; and although that was a summer of heavy precipitation, the high water was as much the result of the method of operation as of the climatic conditions. Any elevation in excess of 385.0 results in damage to property surrounding the lake and particularly to property in Ithaca at the head of the lake.

The term, "method of operation," is used advisedly. In 1922, the control of the navigable waters of the State were under a bureau separate from the State Engineer Department. Since 1927 both the Division of Canals and Division of Engineering have been included in the Department of Public Works under Frederick Stuart Greene, M. Am. Soc. C. E., as Superintendent. Not only has there been an increase of accurate records kept by Mr. Crocker; but the long-continued sequence of the records has resulted in a sound engineering basis on which a proper method of operation has been predicated.

Flood of March, 1936; Oswego River Water-Shed.—The climatic conditions causing the flood of March, 1936, seem to have been similar in many respects to those that occurred in March, 1865, as shown by the

report[63] of J. P. Goodsell, Division Engineer of the Middle Division, New York State Canals, including the Erie, Oswego, Cayuga, and Seneca Canals and the Oneida River improvement. Mr. Goodsell reported that the measured flow at Jack's Reef was 19 435 cu ft per sec on March 22, 1865, and that this was the greatest flow ever experienced in the Seneca River. The greatest flow preceding this was in March, 1857, when the discharge at Jack's Reef was 15 269 cu ft per sec.

There was another great flood on April 10 and 11, 1873, in which Cayuga Lake reached the same elevation as was reached in 1865, but no discharge measurements were reported. The peak discharge at Oswego for the 1865 flood has been estimated as 42 000 cu ft per sec. The peak discharge at Phoenix has been stated (Empire Street Railway Company *vs* the State of New York) as 30 000 to 35 000 cu ft per sec. The Town of Phoenix is below the junction of the Oneida and Seneca Rivers and has a drainage area of 4 940 sq miles, about 500 sq miles greater than at the gaging stations on the Seneca and Oneida Rivers.

The peak discharge at Phoenix is usually about equal to that at Oswego. However, in the 1865 flood, 170 ft of a dam situated on the river 4 miles up stream from Oswego, was washed out which could well account for the difference of 7 000 cu ft per sec. The highest flood elevation recorded in Oneida Lake prior to the construction of the Barge Canal was 376.0, with an estimated peak outflow of 12 500 cu ft per sec. This peak probably occurred in the flood of 1865 and checks very well with the peak at Phoenix and the discharge from the Seneca River, at Jack's Reef. The effect of the control, incidental though it was, is disclosed in Table 13.

TABLE 13.—COMPARISON OF ELEVATIONS AND DISCHARGES, FLOODS OF 1865 AND 1936

Date	Cayuga Lake elevation, in feet above sea level	Seneca River discharge, in cubic feet per second	Oneida Lake elevation, in feet above sea level	Oneida River discharge, in cubic feet per second	Oswego River discharge, in cubic feet per second
1865	389.3	19 435	376.0	12 500	35 000
1936	386.7	22 500	375.0	11 700	37 000

The low navigable stage of Cayuga Lake is 381.5 and that of Oneida Lake is 369.9. During the winter of 1936 the level of Cayuga Lake was drawn down to 380.3 and that of Oneida Lake to 368.5, in preparation for the expected high water that did occur. Damage did result from high water in the Oswego River drainage area, and claims have been filed against the State, but it is anticipated that they will be dismissed.

Messrs. Harrington and Johnson call attention to the importance of basic data depending on long-continued records. This discussion is primarily for the purpose of illustrating the value of numerous long-continued records and their intelligent use by a skillful engineer. The reduction of a flood elevation by 1 ft or 2.5 ft may appear negligible in sections where

[63] Annual Rept., State Engr. and Surv., January 23, 1866.

flood rises greatly exceed those in New York State; but this lowering reduced by hundreds of thousands of dollars the damage suffered. The streams discussed traverse regions of long settled and thickly populated communities and the last foot of rise usually causes the maximum damage.

To spend great sums on construction without the help of basic data is to insure not only waste, but greater injury than the benefits realized.

The reference by Messrs. Harrington and Johnson to the importance of considering the engineering economics of each problem is often lost sight of by some enthusiastic advocates of only one phase of the problem of flood control. Thus, there are the reforestry proponents claiming deforestation to be the primary cause of floods. This cannot be true of the discharge areas discussed herein. There is a greater forest cover now than there was in 1910 (from the writer's personal observation). The 1935 Graphic Compendium of the New York State Planning Board indicates that lumbering reached its peak in New York State in 1869, and that there has been a great reduction in improved farm areas since 1875.

Man-made changes, apparently, have relatively little or no effect in these two instances compared to climatic disturbances over which Man has no control and no definite knowledge of the cause.

EDWARD W. BUSH,[54] M. AM. SOC. C. E. (by letter).[54a]—Combined flood control and hydro-electric developments are engineering subjects that will keenly interest the profession for some time to come. The papers of this Symposium are very valuable contributions on the subject. Many schemes are under consideration for building groups of reservoirs for the dual purpose of flood control and electric development, but practically no information has been presented by their proponents as to the procedures that necessarily must be followed by those operating such dual systems so that the flood-control protection will receive proper safeguarding and, at the same time, those relying on the generated electricity will be satisfied with the services obtained.

In the few cases where procedures have been briefly mentioned the proponents have stated, what to them seems obvious, that the reservoirs will be drawn down just before the freshet seasons arrive and after they are filled by the freshets the impounded waters will later be used to generate electricity. This is quite a simple solution if the rain storms and the weather could be foretold accurately, or could be regulated, but seemingly quite fantastic when the actual facts are recognized. Those wanting protection against flood damages need the reservoirs empty except when in use for holding back flood waters, and when so filled they should be drawn down as soon as the outflow would not cause damage so they are set ready for the next freshet when it comes. Those using the electricity and those selling it to obtain funds for retiring the cost in whole

[54] Engr., Aetna Casualty and Surety Co., Hartford, Conn.

[54a] Received by the Secretary May 25, 1937.

or in part of the reservoirs will desire plenty of water in the reservoirs at all times. It would seem that these conflicting interests cannot be reconciled.

The Connecticut River well illustrates the uncertainty of the time of high freshets. During the past 300 yr, the highest freshet came in the middle of March, 1936, the next highest freshet came in the first week of May, 1854, and the next highest came in the first week of November, 1927, at a time when very low water has often occurred in past years. Fairly high freshets have occurred at one time or another in about every month of the year.

It is highly improbable that any official or board of officials in charge of the operation of a system of reservoirs constructed for a dual purpose could at any critical time determine in advance whether or not to empty the system, or to keep it filled or partly filled if it happened to be in that condition. At such a time the power users and sellers could readily estimate the dollar value to them of any stored water and would strongly urge their claims for such, but those relying on the reservoirs for the protection against flood damages would probably not be organized and, therefore, not able to present forcefully their advantage from having empty reservoirs when what might be termed, "border line" situations arose; and any freshet, when it reached its crest, might only be one of moderate size.

After several of such false alarms it is expecting too much of human nature to believe that those in control would operate a system so that the flood prevention side would always receive the full protection needed. This seems to be a fundamental consideration that should not be lost sight of. In most of the cases when a critical situation is forming there will not be time to convene a board of consulting experts to determine whether or not the reservoir gates should be opened or closed, and even if convened the experts would probably not know the correct answers.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE PASSAGE OF TURBID WATER THROUGH LAKE MEAD

Discussion

By Messrs. O. A. Faris, Paul A. Jones, Carl S. Scofield, and Ivan E. Houk

O. A. Faris,[14] M. Am. Soc. C. E. (by letter).[14a]—Turbid water showing in the discharge from large reservoirs, in the opinion of the writer, is the result of disturbance of silt which was virtually in place; that is, it was no longer in suspension, but had settled to the reservoir bottom soon after entering the slack-water and formed a "flocculent" or "honeycomb" structure of a consistency, ranging from thick cream to heavy molasses. It flows readily on the slope of the reservoir bottom, by virtue of a greater specific gravity than water, and, finally, reaches the dam.

Water is discharged from Lake Kemp, a reservoir on Wichita River, near Wichita Falls, Tex., through a battery of concrete pipes with inverts at or below the elevation of the river bed. The control gates are about midway the length of the conduits, allowing the up-stream half of the pipes to be exposed to pressure at all times. There is evidence that the liquid mud flows from the place of deposition in the old river channel, and down the channel within the reservoir until blocked by the closed outlet gates. On opening the gates, the first rush of water carries a heavy load of silt, but after discharging all mud within easy reach of the conduits, the discharge becomes clear. Water from this reservoir flows down the river channel several miles where it is diverted into the main canal of the irrigation system.[15]

The general arrangement which provides for the outlet at or below the elevation of the bottom of the stream channel and the diversion of water to canals at some distance down stream from the storage dam is favorable for the discharge of deposited silt of the aforementioned character. In most of the Texas reservoirs the highest elevations of the bottom, in a

Note.—The paper by Nathan G. Grover, M. Am. Soc. C. E., and Charles L. Howard, Esq., was published in April, 1937, *Proceedings*. This discussion is presented in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[14] Engr.-Appraiser, Federal Land Bank, Houston, Tex.

[14a] Received by the Secretary, April 29, 1937.

[15] "The Silt Load of Texas Streams", by Orville A. Faris, M. Am. Soc. C. E., *Technical Bulletin 382*, U. S. Dept. of Agriculture, 1933, pp. 37–39.

cross-section, are on the immediate banks of the old stream channel; therefore, none of the silt that is deposited on the wide flat areas between the elevated ridge near the channel and the toe of the slope from the upland, can flow into the old channel and ultimately reach the outlet structure. It is believed to be advantageous to clear reservoir sites of brush, trees, and stumps, and to construct channels with a view to conveying the liquid mud, resulting from the deposition of suspended silt, into the old stream channel and thence through the outlet gates.

As further evidence that deposited silt, in the form of liquid mud, flows down the slopes on reservoir bottoms, attention is called to conditions in Medina Reservoir, near San Antonio, Tex. In September, 1930, this reservoir was empty. A careful inspection revealed that the hummocks on the large flat areas in the reservoir bottom were void of silt deposit although they had been submerged by water to depths ranging from 40 to 100 ft. for long periods of time. On the other hand, all depressed areas on the flats contained silt with maximum depths in the lowest places and tapering to zero depth up the slopes.

The dry material per cubic foot of deposit of "flocculent" and "honeycomb" structure, taken from the bottom of Lake Worth, near Fort Worth, Tex., and composed largely of colloidal sizes, contained 18.7 lb. of oven-dry material per cu ft of deposit. On the basis of 2.65 for the specific gravity of the dry material, water occupied 88.59% of the space and the absolute voids amounted to 88.66 per cent. A sample from the bottom of Lake Kemp, consisting of sizes crresponding to clay particles, contained 37.05 lb of oven-dry material per cu ft of deposit. On the basis of 2.658 for the specific gravity of the dry material, the space occupied by water was 76.76% and the absolute voids amounted to 77.63 per cent.

The water capacity, by volume of undisturbed soils, below the water-table, ranges from 31.3% for very fine sand to 49% for marly loam. Comparing the water capacity of marly loam with that of the sample silt deposit which contained about 89% water, by volume, it is seen that the latter has a greater capacity by 40%, than the former. This greater capacity is due to the difference in structure of the mass of soft silt deposited, from suspension, under water and remaining submerged.

The excess water held in the silt deposit by virtue of structure is liberated so slowly upon exposure of the deposit to the sun and atmosphere that it is practically all dissipated by evaporation and is not available as storage water.

The unconsolidated character of material deposited from suspension, under water and remaining below the water-table, will continue indefinitely. This is evident from the soft character of low-lying material in peat marshes. This soft material was deposited ages ago and still retains the "honeycomb" structure on account of its position below the water-table.

The release of the excess water can be accomplished only at the surface of the deposit, therefore, in large reservoirs, consolidation of such deposits can not occur until the stored water is drawn down exposing the soft material to the action of the sun and atmosphere.

PAUL A. JONES,[16] M. AM. SOC. C. E. (by letter).[16a]—The studies regarding transportation of silt through large reservoirs will, no doubt, be of great interest to engineers, and it is hoped that the work begun by the authors may be continued. However, it is the writer's opinion that studies made concerning the turbidity of water passing Boulder Dam in 1935, or even until the lake has been completely filled, should not be given much weight in connection with transportation of silt in the future.

There were certain topographic conditions that, no doubt, greatly affected the turbidity of the flow at Boulder Dam, especially during 1935. The canyon above the dam, in general, is in the shape of an arc, with the inside of the arc on the side of diversion through Tunnel No. 1, on the Nevada side. It is reasonable to assume that currents of water act similarly to currents of air, and will move toward the inside of the curve instead of following the original channel. Cold or heavy air will follow the inside of a curve, or the shortest distance, as may be observed in a tunnel constructed on a curve. It would seem fair to expect the greatest number of contraction cracks in the concrete lining of a tunnel on the outside of the curve, but the reverse is true, as the cold air in a draft follows the inside of the curve.

At Elevation 720 at Boulder Dam, which is about 30 ft above the top or back of Tunnel No. 1, the contractor constructed a railroad grade down the canyon on the Nevada side, principally from tunnel muck which was of a reddish brown color. Below this grade, on the same side of the canyon, at about Elevation 650, was a comparatively large sand-bar. Just above the entrance to the canyon, and also on the inside of the arc, blow-sand drifts extended in general to approximately Elevation 800. About three miles above the entrance to the canyon, there are at present (1937), or there will be, two rock islands which were connected by a blow-sand ridge, and the original current flowed east of the island farthest east.

If a check is made of the dates of turbidity of flow through the dam and lake-level elevations, it will be found that the first turbidity occurred soon after the sand-bar was well covered; the second turbidity occurred soon after the railroad grade was well covered, and was much redder in color than the usual Colorado River water; and that, a turbidity occurred soon after the ridge between the two rock islands was well covered, and at about the time that a change in current from the east side of the small island to the west side, would be expected. At the level of diversion through the towers, the canyon has little blow-sand or silt, and it is not likely that turbidity below the dam will again occur to the extent observed in 1935.

This discussion is not written to contradict any of the statements made in the paper, but as a possible explanation for the amount of turbidity in 1935 when the reservoir was being filled for the first time, and the current was being changed gradually to new and unstable ground, due to the fact that the canyon immediately above the dam takes the form

[16] Engr., U. S. Bureau of Reclamation, Yuma, Ariz.

[16a] Received by the Secretary May 4, 1937.

of an arc, with the inside of the arc on the side of diversion, or the Nevada side.

It is well understood that river water carries large quantities of collodial silt in suspension, and studies relative to this material, no doubt, would be of great benefit to the Engineering Profession. For example, there is a shoal in San Francisco Bay, just west of Mare Island, which is composed entirely of collodial silt deposited where the fresh water of the rivers come in contact with a certain percentage of saline water from the ocean. Since the effect of tides is noted as far up the Sacramento River as the City of Sacramento, and the collodial silt is not deposited until it reaches a point nearly 100 hundred miles distant, and is there precipitated by salt, it is reasonable to expect that a large quantity of fine silt will be transported through Lake Mead.

Carl S. Scofield,[17] Esq. (by letter).[17a]—Hydraulic engineers are fortunate in having made available in precise form the information concerning a phenomenon that has been reported hitherto only in general terms. Observations that different waters may move together in the same stream channel with little blending, and that water may flow into, or through, a reservoir with little or no mixing, have been reported before but only circumstantially. The findings reported by Messrs. Grover and Howard, therefore, challenge attention and invite further inquiry. The reasons underlying the phenomenon described remain obscure.

The assumption that two different solutions brought together in the same container would automatically blend into one homogeneous mixture runs counter to daily experience. One naturally uses a spoon to stir a cup of coffee after the cream and sugar are added. In the case of reservoirs such mixing as occurs may be due in part to the effects of currents set in motion by inflowing water, by wind action on the surface, or in part to "inversions" caused by temperature changes that affect the specific gravity of the water. For example, if the surface water of a reservoir becomes colder than the deeper water, its specific gravity becomes higher and inversion occurs.

When turbid stream water enters a reservoir in the summer, if its temperature is higher and its salinity is lower than that of reservoir water, the normal behavior is for the silt to settle to the bottom and for the clear water to spread over the reservoir surface. On the other hand, if the inflowing turbid water is much more saline than the reservoir water the higher specific gravity due to salinity tends to offset the lower specific gravity due to temperature so that the water tends to stay with the silt and to move into the reservoir along the bottom.

Furthermore, the behavior of silt in relation to its transporting water appears to be influenced by the relative conditions of salinity. If the concentration of dissolved salts in the inflowing water is less than that of

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[17] Agriculturist in Chg., Div. of Western Irrig. Agriculture, Bureau of Plant Industry, U. S. Dept. of Agriculture, Washington, D. C.

[17a] Received by the Secretary May 12, 1937.

the reservoir water, flocculation and deposition of the silt appear to be hastened. If, conversely, the concentration of dissolved salts in the inflowing turbid water is higher than that of the reservoir water, the tendency is to retard the flocculation and deposition of the silt. The combined effect of silt and of dissolved salts on the specific gravity of water, of course, is much greater than the effect of temperatures, at least within the ranges that ordinarily occur in streams and reservoirs.

The foregoing are largely theoretical considerations. Engineers do not, as yet, have enough facts to explain convincingly why the incoming silt is sometimes deposited at the head of a reservoir and, at other times, is carried far along the bottom or even all the way through the reservoir. One phenomenon would probably be regarded as quite as "normal" as the other if one knew more of the facts and factors involved. Both are known to have occurred at Boulder Dam and at Elephant Butte Dam.

It is to be hoped that this paper may stimulate interest in the subject and lead to further investigations.

IVAN E. HOUK,[18] M. AM. SOC. C. E. (by letter).[18a]—The authors have contributed some valuable information regarding the character and quantity of suspended material carried through Mead Lake. The paper is especially noteworthy since outflow conditions at the reservoir during the period covered by the silt observations will never again be experienced. During 1935 lake water was released through one of the diversion tunnels at about

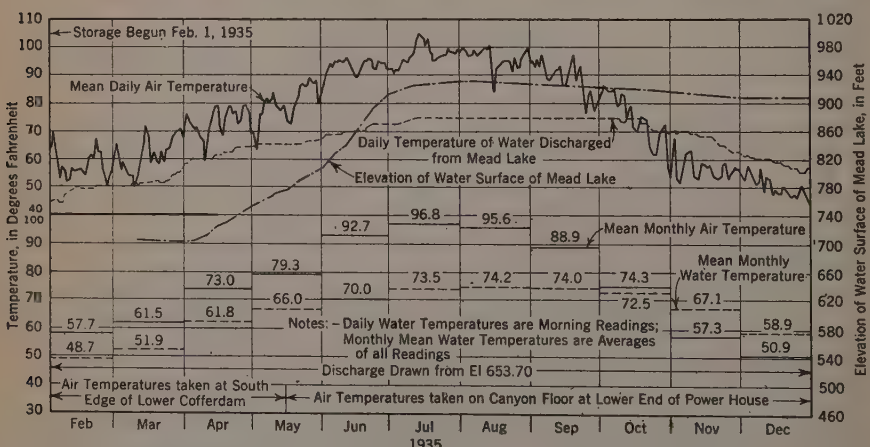

FIG. 3.—AIR TEMPERATURES AT BOULDER DAM COMPARED WITH TEMPERATURES OF WATER DISCHARGED FROM LAKE MEAD DURING PERIOD FROM FEBRUARY 1 TO DECEMBER 31, 1935.

the normal low-water surface of the Colorado River. About May 1, 1936, the tunnel flow was shut off; not only by closing the regulation gates, inside the tunnel, but also by permanently sealing the up-stream end of the tunnel with a 50 by 50-ft Stoney gate. Since that time water has been released

[18] Senior Engr., Technical Investigations, U. S. Bureau of Reclamation, Denver, Colo.

[18a] Received by the Secretary May 17, 1937.

through cylinder gates in the intake towers. These gates are located at heights of approximately 250 and 400 ft above the tunnel gates. They will be used in all future regulation of reservoir outflow for irrigation and power purposes.

Unfortunately, no measurements of river water temperature were made at the up-stream end of Mead Lake during 1935. However, temperatures of river water at the dam site have been observed since April, 1930. These measurements were being made, daily, during the period covered by the authors' silt observations. After storage began on February 1, 1935, the temperature measurements were made in the reservoir outflow. Fig. 3 shows the results of the observations; also the daily air temperatures, mean monthly air temperatures, and mean monthly water temperatures.

A comparison of the daily water temperatures during 1935 with those measured during previous years shows that the reservoir inflow temperatures must have been approximately the same as, or slightly lower than, the lake temperatures at the elevation of the tunnel gates, during the periods of silt movement through the reservoir. Consequently, the inflow densities must have been approximately the same as, or slightly greater than, the lake densities at the tunnel level; and the silty inflow undoubtedly followed the original river channel along the lake bottom to the open gates as assumed by the authors. Since the gates were closed silty inflow probably has moved along the lake bottom to the level of its density, then spread out, and gradually come to rest at that level. The fine silt particles, which might have been discharged with the outflow had the tunnel gates remained open, are now settling to the lake bed and are gradually becoming consolidated under the lake pressures.

Measurements of water temperature at different depths in Mead Lake were begun in June, 1936, and are being continued at monthly intervals. The securing of samples of the lake water at different depths, for chemical and silt analyses, is now (1937) being considered. When such data become available, it should be possible to discuss the movement of turbid water through the reservoir much more adequately. The writer believes that, some time in the future, silty flood water, having the same density as the lake water at the elevation of the cylinder gates, may enter the reservoir, flow through the lake at the level of its density, and be noticeable in the reservoir discharge.

The conditions under which turbid water may flow through a reservoir instead of along the bottom, or instead of being rapidly diffused at the up-stream end of the lake, may be discussed on the basis of the temperature distribution within the lake. Fig. 4 shows the water temperature at different elevations in Elephant Butte Reservoir on June 18, 1934. Assuming that the water had the same salinity and was free from silt at all elevations, the density curve can be calculated from the temperature curve. Such a curve will show approximately constant densities from the bottom of the reservoir up to about Elevation 4 270, then gradually decreasing densities as the temperature increases toward the surface.

Silty flood flows, entering the reservoir with a density greater than the water at the lake bed, would be expected to flow along the original river channel beneath the body of the lake. Similar flows, entering the reservoir with a density the same as the lake water below Elevation 4 270, would be expected to spread out laterally and vertically through the entire reservoir section below that elevation. However, silty floods entering the reservoir with a density equal to the lake water at some elevation above 4 270, as, for instance, at Elevation 4 300, would be expected to spread out laterally at that level and to flow through the reservoir at approximately that elevation without diffusing through appreciable vertical distances, just as the water did in the laboratory experiments described by Mr. Freeman, and cited by the authors.[3] In any case, the appearance of silt in the reservoir outflow is probably dependent upon the continuance of a relatively large silty inflow at the upper end of the reservoir until the silty flow has had time to reach the outlet gates.

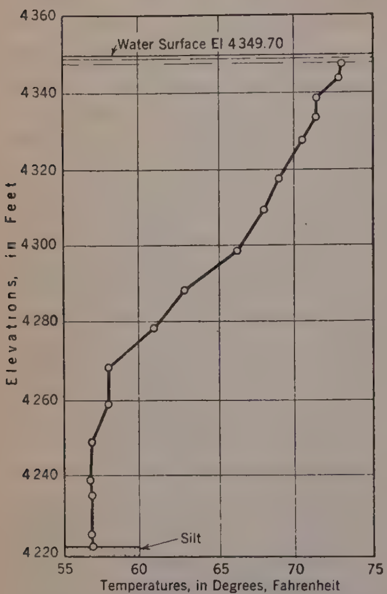

FIG. 4.—TEMPERATURE DISTRIBUTION IN ELEPHANT BUTTE RESERVOIR ON JUNE 18, 1934.

Temperature measurements in existing reservoirs show that during some periods of the year, usually during the winter months, lake densities are approximately constant through the entire vertical section. At such times flood flows, entering the reservoir with a density the same as the reservoir water, would be expected to diffuse laterally and vertically through the entire cross-section of the reservoir. Consequently, under such conditions, only exceptionally large and long-continued inflows would cause movement of the turbid water to the outlet gates.

The writer doubts whether it will ever be desirable to design irrigation reservoir outlets so as to secure a maximum outflow of turbid water. As mentioned by Mr. Fiock, in the reference cited by the authors,[6] the silt discharged at such times would be deposited in the canals, or carried through the canal systems, and deposited on the farm lands, either of which would be objectionable. However, investigation of the problem is well worth while from the scientific standpoint, even though the studies may produce no results of great economic value. The authors have rendered a service to the Engineering Profession in bringing the matter up for discussion.

[3] "Hydraulic Laboratory Practice", John R. Freeman, Editor, p. 322, A. S. M. E., 1929.

[6] *Transactions*, Am. Geophysical Union, 1934, Pt. 2, p. 472.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STRUCTURAL APPLICATION OF STEEL AND LIGHT WEIGHT ALLOYS A SYMPOSIUM

Discussion

BY MESSRS. A. V. KARPOV, LEON S. MOISSEIFF, A. V. KARPOV, R. L. TEMPLIN, J. H. A. BRAHTZ, M. J. R. MORRIS, ZAY JEFFRIES, C. F. NAGEL, JR., AND R. T. WOOD, JAMES ASTON, AND E. C. HARTMANN.

A. V. KARPOV,[172] M. AM. SOC. C. E. (by letter).[172a]—The purpose of the Symposium is exceptionally well defined in the seven terse questions given in the "Foreword" by Mr. Jones. The papers and the discussions give at least partial answers to each of these questions, although they show also that, at the present stage of engineering development, and in all probability for many years to come, no final answer can be given on many of these problems. Engineering development forges ahead, and it seems that no better goal can be accepted by the Structural Division of the Society than to keep up to date the valuable data collected in this Symposium. The Committee on Fundamentals Controlling Structural Design, appointed by the Executive Committee of the Structural Division in October, 1936, should shape its work along the lines laid down by this Symposium.

The most fundamental conclusion that can be drawn from the Symposium is the impossibility of limiting the field of advanced structural engineering to stationary structures of fundamental character. The broader viewpoint is necessary, expanding into border fields of structural design which are commonly assigned to mechanical engineering.

NOTE.—This Symposium was presented at the meeting of the Structural Division at Pittsburgh, Pa., October 14–15, 1936, and published in October, 1936, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1936, by Messrs. E. Mirabelli, R. W. Vose, Raymond H. Hobrock, William F. Clapp, J. C. Hunsaker, Horace C. Knerr, and F. T. Sisco; January, 1937, by Messrs. J. Charles Rathbun and D. M. MacAlpine, Fred L. Plummer, C. F. Goodrich, G. K. Herzog, John H. Meursinge, P. C. Lang, Jr., and W. L. Warner; February, 1937, by Messrs. Elmer K. Timby, Werner Lehman, Otis E. Hovey, and R. G. Sturm; March, 1937, by Messrs. E. Robert deLuccia, O. J. Horger, A. W. Demmler, Theodore Beizner, J. P. Growdon, Karl Arnstein, A. Christianson, Robert E. Glover, Arthur C. Ruge, Alexander Klemin, Paul E. Gisiger, Russell C. Brinker, Arshag G. Solakian, and H. D. Hussey; and April, 1937, by Ralph Freeman, M. Am. Soc. C. E.

[172] Chairman, Committee of the Structural Division, on Fundamentals Controlling Structural Design; Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[172a] Received by the Secretary June 4, 1937.

The determination of actual stresses in structural elements is leading into fields which are different from conventional design practice. New methods of attack are developing both in the theoretical and experimental handling of structural problems. Fatigue, impact, and creep test are supplementing the work of the ordinary metal testing laboratories; small and large scale models are being used extensively; indirect methods of strain and stress measurements have been added to the direct strain measuring methods that were used in the past; and the vibrator test is being used in the laboratory as well as in the field.

This very extensive theoretical and experimental work, which at present is being conducted in a large number of laboratories in the United States, seems to suffer from lack of co-ordination and insufficient exchange of information concerning the work proposed or being done. The different testing programs in the field of structural research that were mentioned in the Symposium, suggest the necessity for steps to be taken to remedy these conditions. The Society, which by its very nature is interested in promotion of engineering knowledge without paying undue attention to the commercial aspects, is the most logical organization which could undertake successfully such a co-ordination and co-operation and could bring closer together the different educational and industrial organizations working in the field of structural research. Such a co-operation would result in a properly planned and co-ordinated research program, eliminating unnecessary overlapping and wasteful repetition of work.

Part II of the Symposium brought out forcefully the fact that structural engineering is no longer a single-metal field. As in all fields of engineering, materials must be chosen, depending upon the suitability for each particular design. The requirements are becoming so exact that it does not seem probable that in the near future a universal alloy will be introduced that can take the place that open-hearth mild steel took in the past.

In line with the papers and discussions of the Symposium there seem to be at present three outstanding problems that should be solved:

First.—A more suitable definition of terms that have a clear meaning in their application to ordinary structural steel, but which lose this clearness if applied to high-grade steel alloys or to light-weight alloys. More appropriate definitions of ultimate strength, yield point strength, and related physical properties of materials should be developed. The particular and most pressing objective would be the proper definition of yield point strength and its relation to other fundamental properties.

Second.—The problem, made clear in the Symposium, concerns the necessity of introducing additional characteristics of structural materials. The time-honored coefficients such as ultimate strength, yield point, and elongation are still the basic coefficients that are necessary to form engineering judgment as to the suitability of a material; but they are obviously insufficient: Fatigue, creep, impact, corrosion, and welding properties must also be considered. To give a practical definition of these properties and to outline the extent to which they should be used in structural designs is an important field that should be properly covered.

Third.—Welding is rapidly growing in importance in structural designs. The problem of weldability of alloys is encountered more in the field of metallurgy; but the manner in which welds are made is becoming a problem in structural engineering. The researches that were and are being made have disclosed the importance of fatigue considerations in welded joints. The remarkable increase in fatigue strength that may be obtained by changes in the design of welded joints, the comparison with riveted joints, and the determination of conditions under which different kinds of joints should be considered, are fields which undoubtedly will occupy the attention of the structural designer.

Part III must be considered in light of the theoretical, Part I, and the metallurgical, Part II. "The picture" that can be gained from Part III, gradually emerging from a single material stage and developing into multiple material designs, can be taken as a guide to an understanding of the development trend in structural engineering. The self-sufficiency of the structural designer of the past is gone, probably forever. The best handbooks available cannot keep a structural designer abreast of the latest practice. A correct design may result only from the correlated work of the theoretical designer, structural laboratory engineer, and metallurgist. If the structural designer wishes to maintain a leading part in this work, which naturally is his, he must possess the insight and ability to understand the problems of all the professions involved, as was evidenced to such marked degree by the author of one of the papers in Part III.

Part IV is the logical conclusion of the entire Symposium. Light-weight designs are encountered in the most recent and most modern field. The importance of this field is not often realized. Many such designs cannot be rationalized due to lack of engineering knowledge. The stability problems, which were more or less neglected in the past, are becoming of the same importance as the stress problems. Corrosion, expected life span of a structure, fatigue and creep properties, and stress concentrations, are all problems whose importance is considerably accentuated in these designs. It is interesting to note that these problems are of particular importance in two structural fields which are as far apart as can be imagined—in aircraft and in long span bridges. The modifications of standard design formulas that were proposed during this session of the Symposium may be accepted as a clear indication of the necessity to consider the difference in properties of new alloys, as compared with structural steel. Different treatment is necessary to obtain the entire advantage of the light-weight design.

A useful field of endeavor would be to watch the trend of light-weight designs and to bring to the attention of the structural designer the most important and practical developments.

LEON S. MOISSEIFF,[173] M. AM. SOC. C. E. (by letter).[173a]—The valuable papers presented in the Symposium and the many discussions it has brought forth testify to the timeliness of the undertaking. The Society, through its

[173] Cons. Engr., New York, N. Y.

[173a] Received by the Secretary June 9, 1937.

Structural Division, has succeeded, at an opportune time in gathering and presenting to the Engineering Profession the knowledge, the information, and the experience of engineers active in the making of structural metals and in fabricating them, in the planning of structures, and in their design, erection, and utilization. The Society, moreover, has been fortunate in securing a group of papers which has proved interesting enough to provoke a many-sided and illuminating discussion by men of information and reflection.

The authors of the Symposium wisely did not attempt to offer complicated mathematical analyses and subtle discussions of the various phases of structures built of metal. Instead, they have presented to the profession the thoughts which have matured in their minds during many years of experience, in making and forming of metal and in designing and building structures. The Symposium has been successful in producing a representative cross-section of current American engineering thought. A comprehensive study of the papers reveals the present trend of structural development.

The Symposium has brought forth a symphony of many instruments. The low tones of the heavy metals and the high ones of the light metals unite in one sustained theme. They announce that to create structures for the wants and comforts of the many, such structures must be efficient and economical. They must be built of materials which possess suitable qualities to fulfill the special requirements and functions, and they must be workable, dependable, uniform, and enduring in order to justify the trust placed in them. It is the constant business of producers and fabricators to search and test the products of the mills and shops, and to establish the capacities and limitations of these products to sustain and endure the attacking forces and agents in space and time.

These capacities must be utilized further by the planners and designers to the best advantage so as to produce efficient and economical structures. It is the task of engineers, therefore, to strive to approach the fullest utilization of their materials in strength and endurance, in economy and comfort. Only then when the fullest knowledge of materials and structures will be gained will this aim have been attained. To acquire this knowledge it devolves upon engineers to study the true behavior of structures under the action of forces by analysis and strain measurements on models, by tests in laboratories, and by observations in the field.

The Symposium shows that engineers are realizing that the elementary approach in analyzing structures was well enough for the smaller dimensions where cheaper materials could be used and some of it could be wasted, but that for large structures the most suitable material is demanded and almost all of it should be utilized. Engineers realize that a deeper insight into the behavior of structures is required and that much of it can be attained by modern methods of analysis and search, and they feel that the application of such knowledge is imperative in the design of important structures. It must not be forgotten that the admission of higher unit stresses and larger strains demand more care in analysis and proportioning and that all advance in this direction is based on the mental probity of the engineers.

A. V. KARPOV,[174] M. AM. SOC. C. E. (by letter).[174a]—The purpose of the paper was to bring to the attention of the structural designer the fact that the theories on which structural designs are based by no means can be considered as perfect. These theories are based not only on approximations but in many instances on assumptions that are contrary to present knowledge. The result is that instead of developing theoretically correct methods, structural research is leaning toward a larger use of empirical and not correlated coefficients. The developments of fatigue data and stress concentration factors are probably good illustrations of this thought.

Professor Mirabelli's discussion gives an interesting method of a more satisfactory correlation of the experimentally determined stress-strain relation and the actual behavior of structural elements. Such investigations should be conducted extensively. Mr. Horger raises a number of most important questions in his short discussion. The boundary conditions and their influence on stress distributions is a problem that requires considerable additional study. If the influence of boundary conditions is accepted then there should be a different stress distribution within the thickness of the material and at the boundary. Figs. 3(a) and 9 are indicative of the most probable change of stress distribution at the boundaries for specimens subjected to tension-compression and bending, respectively. The additional data with reference to fatigue strength, size effect, and stress concentrations are not yet available and, therefore, the last question asked by Mr. Horger can not be answered at present.

The writer does not agree with Mr. Glover's conclusions that the investigators of the past gave all the necessary information that can be further developed. Theories based on incorrect assumptions cannot be satisfactorily developed beyond very narrow limits. His work on trial load arch dams and supporting rings for penstocks, mentioned in his discussion, would seem to be of very considerable interest.

R. L. TEMPLIN,[175] M. AM. SOC. C. E. (by letter).[175a]—Not only has Professor Timby amplified many points raised in the paper but he has also indicated some additional uses of models. In commenting on the merits that he has emphasized, which obtain when using stainless steel models fabricated by "shot" or spot welding, the writer wishes to state that he has used numerous spot-welded aluminum alloy models of structural units with very satisfactory results. Although it is scarcely probable that any one individual would be familiar with all the recent developments in model testing and analysis, it is apparent that Professor Timby has assembled information on a group of model studies which covers the apparent range of application quite thoroughly. The examples cited both in the paper and in the discussion, represent considerable evidence to show the increasing extent of the use of models in various engineering fields.

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[174] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[174a] Received by the Secretary June 4, 1937.

[175] Chf. Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

[175a] Received by the Secretary April 3, 1937.

Many improvements have been made on strain-gages since the time the Howard gage (mentioned by Mr. Belzner) was introduced. Nevertheless, as was stated in the paper, there is still considerable of the personal equation involved in the manipulation of the modern strain-gage and this presents a factor which must be considered in obtaining and interpreting strain-gage results.

The model girder referred to by Dr. Arnstein would seem to offer some interesting advantages in certain model studies.

A problem very similar to that mentioned by Professor Ruge was encountered in the model of the Santeetlah Pipe Line.[14] The load was applied by gravity and although the model could be scaled down photographically, the result was that the stresses were also reduced photographically. Consequently, it was not sufficient simply to rely upon the model test directly, but to use it to substantiate the theoretical development for this type of loading. The example suggested by Professor Ruge appears to be likewise of this type and, therefore, an excellent one to illustrate the point raised. The gusset-plate problem that he has mentioned will not give exact similitude because the modulus of elasticity for the model would not be reduced from that of the prototype in the same proportion as the scale ratio.

Professor Klemin's reference to full-sized structural tests of aircraft is an excellent example of a case in which the load conditions on the actual structure are so indefinite as to make model testing relatively unreliable, except in the study of component parts of the structures.

J. H. A. BRAHTZ,[176] ESQ. (by letter).[176a]—In closing this paper, the writer wishes to express his appreciation for the many interesting discussions that have appeared. These discussions have brought out many points of interest and valuable details which were not included, or were insufficiently elaborated, in the paper. In reviewing the discussions briefly, these will be taken up in the order in which they appeared.

In his excellent discussion, Mr. Vose emphasizes the difficulties in producing a truly two-dimensional state of stress. It is a fact that it never really occurs. In order to have a plane state of stress the plane plate $(x\,y)$ should be infinitely thin, and in the case of plane strain the plate should be infinitely thick in order to comply completely with the definitions of those states, in which the stresses σ_x, σ_y, and τ_{xy}, are independent of the elastic constant of the material. In photo-elastic experimentation, of course, a small finite model thickness is necessary. Consequently, the average stresses over the thickness of the model are measured. It is true that these stresses will be effected to some extent by the lateral stresses near discontinuities on the boundaries. On the other hand, the same effect occurs in the prototype so that the deviations between experimental and actual stresses in this respect are practically only caused by the ratios of the elastic properties in the two materials. This is one of the many reasons why the maximum errors of experimentation

[14] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 154.
[176] Director, Photo-Elastic Laboratory, U. S. Bureau of Reclamation, Denver, Colo.
[176a] Received by the Secretary May 24, 1937.

in such regions cannot be expected to be less than 10% and often may be even greater.

Mr. Vose calls attention to the convenient portable reflecting polariscope designed at the Massachusetts Institute of Technology, which should be of great help in the type of observations mentioned in his discussion.

In a very lucid discussion Professor Rathbun describes a number of experiments with riveted connections. The very ingenious technique of multi-materials of varying optical sensitivity no doubt will prove extremely useful in the future. The aforementioned three-dimensional difficulties now become acute. It was this problem that the writer had in mind when stating in the paper that the elastic properties of the model materials should be such as to imitate as closely as possible the conditions in the prototype. Due to plastic flow, the writer has had difficulties in calibrating the highly sensitive materials, accurately. Professor Rathbun's discussion brings out clearly the truth of the statement that "photo-elasticity has proved its value, is here to stay, and new apparatus, model materials, and technique are being developed continuously."

In his very valuable discussion Mr. Timby explains clearly the three contributing reasons for an adequate safety factor. The writer is in full agreement with these viewpoints. The live loads, including wind and temperature effects, are seldom accurately known and can only be assumed for many types of structures, as for example, bridges, buildings, ships, etc. In such structures as dams these effects are defined more clearly, but many other uncertainties enter, among which may be mentioned earthquake effects and uncertainties in the quasi-elastic properties of the materials in the dam proper relative to those of the foundation. The photo-elastic laboratory of the U. S. Bureau of Reclamation has proved extremely useful in connection with designs of hyperstatic structures (for example, large drum-gates) in obtaining the most economical sections.

The interesting discussion by Mr. Solakian deals to a large extent with the history of the fundamental physical phenomena, which enables present-day photo-elastic experimentation. He argues that the polariscope designed and constructed by the writer at the California Institute of Technology and having the large field lenses replaced by aluminum-coated parabolic or spherical mirrors, will be less popular since the introduction of new "polarizing media." These polarizing glass plates will successfully replace the far more expensive Nicol prisms. The writer fails to see the point since it is the high cost of high-grade lenses *versus* the comparatively low cost of high-grade reflectors that matters.

In conclusion, the writer can state that the new interferometer of the Favre type which was recently designed and constructed by the engineers of the U. S. Bureau of Reclamation, has proved highly satisfactory in its application. The polariscope is now being used only in: (1) Such experiments, in which merely the boundary stresses are needed; (2) for preliminary design studies in order to obtain an optimum section, which is then analyzed in detail by the interferometer; and (3) for checking the principal stress differences obtained in the interferometer.

The interferometer differs from the instrument described by Favre in several respects. For example, it gives both principal stresses and their direction in one operation, whereas Favre obtains the direction and magnitude of the principal stresses in separate instruments.

M. J. R. MORRIS,[177] ESQ. (by letter).[177a]—Referring to Mr. Herzog's discussion, the writer agrees that he has not gone into detail as far as possible, as it was the primary intention to cover as large a field in general as possible; his comments, therefore, are very well taken. There is no doubt that the addition of alloying elements, such as molybdenum and columbium, will open up new fields to the stainless alloys, and will help greatly in the actual fabrication of some of these alloys by making them more easily handled and possibly more nearly "fool-proof." As time goes on and improvements follow experimental work, these alloys will come into more and more prominence and broader use with the civil engineer just as they have already been a boon to chemical and industrial engineers in general.

Mr. Gisiger cites the application of stainless bars in connection with trash screws protecting the turbine intakes against floating logs, trash, etc., of the Holtwood Plant, of the Pennsylvania Water and Power Company, at Holtwood, Pa. This is an excellent example of the reduction in weight made possible through the use of corrosion-resistant stainless steel over structural carbon steel; also, the increased efficiency provided by decreasing the area covered by the bars, themselves, which obstruct the passage of the water.

In connection with welding these stainless alloy bars to structural carbon steel, the writer has found that, in general, a heavy, flux-coated alloy electrode will give the most satisfactory weld.

If mild steel electrodes are used, the weld metal (which will have a smaller cross-section than the structural steel) will be subject to any localized corrosion that may be set up as a result of the stainless alloy being in contact with mild steel. If the weld, itself, is stainless, then the corrosion will be transferred to the heavy, structural shape, which will be much more capable of distributing and minimizing its effect so as to be of very little consequence.

In laying a stainless weld on mild steel, it is best to deposit at least two beads on top of each other to eliminate the effect of pollution from the steel. The lower bead fused to the steel will be less corrosion resistant than the record on subsequent beads. Such applications well illustrate previous statements as to the broad use of stainless alloys in the hydraulic field.

The writer would like to take this opportunity to thank the several commentators for their very interesting discussions.

ZAY JEFFRIES,[178] ESQ., C. F. NAGEL, JR.,[179] ESQ., AND R. T. WOOD,[180] ESQ. (by letter).[180a]—In commenting on the writers' paper, Mr. Hobrock has

[177] Chf. Metallurgical Engr., Central Alloy Dist., Republic Steel Corporation, Massillon, Ohio.

[177a] Received by the Secretary May 24, 1937.

[178] With Incandescent Lamp Dept., General Electric Co., Nela Park, Cleveland, Ohio.

[179] Chf. Metallurgist, Fabricating Div., Aluminum Co. of America, Pittsburgh, Pa.

[180] Chf. Metallurgist, American Magnesium Corporation, Cleveland, Ohio.

[180a] Received by the Secretary, April 23, 1937.

given emphasis to the desirability of consulting the manufacturers of materials before making final selection of a particular alloy. Quite correctly, he points out that this is not a feature peculiar to aluminum, or to any one metal, but applies to all. However, it probably applies less to those older metals for which many years of use in a particular service have built up a present satisfactory practice by the process of "cut and try," the soundness of which has been reasonably well proved or at least accepted. In passing, it might not be out of order to suggest that following "tradition" carries with it certain dangers—namely, what was quite adequate yesterday may be inadequate tomorrow—if not to-day. Engineers should not necessarily select alloys or materials to-day because they were satisfactory a few years ago.

Materials suppliers are constantly making progress, both in the greater uniformity of the properties of materials, and in the development of improved products possessing superior qualities, range of sizes and forms, and closer tolerances. Furthermore, the manufacturer is constantly enlarging his knowledge regarding the properties of those materials.

It is the latter point that Mr. Hobrock has emphasized. Some danger, from a commercial angle, may lurk in the possibility that a purchasing agent, becoming aware of the existence of characteristics other than those commonly mentioned in purchase specifications, may unnecessarily include specifications covering non-pertinent properties. Nevertheless, as Mr. Hobrock properly states, the satisfactory performance of a structure may not depend upon the more commonly published properties, but possibly upon other characteristics such as, for example, the strength at certain elevated temperatures. Although these other fields of properties have not been fully charted, a substantial beginning has been made, and the knowledge available should be put to use. This should not result in every writer of materials specifications attempting to state and define the limits of every conceivable characteristic (that would be utterly unwarranted and abusive of human intelligence), but it does mean that the engineer should ferret out to just what actions his particular structure will be subjected, and then ascertain from the manufacturer what information is available as to the properties of his product with respect to those conditions. One should not expect that the desired "road map" will be forthcoming in all details, but in many cases numerous useful "land marks" can be shown.

Mr. Hobrock's contribution to this subject of "consulting the supplier" deals mainly with the uncommonly published properties of alloys. This is important. The writers' main reason for presenting this thought was somewhat different, although not at all contradictory. Progress is continually being made in the perfection of range of dimensions, both as to maximum sizes and tolerances and, more important, in knowing just what can and cannot be accomplished from an economical viewpoint. This feature is of special importance in the field of castings, although it is not confined to that field.

Slight changes in the detail design of a casting, for example, or a more complete understanding on the part of the foundryman as to how the prod-

uct is to be used, may permit of a more favorable method of gating, which, in turn, may lower the cost of the product, or may permit the production of a casting in which the metal at critical locations will possess more favorable characteristics.

An alloy is defined by its composition alone. The properties of Alloy *A*, for example, may well be different when in the form of a thin sheet, as compared with a rolled shape. The processes by which the original ingot is converted into various wrought shapes may differ widely and, hence, the final characteristics may differ. Furthermore, the characteristics of the ingot (caused by the method of manufacture and the dimensions) may vary depending upon the particular wrought product for which it is intended. The structures of those different ingots are not all alike, and the effect of this original ingot difference may affect the final characteristics and, hence, one should not casually refer to "properties of the alloy" when considering the design of a structure but, rather, should think in terms of the properties of the particular type of material concerned. Many published tables fail to reveal whether the properties stated apply to sheet alone, to forgings alone, or to rolled shapes, etc. When using any table for design purposes, therefore, one should know to what particular type of product those properties apply.

JAMES ASTON,[181] ESQ. (by letter).[181a]—The logic of Mr. Clapp's suggestions will be admitted by all who are actively interested in the corrosion problem. Engineers are concerned primarily with the serviceability of materials, and a systematic study which would evaluate their utility in particular types of service, or would enable one to apportion properly the effects of service factors, would be invaluable in enabling a more correct appraisal and use of metals to be made. Service data have been gathered in considerable amounts, largely by individual efforts of competing manufacturers, with release prompted largely by the biased discretion of the compiling interest.

Actually, the task is of great complexity and magnitude. One must have the several metals in indisputably parallel conditions of service; they must remain undisturbed throughout their service life; and, particularly, interest and records must be kept alive through the long life span of many metals in many services. In corrosion testing, the temptation is to accelerate the test, or to attempt to control conditions in a laboratory set-up. With such procedure, unbalancing of the several factors involved will probably seriously distort the results as compared with what might appear to be similar conditions in actual service. Furthermore, results from a controlled or specific type of service are applicable only to that condition, and cannot be interpreted more broadly. Finally, failure of material is the only true end point of a test, with the criterion of failure dependent upon the commodity involved and the service requirements.

[181] Metallurgist, Pittsburgh, Pa.

[181a] Received by the Secretary May 24, 1937.

By way of illustration, one may cite the sheet metal tests conducted by the Corrosion Committee of the American Society for Testing Materials. In this field, one finds a co-operating personnel of outstanding qualifications. The tests were carefully planned, were carried out on a comprehensive scale, and were painstakingly observed and recorded. The contribution to knowledge has been of great value; yet 1937 sees the passing of the twentieth anniversary of the tests, with a clearing up, at best, of only a few points related to atmospheric exposure of bare sheet steel. There is still almost as much controversy as in the beginning due to disputed points in the conduct of the tests; or a doubt regarding the practical value of tests of bare sheets when actual utilization requires paint or metallic protection; and particularly because in a twenty-year period many real or fancied advances have been made in the production of steel, with all the uncertainties and arguments formerly prevailing with respect to their relative merits.

E. C. Hartmann,[182] Assoc. M. Am. Soc. C. E. (by letter).[182a]—Many of those who have discussed the writer's paper have commented on various phases of shop practice. In his excellent description of the design and construction of the emergency bulkheads for the Gallipolis Dam, Mr. deLuccia emphasizes the need for great care in workmanship. He raises a doubt as to whether the average shop practice for structural steel is suited for aluminum alloys. Mr. Christianson also emphasizes the need for careful shop work, but states that aluminum alloys present no new problems to the shop experienced in producing a high quality of workmanship in other metals. Mr. Lehman states that with proper instructions one can rely on the shop to handle the material properly so that no parts are damaged. All these viewpoints may be summarized in the term, "good shop practice". In the writer's experience with fabrication methods in various types of construction, he has yet to find the shop which, when properly instructed, cannot produce a first-class aluminum alloy structure.

The question of riveting aluminum has received some attention by the discussers. Mr. Lehman describes the use of hot-driven steel rivets in aluminum structures and Mr. deLuccia describes cold-driven steel rivets, which is a newer development. To complete this picture a number of examples of the use of aluminum alloy rivets could be mentioned as, for instance the truss-type, traveling cranes of 76-ft spans built in 1930 and fabricated throughout with hot-driven 0.75-in. aluminum alloy rivets.

Professor Hunsaker has questioned the advisability of using steel rivets in aluminum structures because of the possibility of corrosion at the junction of the two metals in cases where the paint protection is not adequate. The possibility of such corrosion cannot be denied, and because of this possibility steel-riveted aluminum-alloy structures must receive adequate paint protection. On the other hand, tests have shown that unpainted full-sized structural joints, involving hot-driven steel rivets, sub-

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[182a] Received by the Secretary May 5, 1937.

jected to sea-coast conditions for two years retain their original strength. It should be remembered in this connection that visible surface corrosion can occur without appreciable reduction of strength, particularly in structural shapes and plates which have the advantage of greater bulk compared to the thinner materials used in aircraft.

Both Mr. deLuccia and Mr. Growdon have commented on the necessity for thinking in terms of aluminum when designing structures of aluminum, especially with reference to the question of deflection. The writer was particularly pleased with this phase of Mr. deLuccia's description of the bulkheads of the Gallipolis Dam. It is quite evident that in these structures arbitrary limitations based on conventional practice were not permitted to interfere with the logical development of the design of a light-weight structure to serve a specific purpose. Aluminum structures, of course, can be designed without this approach to the problem, but maximum economy and minimum weight are rarely attained without it.

Mr. Sturm's discussion presents a refreshing and thought-provoking concept of the relation of bulk of metal to stability. Stability problems are almost certain to be encountered more frequently in light-weight construction than in conventional construction, whether the weight savings are accomplished by the use of light-weight, low-modulus materials, or by the use of thin sections of the heavier materials. The question of bulk of metal is important in both instances. In the structures made up of thin sections of the heavier metals the reduction in bulk compared to conventional construction is largely responsible for the stability problems encountered; and in light-weight materials the relatively greater bulk is advantageous in overcoming the effect of lower modulus.

Professor Plummer has suggested a more complete treatment of the economic factors which influence the selection of materials for light-weight construction. In this connection this writer was pleased to note the comments of Mr. Lehman, Mr. Growdon, and Mr. Christianson, indicating that in three fields as widely separated as construction equipment, bridges, and railway cars, problems in economics arising in connection with specific applications of aluminum alloys have been solved satisfactorily. Mr. Lehman's interesting statement that, "the selling of such a benefit is often more difficult than the proof of economy," no doubt will be echoed with feeling by all who have been associated with the commercial development of the newer materials.

Two of the discussers have described aluminum alloys not specifically mentioned in the paper. As pointed out in the paper, there are several such alloys available, and the selection of the best one for a specific purpose involves a consideration of many factors. The day has passed when each engineer could expect to be familiar enough with all available materials to make his selections entirely on the basis of his own experience. Any logical modern approach to the problem of selection of material should certainly include consultation with the producers of the materials.